Minimal walking technicolor: a case-study for lattice BSM

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Acknowledgements

- work in collaboration with:

  bursa, keegan, kerrane, lucini, moraitis, patella, pica, pickup, rago

- other lattice studies:

  catteral et al, degrand et al, rummukainen et al, deuzemann et al, appelquist et al, LSD collaboration, kuti et al, hasenfratz, belgici et al, kogut et al

- analytical studies:

  armoni et al, sannino et al, rattazzi et al, nunez et al, unsal et al, kaplan et al

  sannino et al, giudice et al

- publications:

  arXiv: 0802.0891, 0805.2058, 0907.3896, 0910.4535

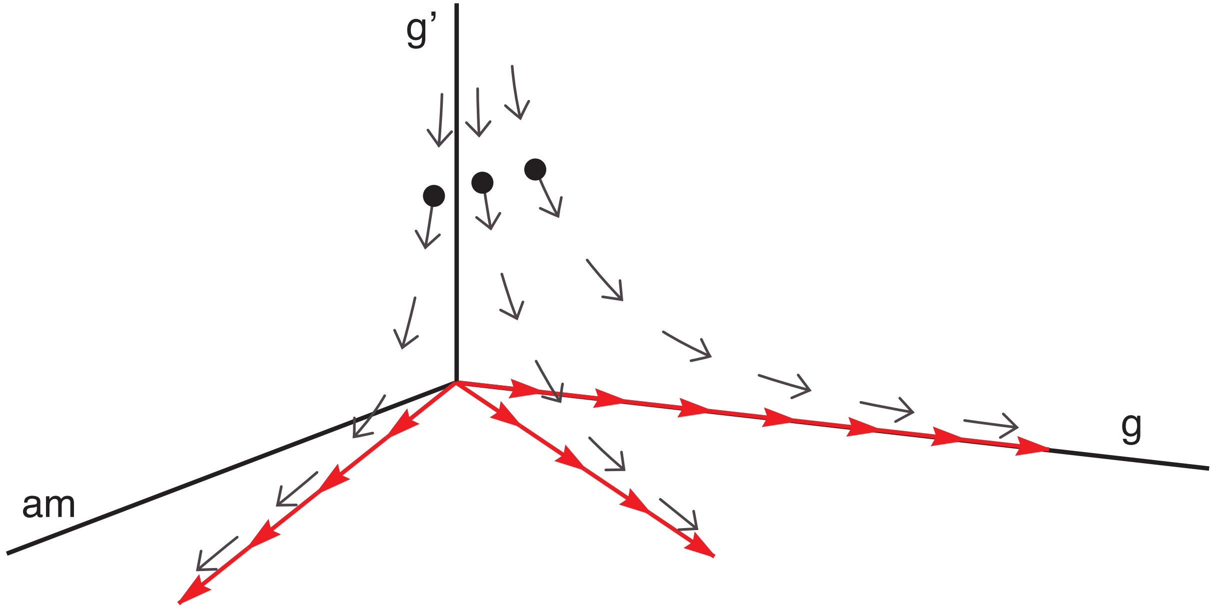
Dynamical Electroweak Symmetry Breaking

- strongly–interacting BSM theory, new resonances $\mathcal{O}(1\ \mathrm{TeV})$
- compute phenomenologically relevant quantities: spectrum, S-parameter, couplings, anomalous dimension
- wide choice of candidates: flavors, colors, representations?
- light dynamical fermions play a key role for **non QCD–like** behaviour
- **understand the effect of systematic errors in lattice results**
- focus on a specific model $\Rightarrow$ extract characteristic features
- minimal walking TC: SU(2) with 2 adjoint Dirac fermions

RG flows for QCD

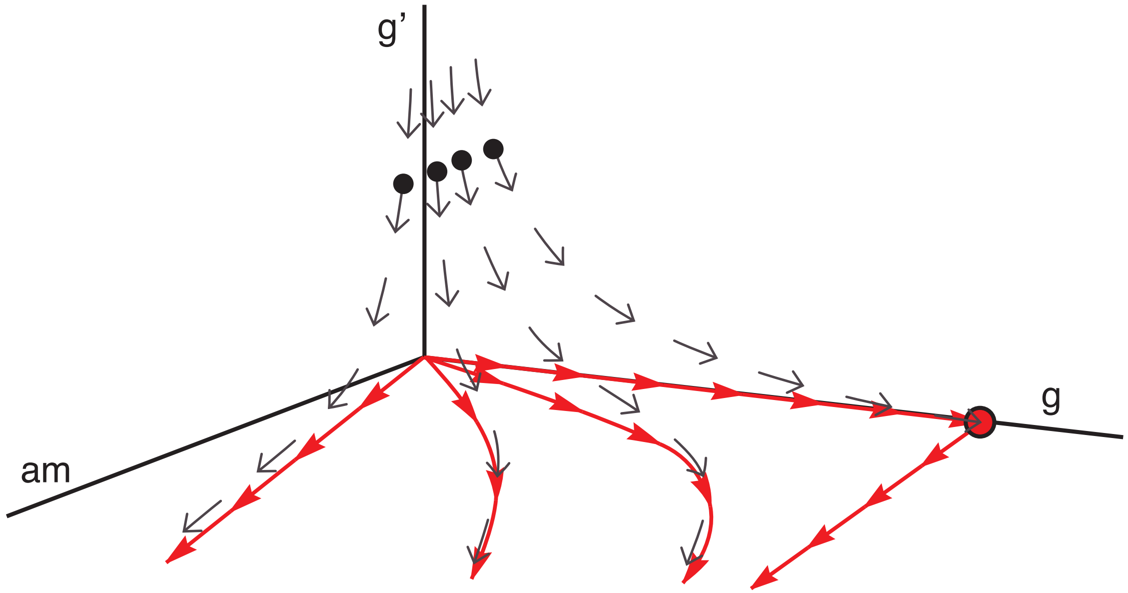
$$\mathcal{L}_{\rm LEL} = \mathcal{L}_0 + a\mathcal{L}_1 + a^2\mathcal{L}_2 + \ldots$$ [Symanzik]

- the theory has **two** UV-relevant parameters g, m
- renormalized trajectories lie in the (g, m) plane
- simulations are performed away from this plane
- $a\mu$ small in order to have small discretization effects

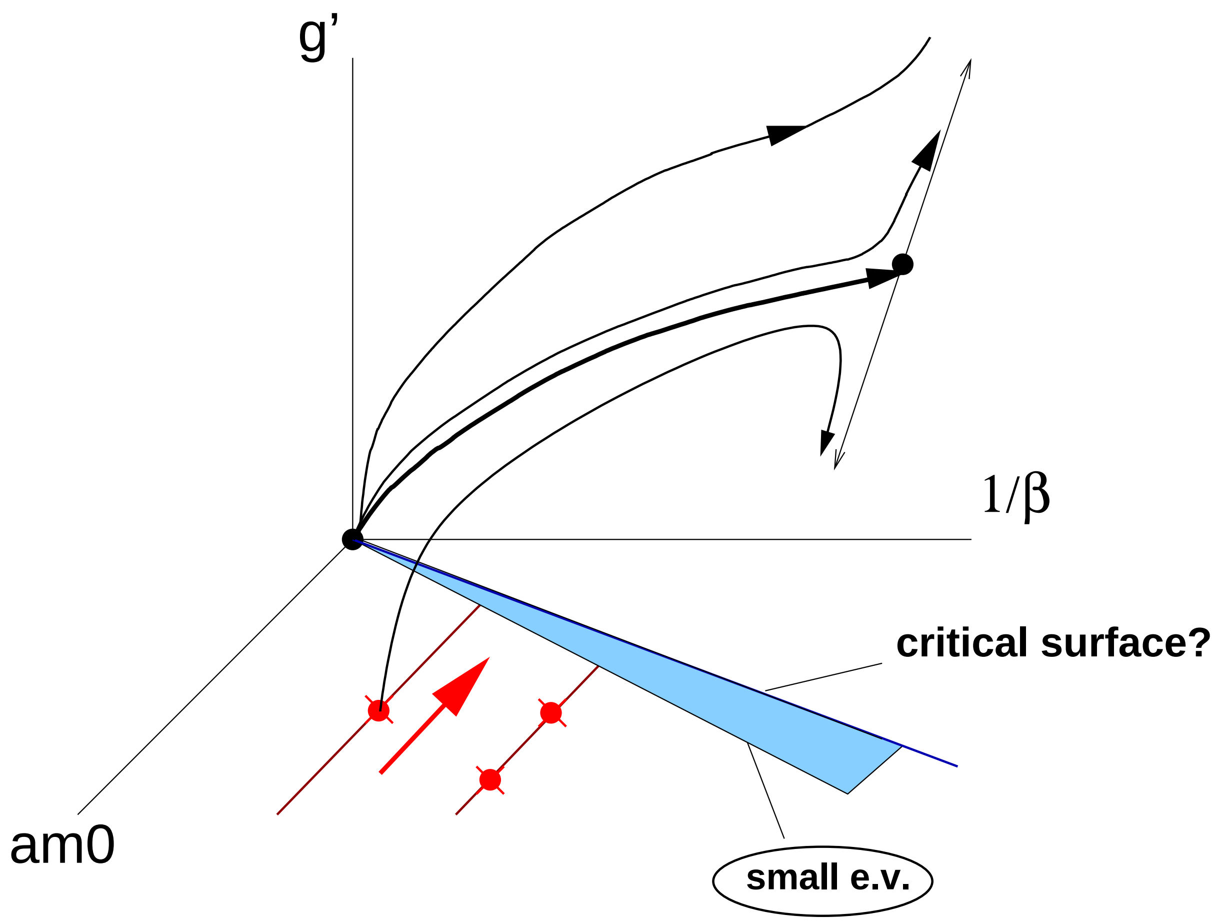


RG flows for IRFP

- scale invariance is broken by m AND $1/L$
- large physical volume + light masses!
- deviations from QCD spectrum
- Schrödinger functional/twisted BC allow $m = 0$



Lattice perspective



How do we observe an IRFP in lattice data?

Conformal scaling

solution of RGE in the vicinity of a FP yield scaling laws

mass scaling:

$$\boxed{M \sim m^{1/1+\gamma}}$$

finite size scaling:

$$\boxed{LM \sim \mathcal{F}(L^{y_m} m)}$$

NB: these formulae are derived in the neighborhood of the FP

Taming systematic errors

In order to keep systematics under control:

$$a/L \ll ma \ll \left(\frac{r_0}{a}\right)^{-1} \ll 1$$

In QCD: $L/a \sim 48$, $r_0^{-1} \simeq 400\ \mathrm{MeV}$, $m_\pi \simeq 250\ \mathrm{MeV}$, $a^{-1} \simeq 2\ \mathrm{GeV}$

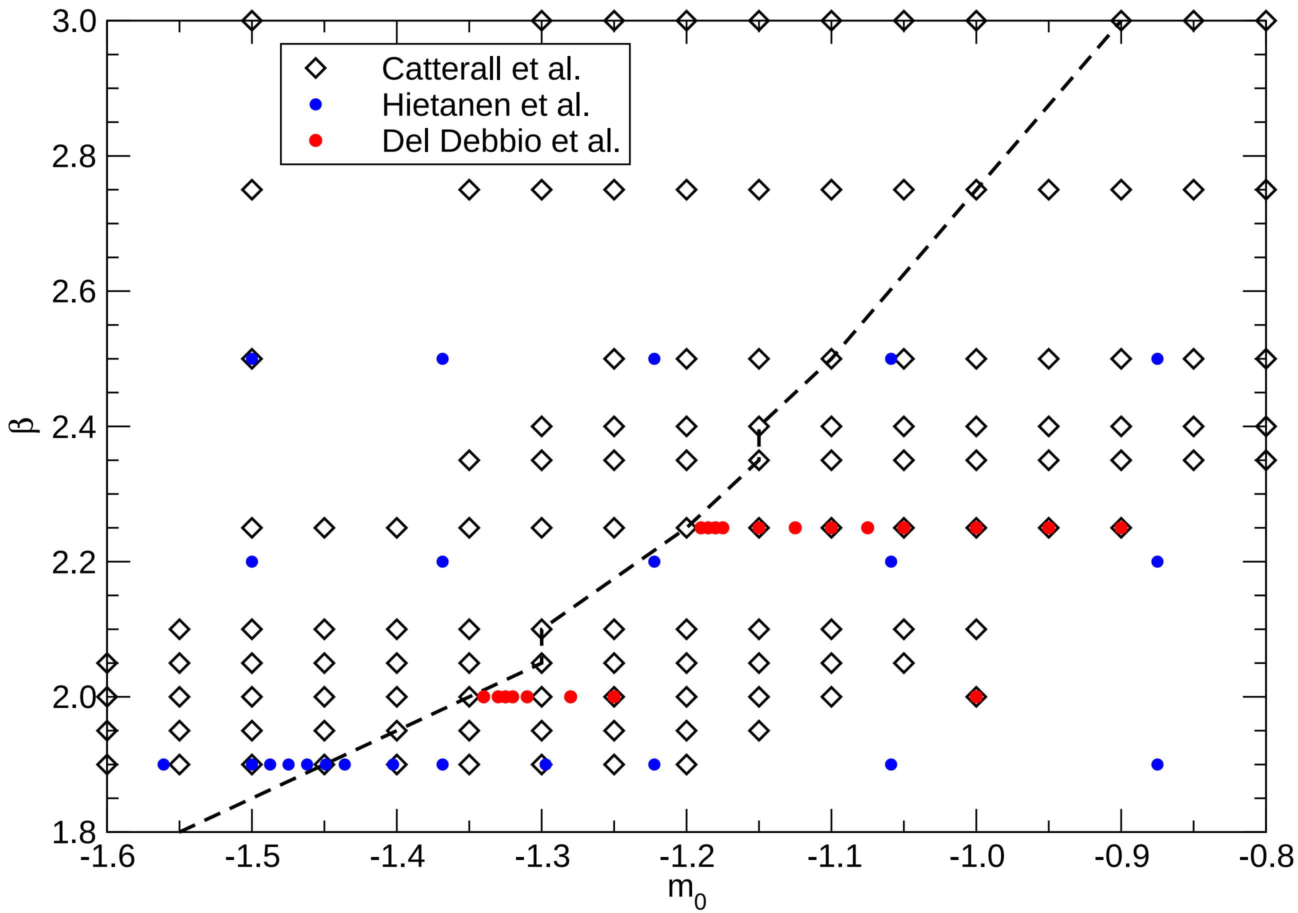
$$0.02 \ll 0.05 \ll 0.2 \ll 1$$

if IRFP is present: conformal symmetry is broken by the mass

- same conditions apply on L,m, and a
- more delicate to define a scale like r_0
- **large** volumes and **small** masses are mandatory

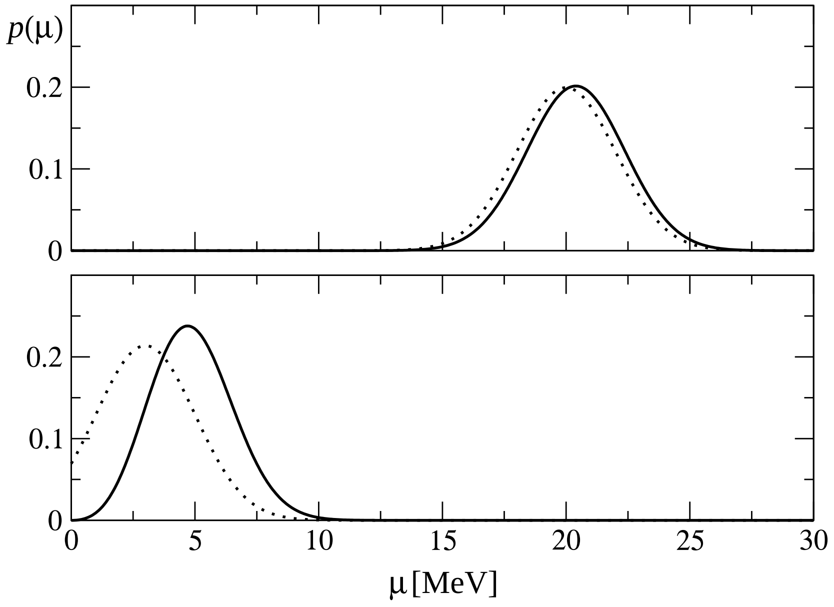
Exploring the chiral regime of MWT

[catterall et al 07, ldd et al 08, rummukainen et al 08]



Stability of the HMC

[ldd, giusti, luscher, petronzio, tantalo 05]



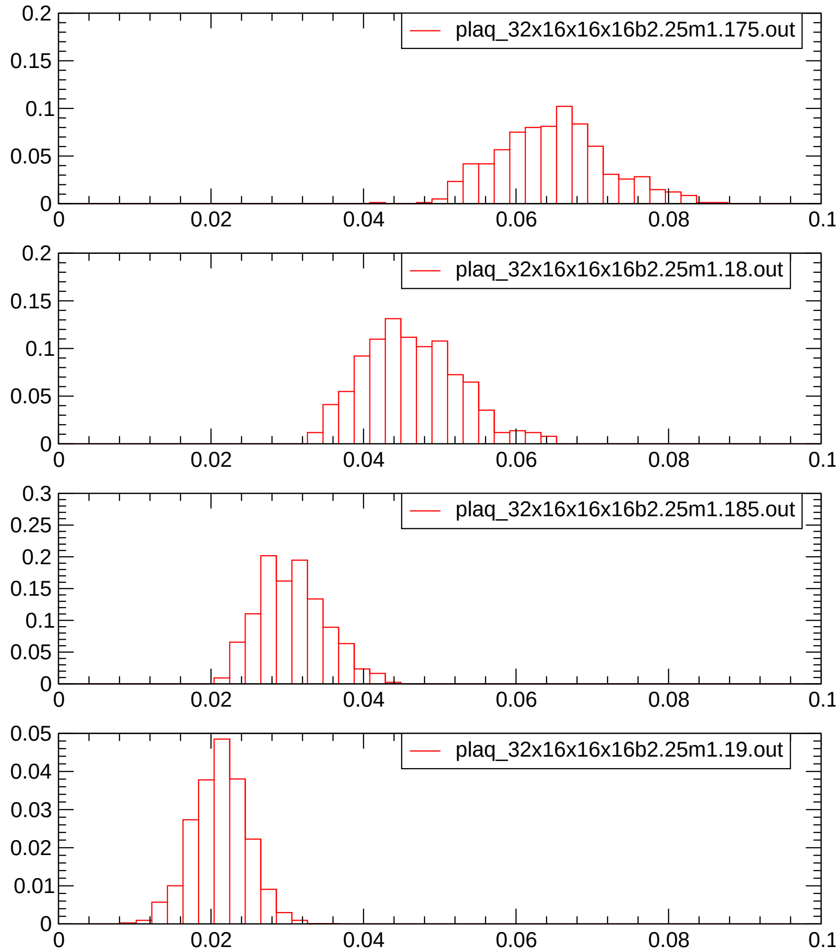
- integration instabilities / reversibility
- ergodicity problem: HMC stuck in a sector with $\eta \neq 0$
- sampling of observables: $p(\mu)/\mu^2$

difficult regime for simulations:

$$m \to 0$$
$$a, V \ \text{fixed}$$

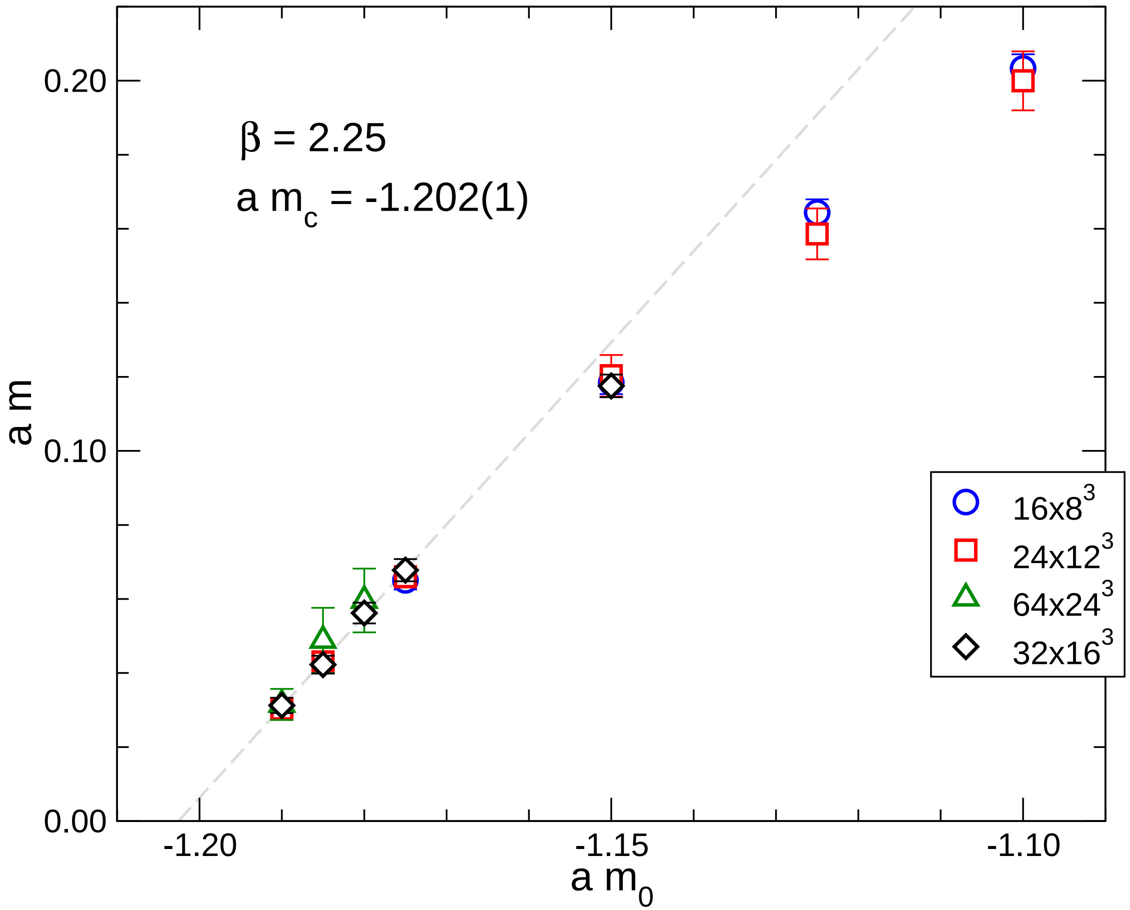
$\hookrightarrow$ check that runs are in a safe regime!!

Lowest eigenvalue for SU(2) adj

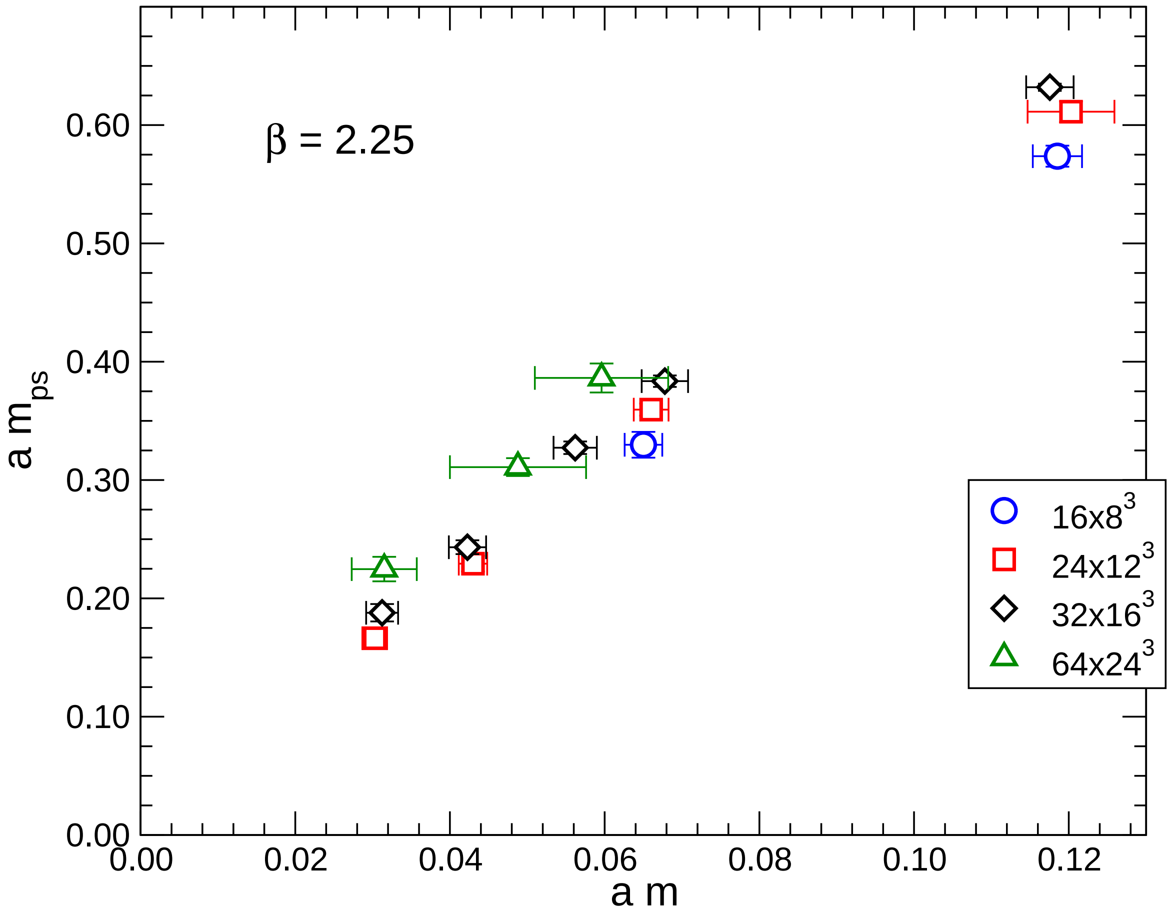


data from 32×16^3 lattice, $\beta = 2.25$

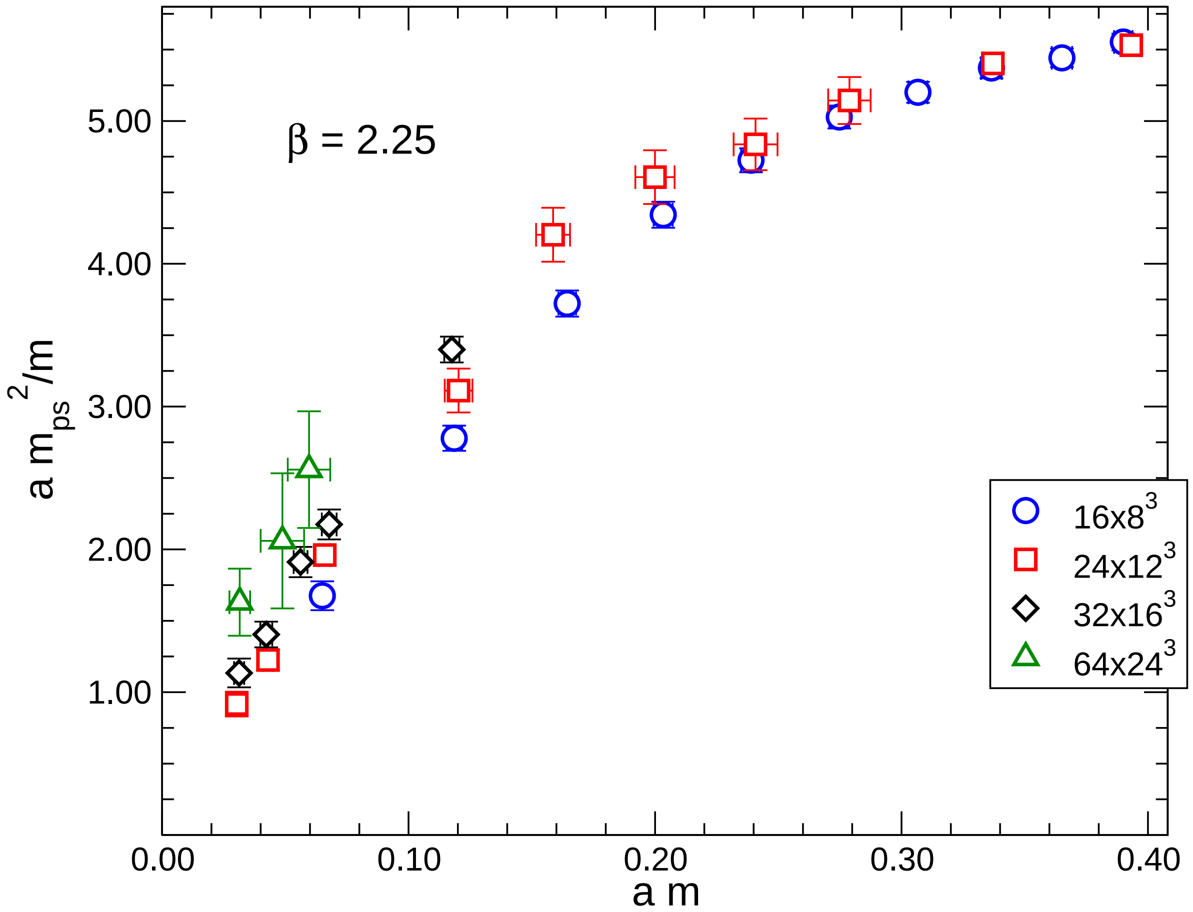
Chiral limit (2)



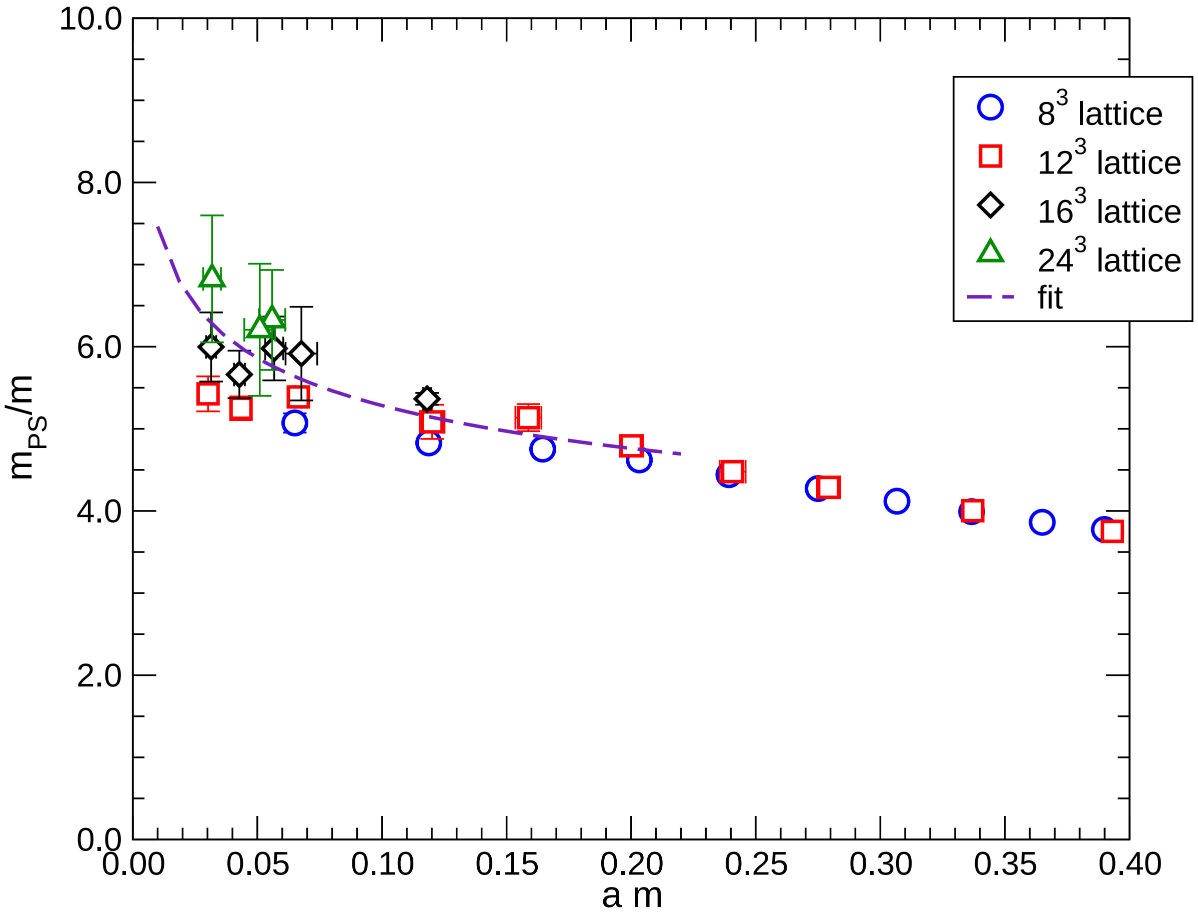
Pseudoscalar mass (2)



Pseudoscalar mass (3)

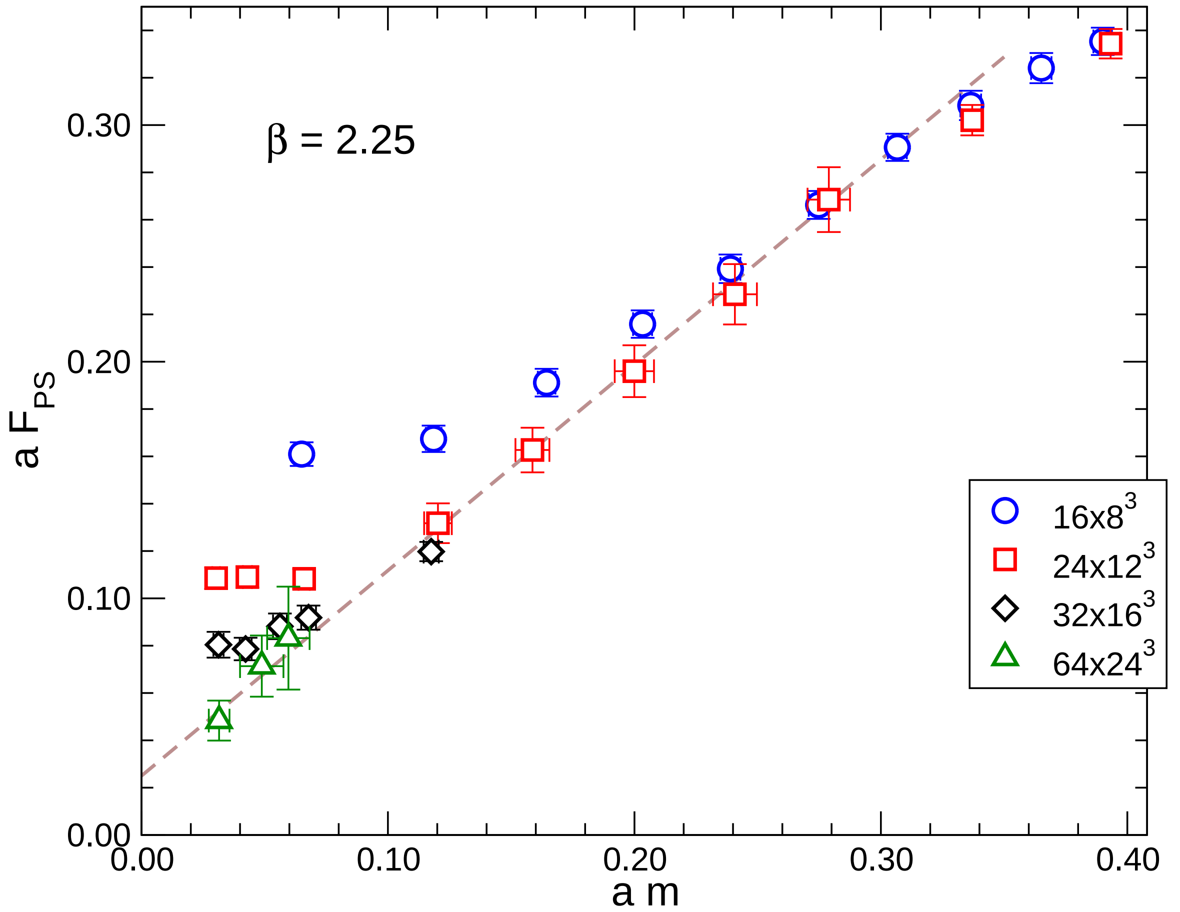


Pseudoscalar mass (4)

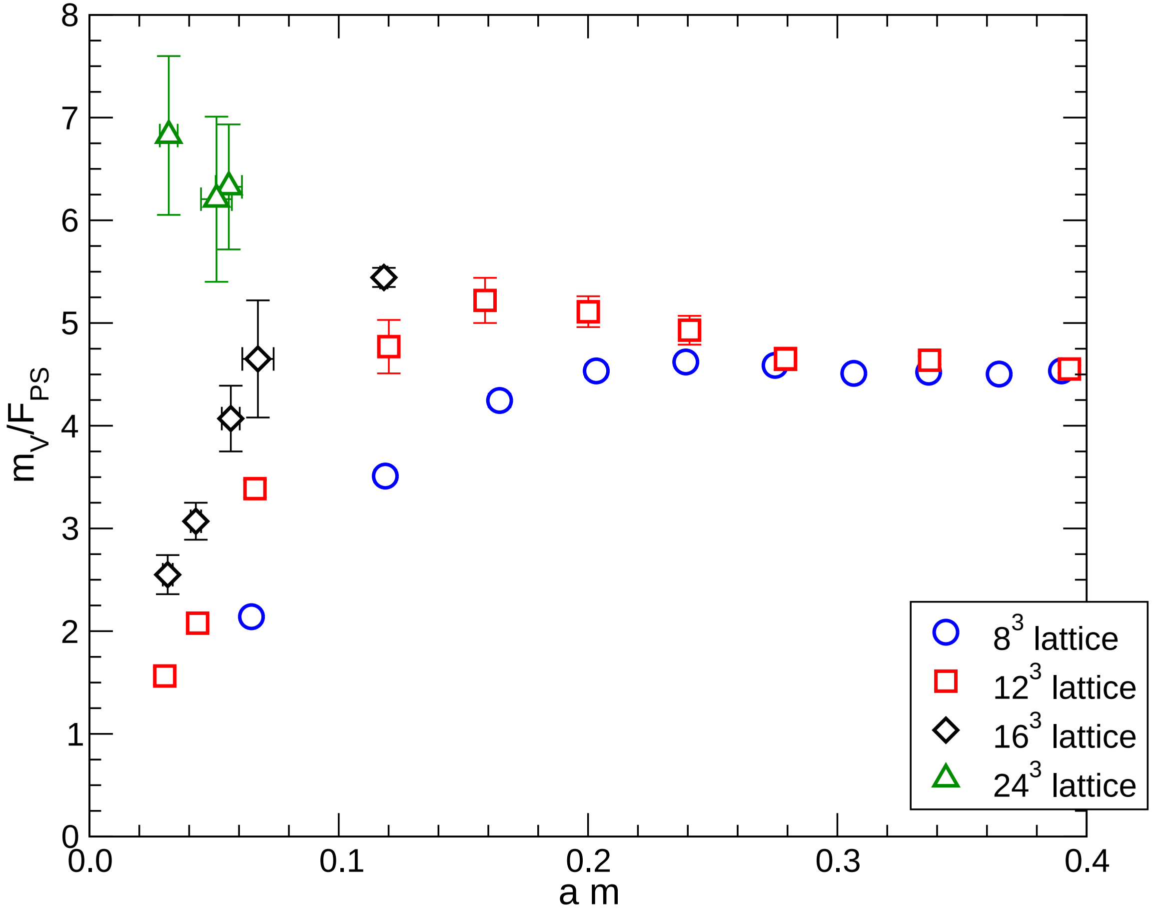


$$\log m_{\mathrm{PS}} = \frac{1}{1+\gamma} \log m + C = 0.85 \log m + C$$

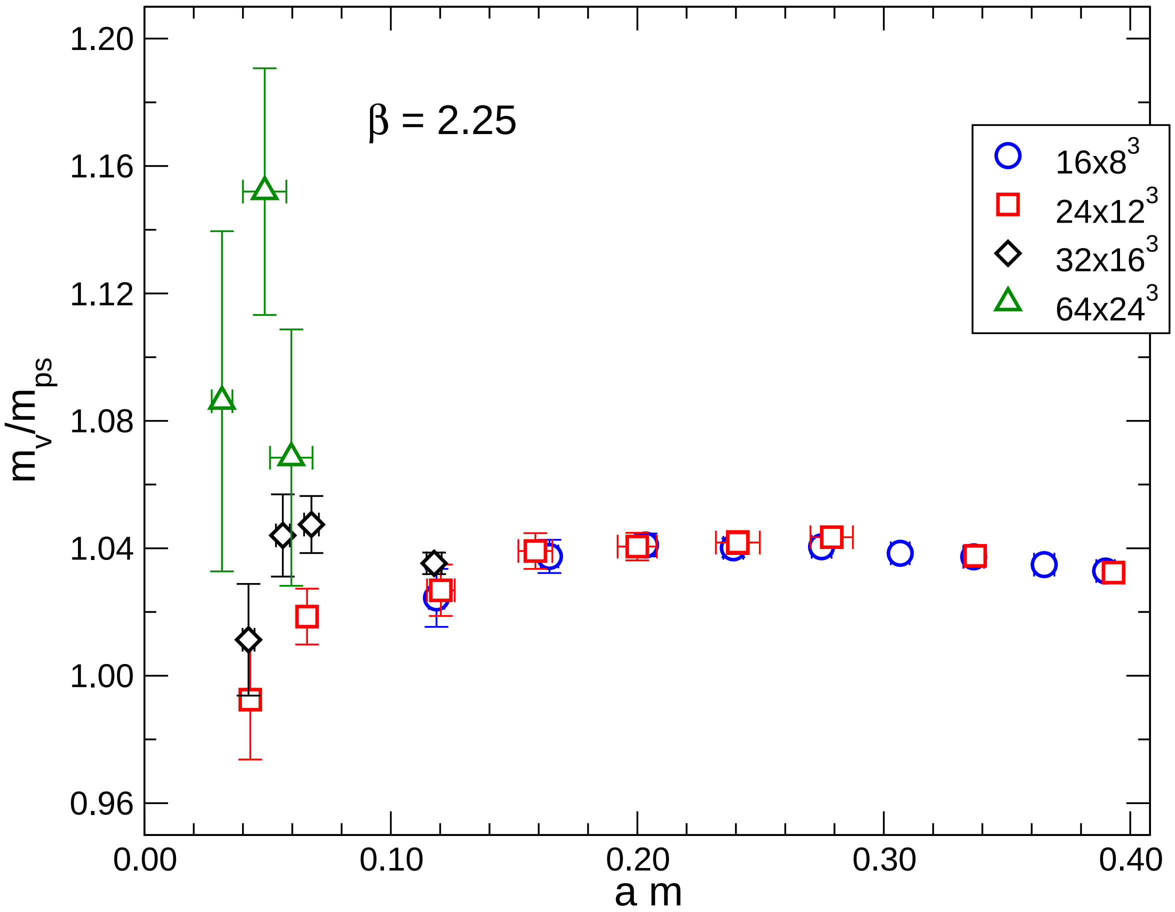
Decay constant



Vector mass (1)



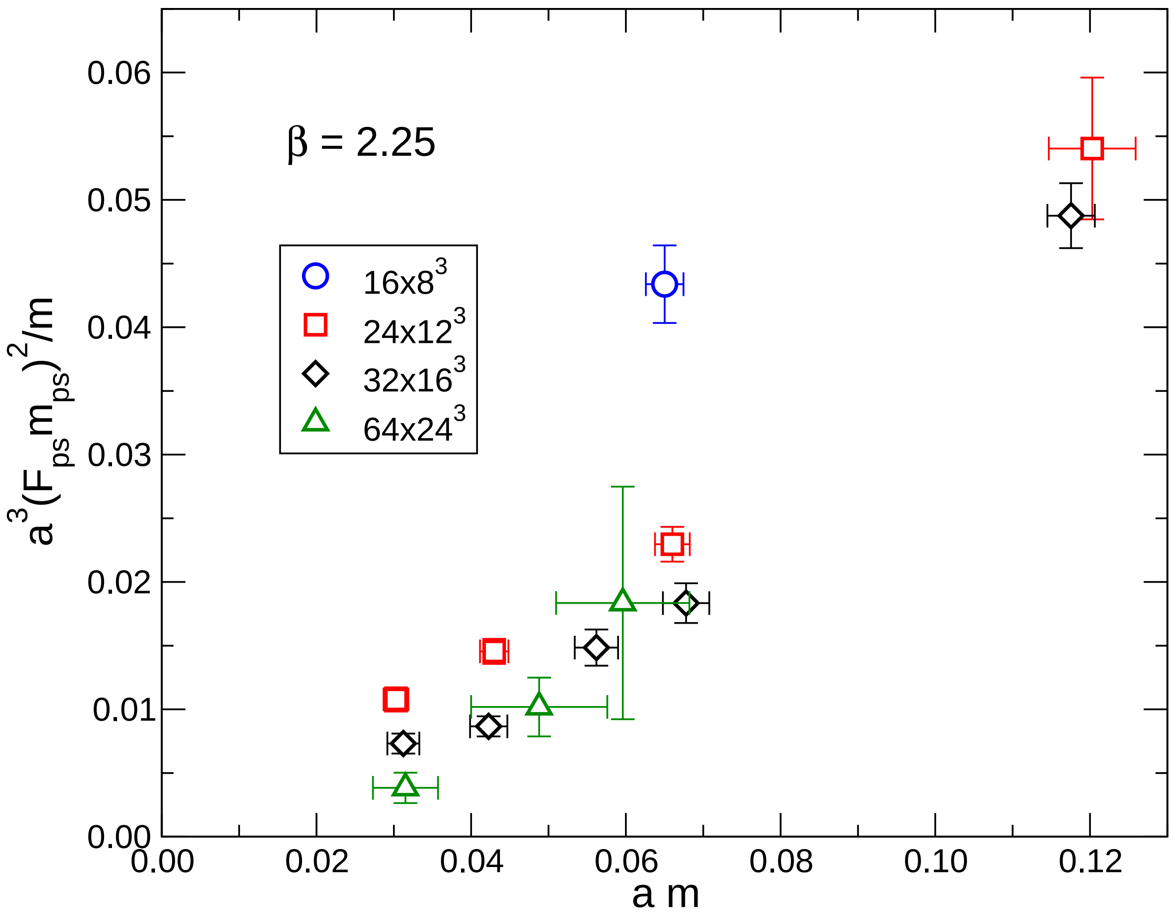
Vector mass (2)



Is this just HQET?

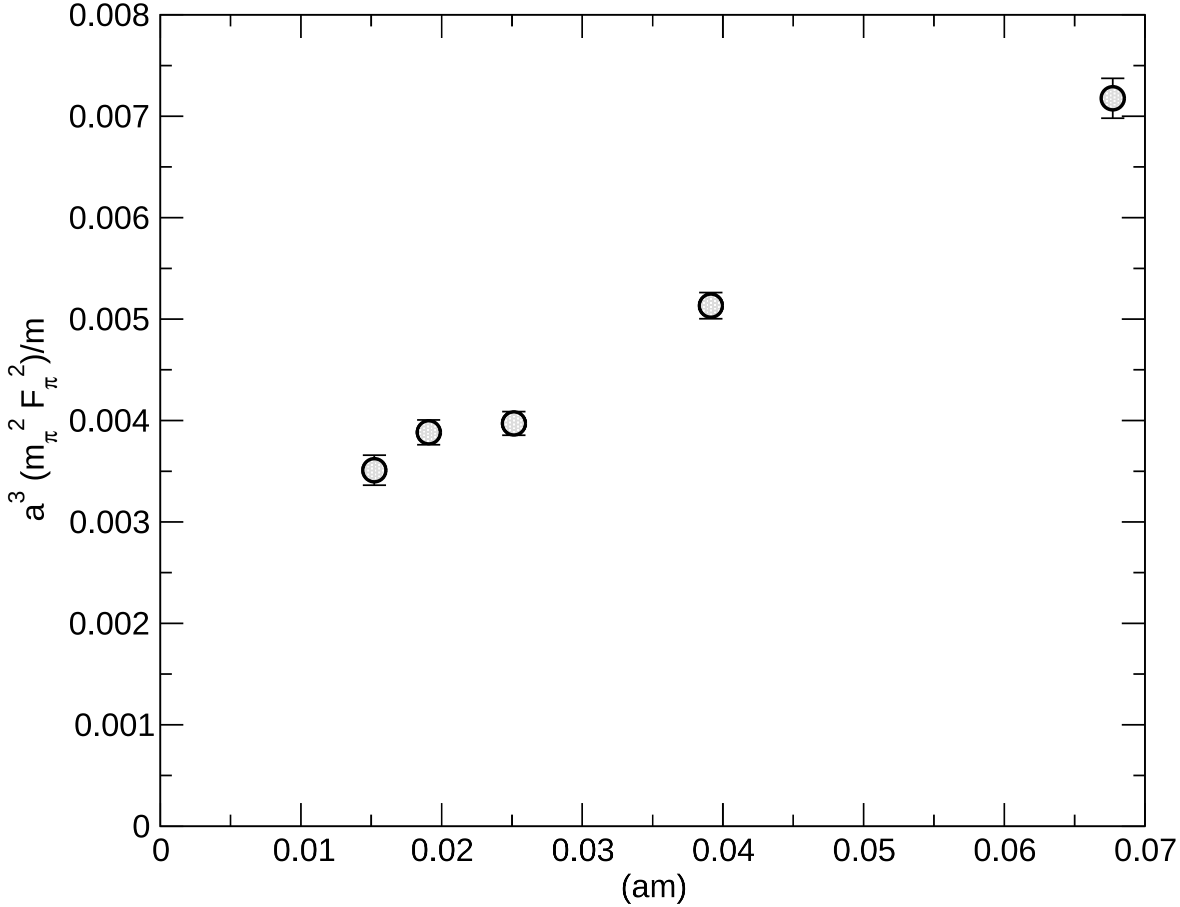
GMOR relation

$$m_\pi^2 F_\pi^2 = B m^x$$



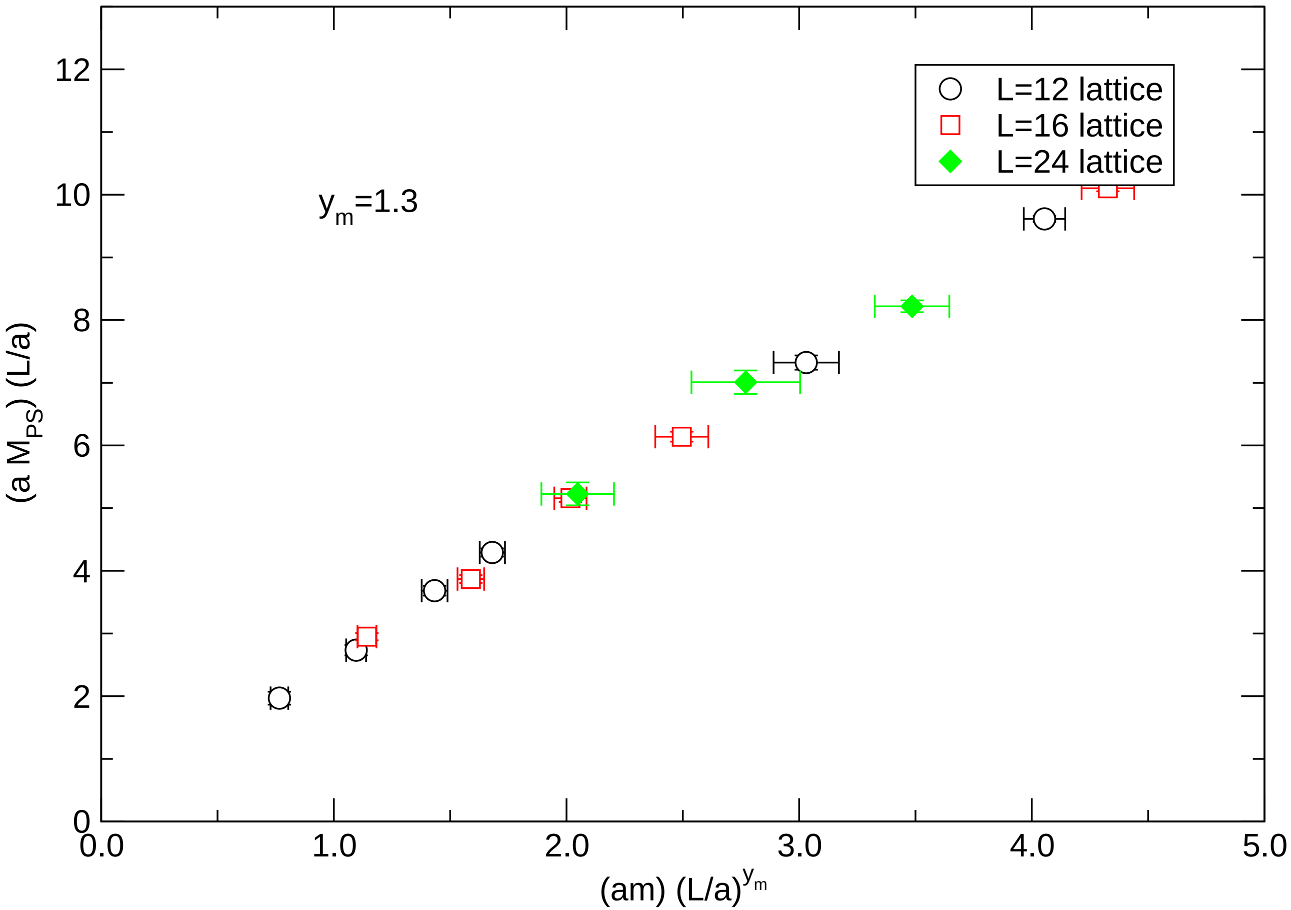
GMOR relation in QCD

$$m_\pi^2 F_\pi^2 = Bm$$

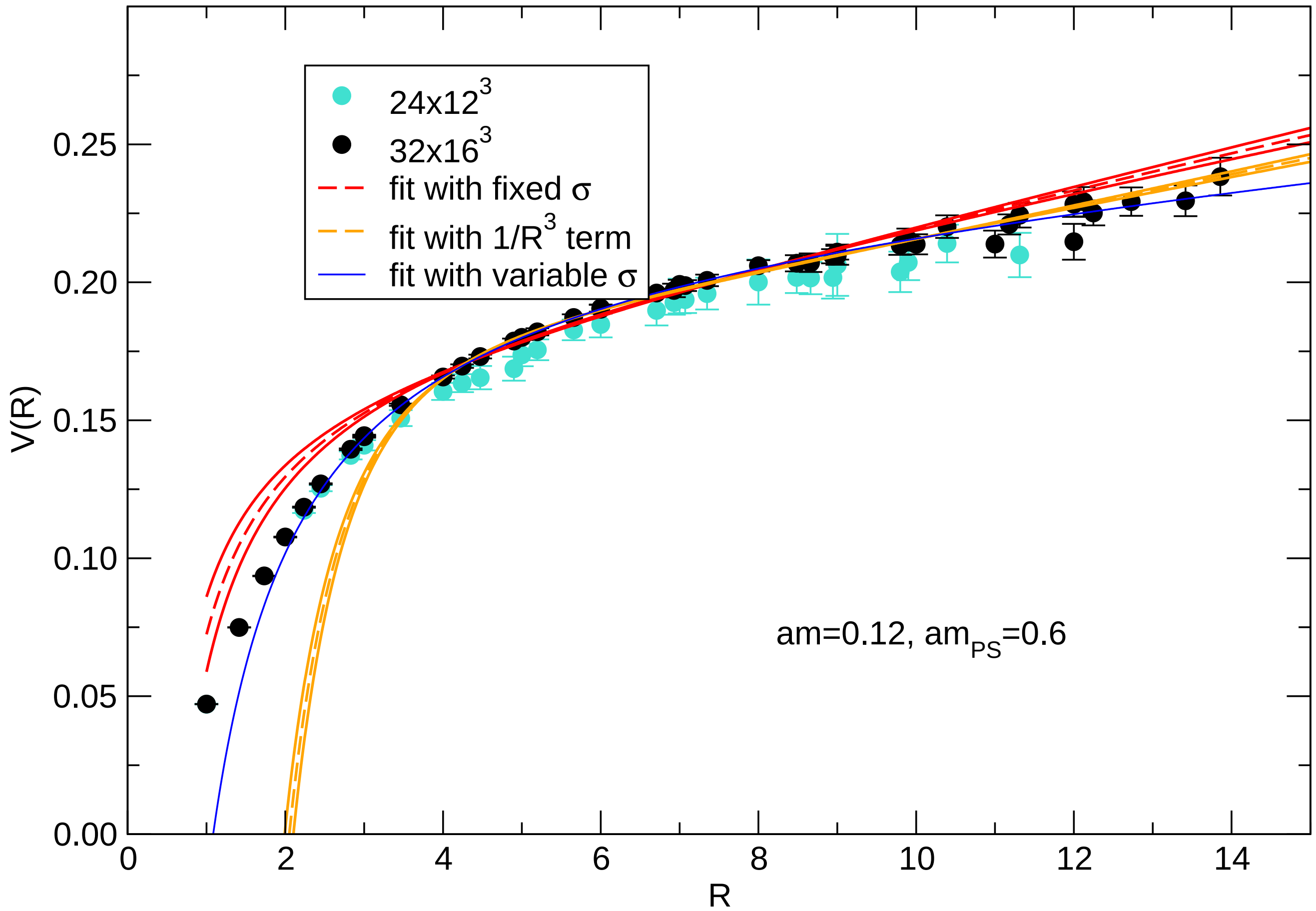


[Idd et al 06]

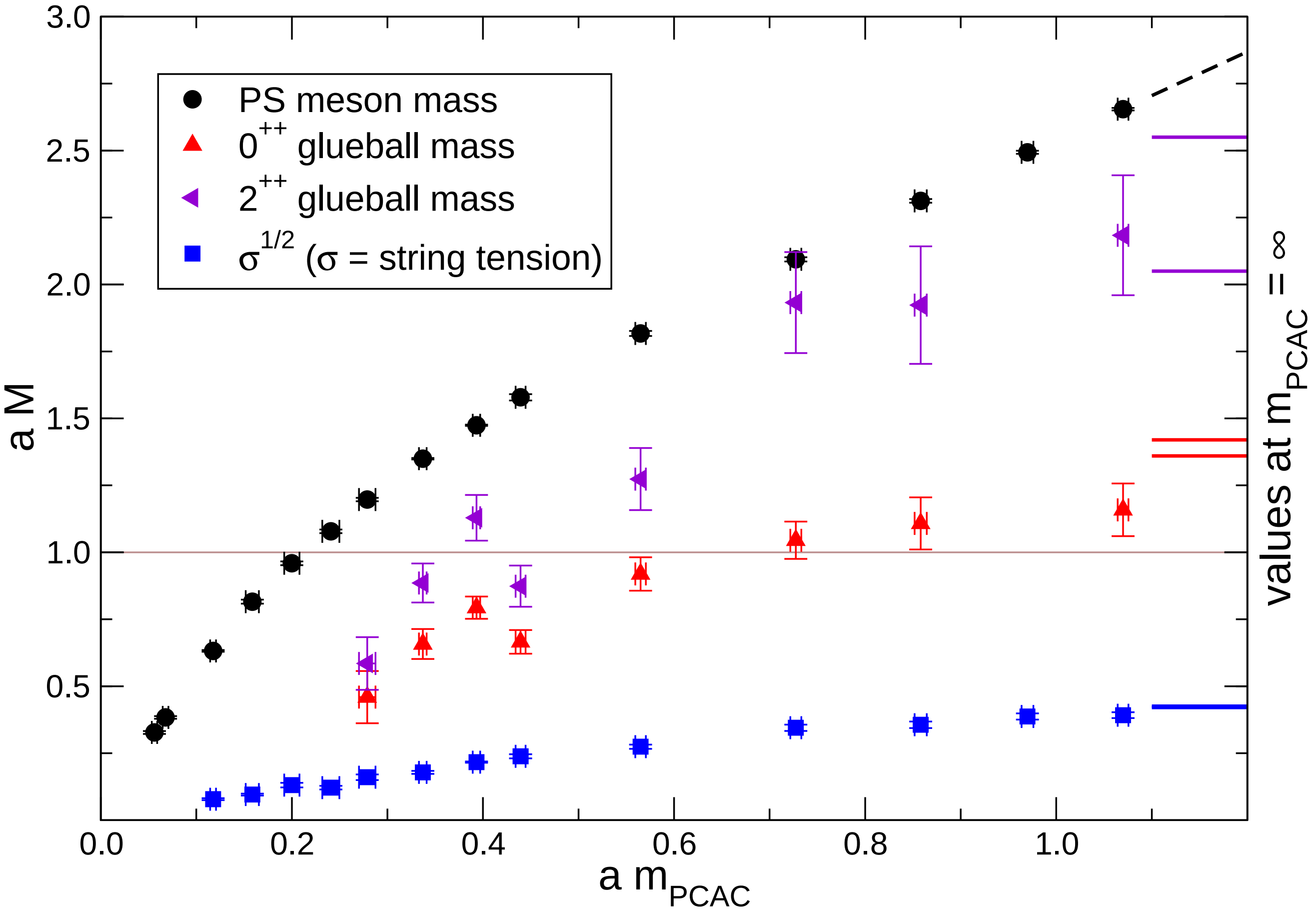
Finite-size effects



Gluonic sector

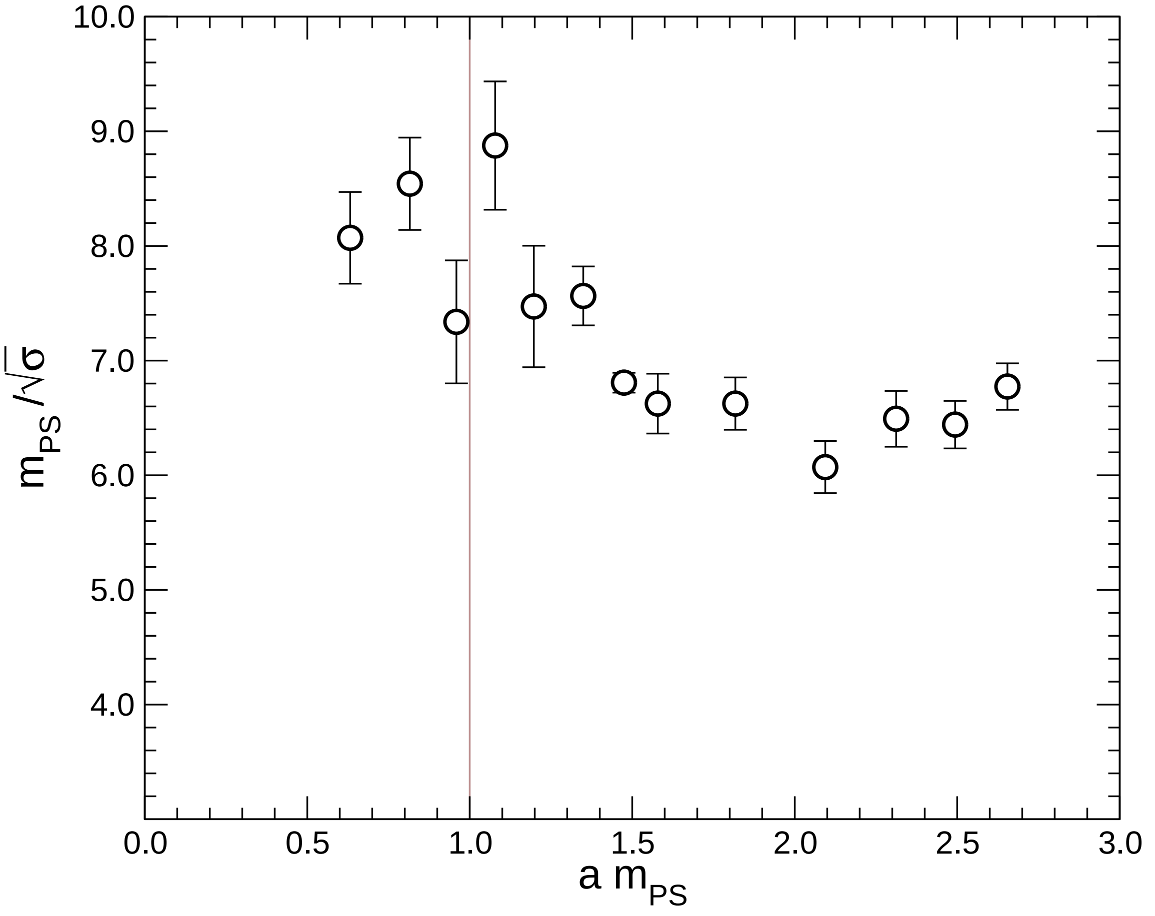


A hierarchy of scales



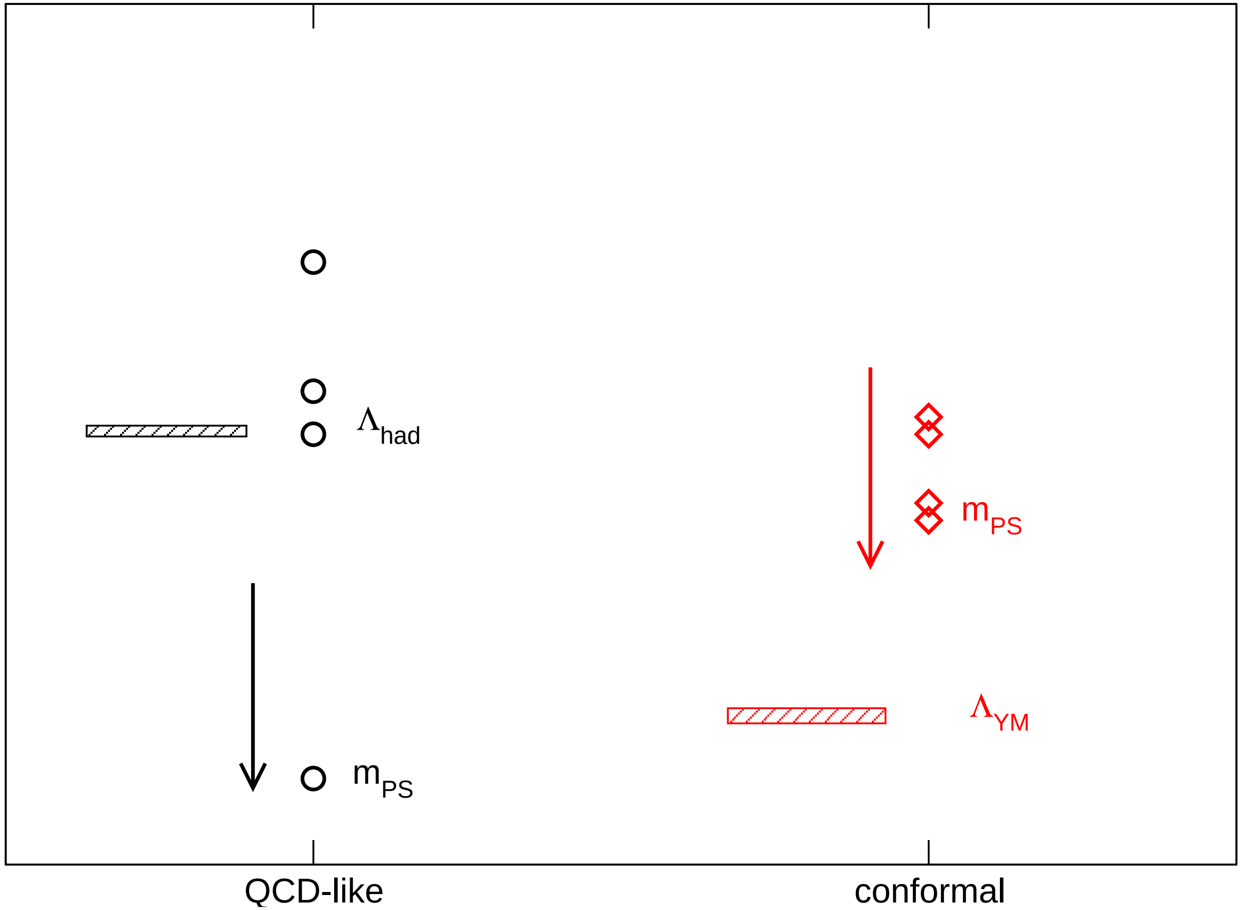
[Idd et al 09]

More on the hierarchy



$$\sqrt{\sigma} \ll m_{\rm PS}$$

Conformal spectrum



[e.g. miranski 95]

Running coupling

Schrödinger functional [ALPHA collaboration]

$$Z[\eta] = e^{-\Gamma[\eta]} = \int_{L\times L^3} \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \exp(-S[U,\psi,\bar{\psi}])$$

Dirichlet boundary conditions at $t = 0, L$, dependent on η

$$\boxed{\left.\frac{\partial\Gamma}{\partial\eta}\right|_{\eta=0} = \frac{k}{\bar{g}^2}, \qquad \bar{g}^2 = g_0^2 + O(g_0^4)}$$

Lattice step scaling function:

$$\Sigma(u, a/L) = \left.\bar{g}^2(bL)\right|_{\bar{g}^2(L)=u, m=0}$$

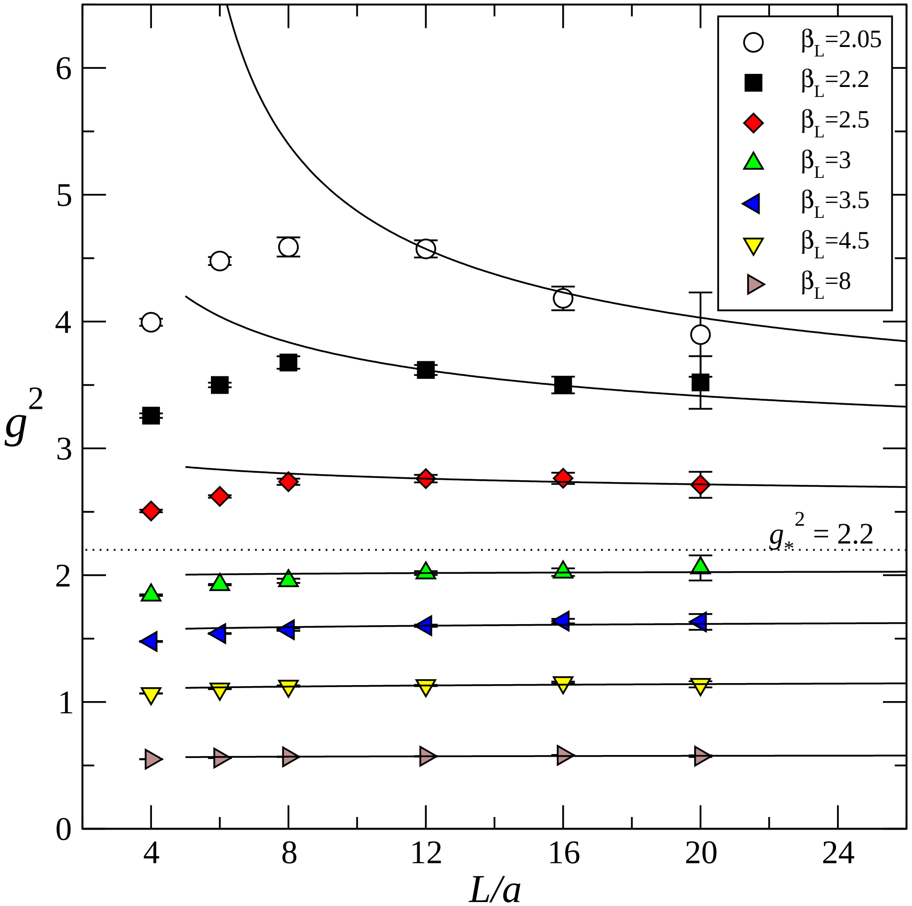
Step scaling function:

$$\sigma(u) = \lim_{a\to 0} \Sigma(u, a/L)$$

$$\beta(\sqrt{\sigma(u)}) = \beta(\sqrt{u})\sqrt{\frac{u}{\sigma(u)}}\sigma'(u)$$

Running coupling

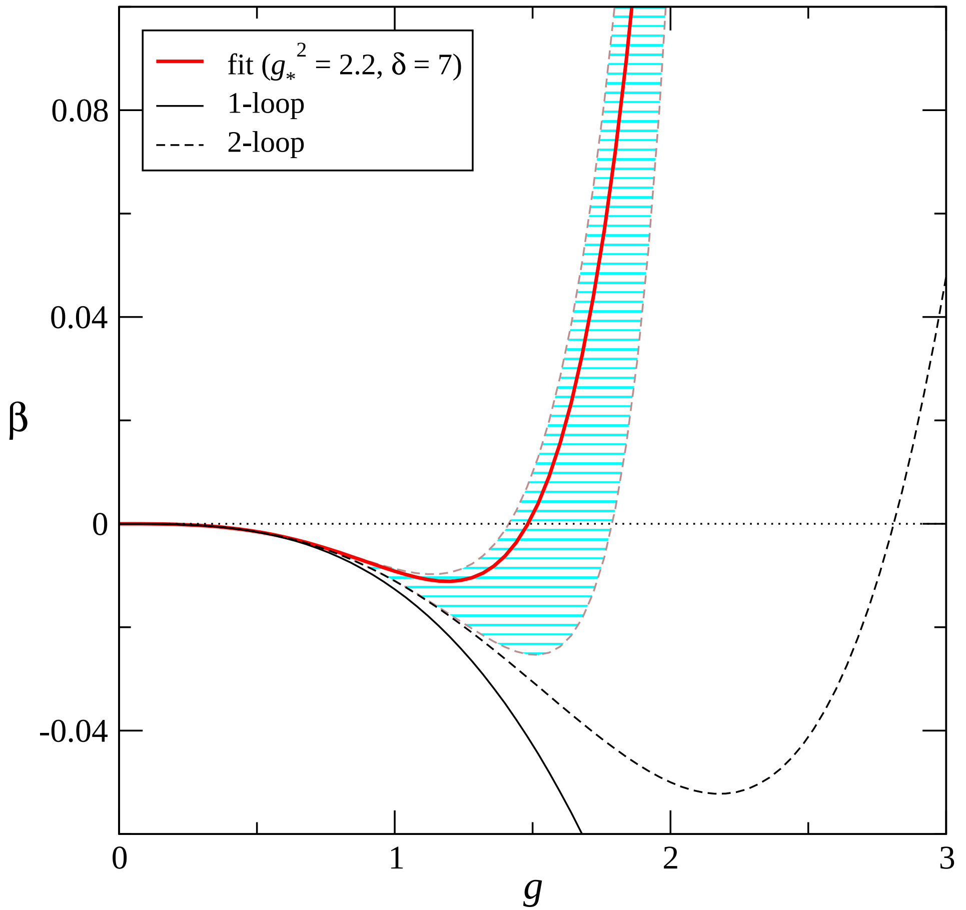
SU(2) with 2 adjoint fermions [rummukainen et al 09]



Running coupling

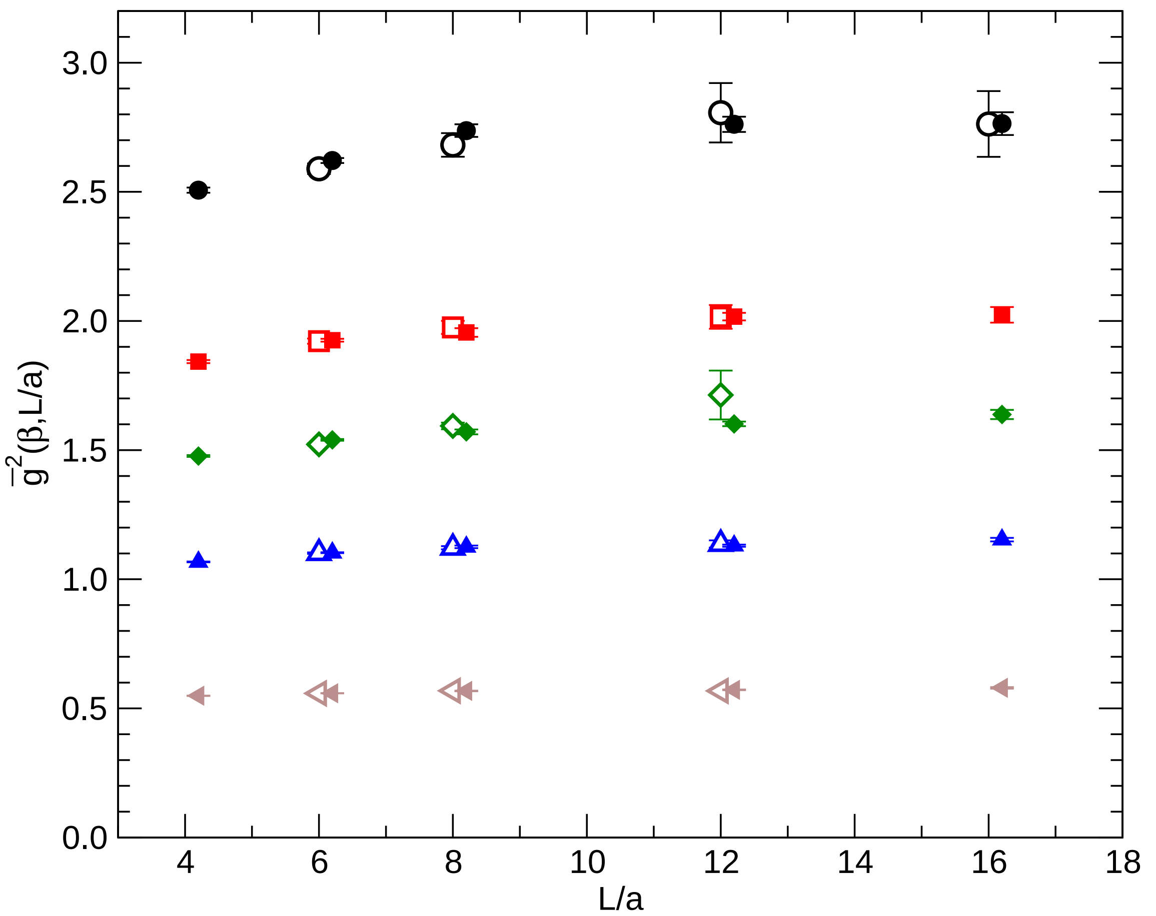
[rummukainen et al 09]

$$\beta(g) = \frac{dg(L)}{d\log L}, \quad \beta(g) = -b_0 g^3 - b_1 g^5 + (b_0 g_*^{3-\delta} + b_1 g_*^{5-\delta}) g^\delta$$



Running coupling

[Idd et al 09]



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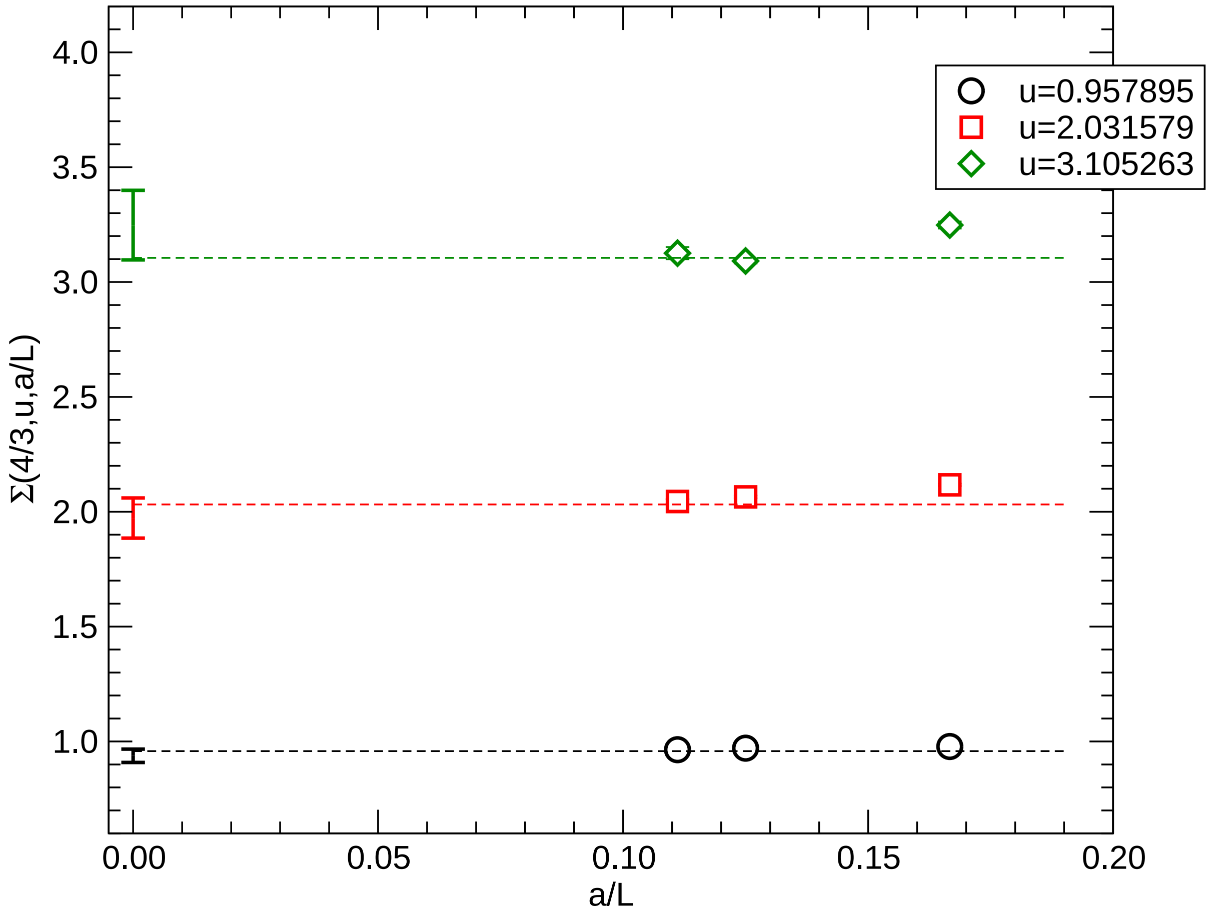

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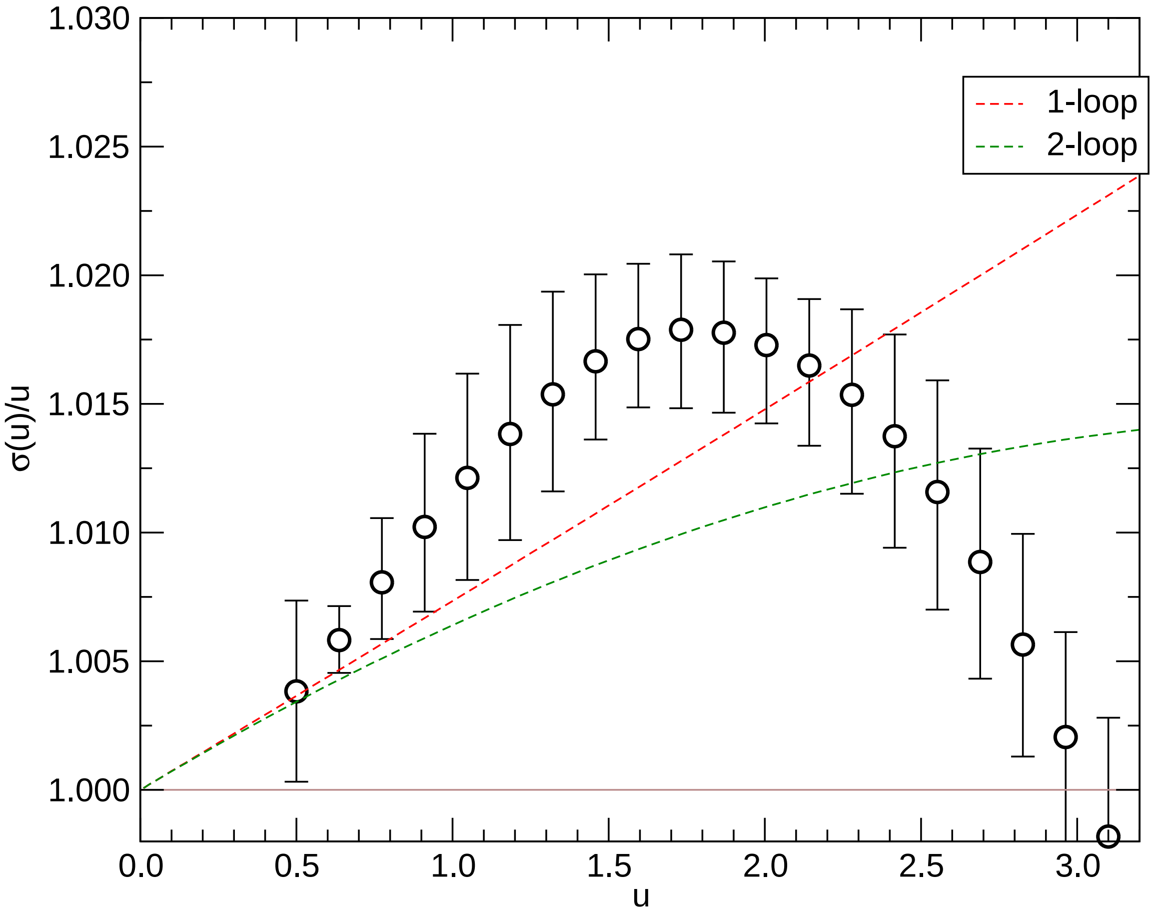

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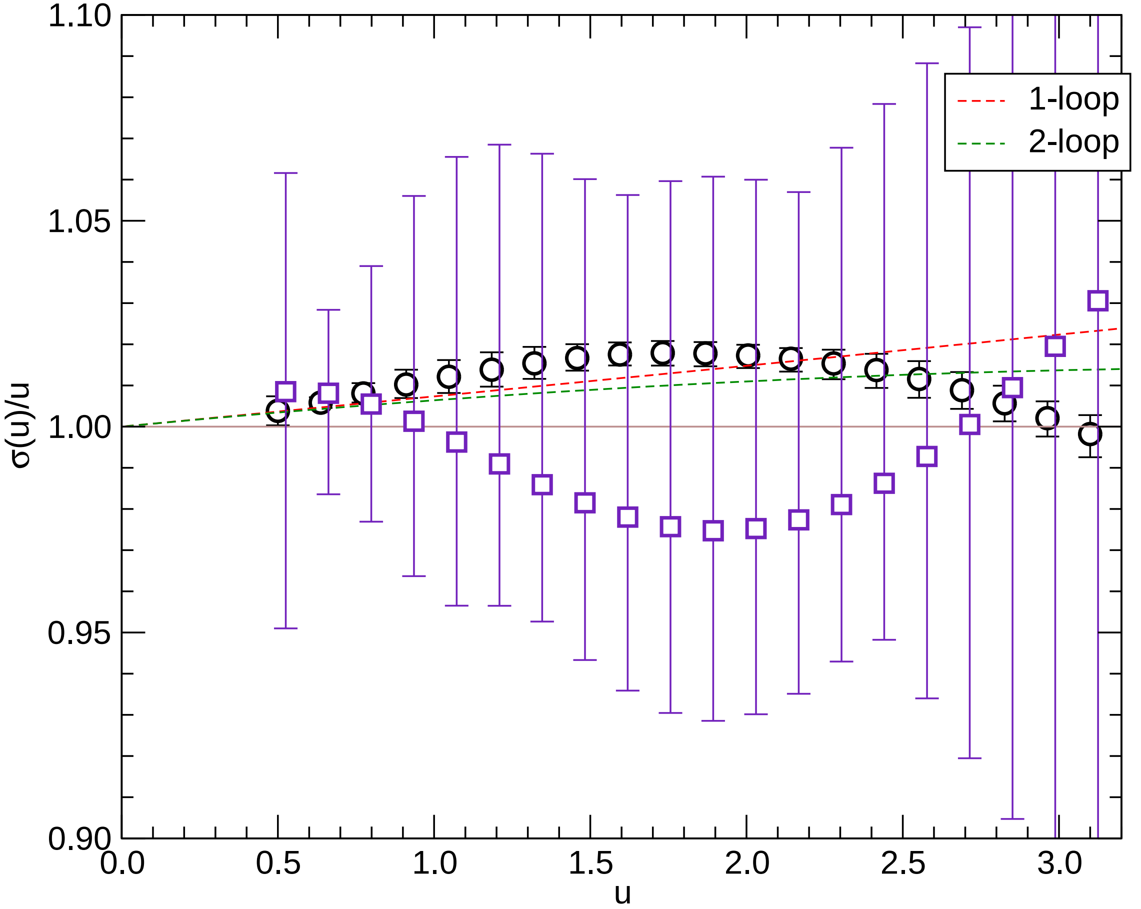


Running mass

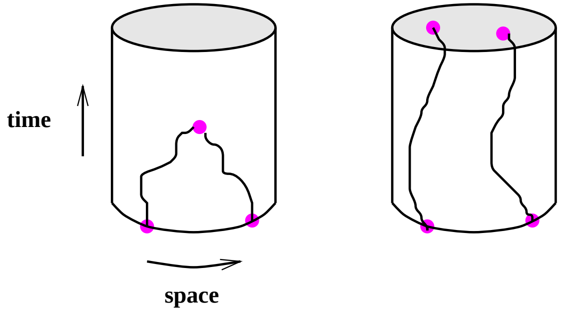
Definition of the renormalized mass

$$\boxed{\partial_\mu (A_R)_\mu = 2\bar{m} P_R}$$

$$\begin{aligned} (A_R)_\mu(x) &= Z_A \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x) \\ (P_R)(x) &= Z_P \bar{\psi}(x)\gamma_5\psi(x) \end{aligned}$$

Lattice step scaling function

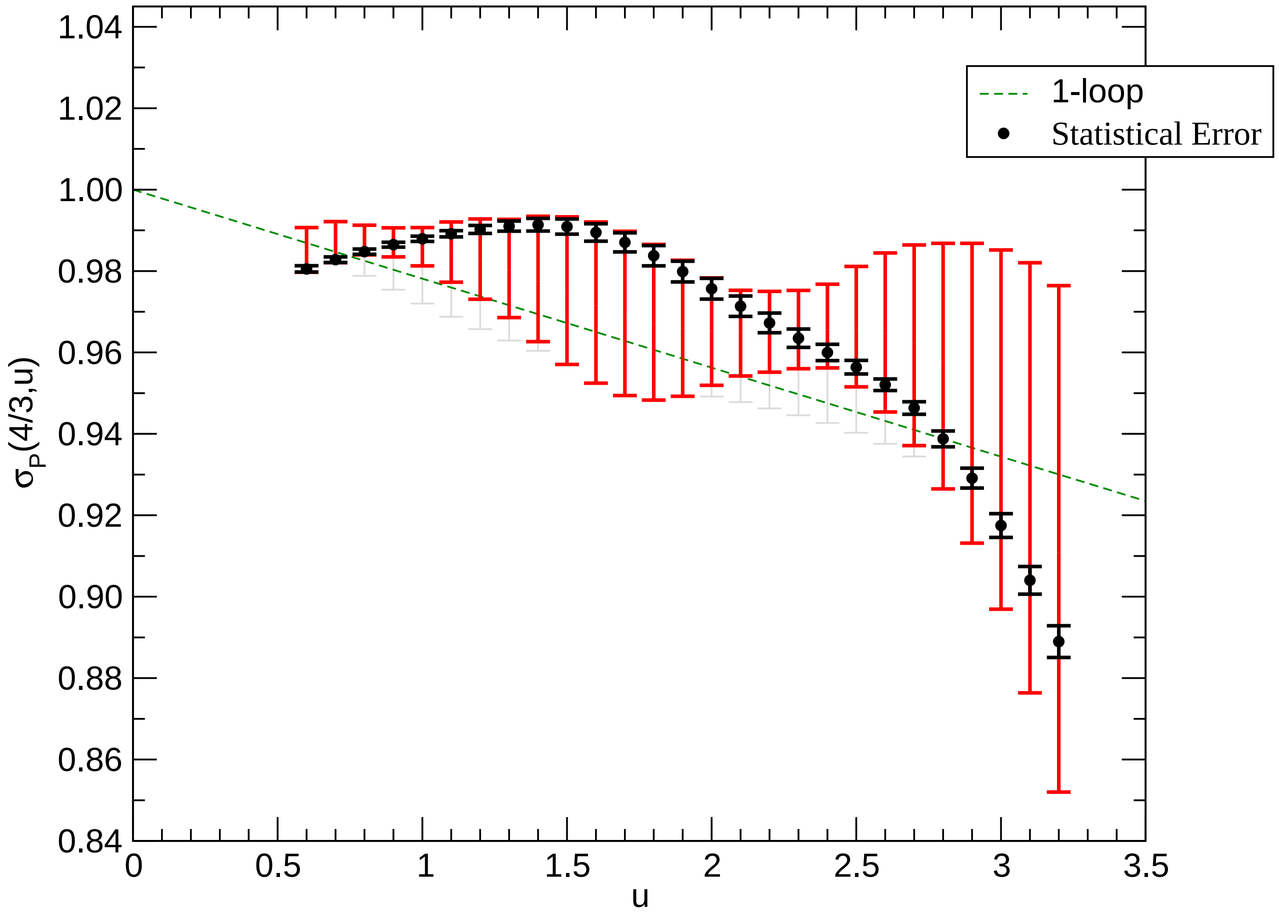
$$\begin{aligned} Z_P(g_0, L/a) &= c\frac{\sqrt{3f_1}}{f_P(L/2)} \\ \Sigma_P(u, a/L) &= \frac{Z_P(g_0, bL/a)}{Z_P(g_0, L/a)}, \quad \bar{g}^2(L) = u \end{aligned}$$



Step scaling function

$$\sigma_P(u) = \lim_{a\to 0} \Sigma_P(u, a/L)$$

Running mass



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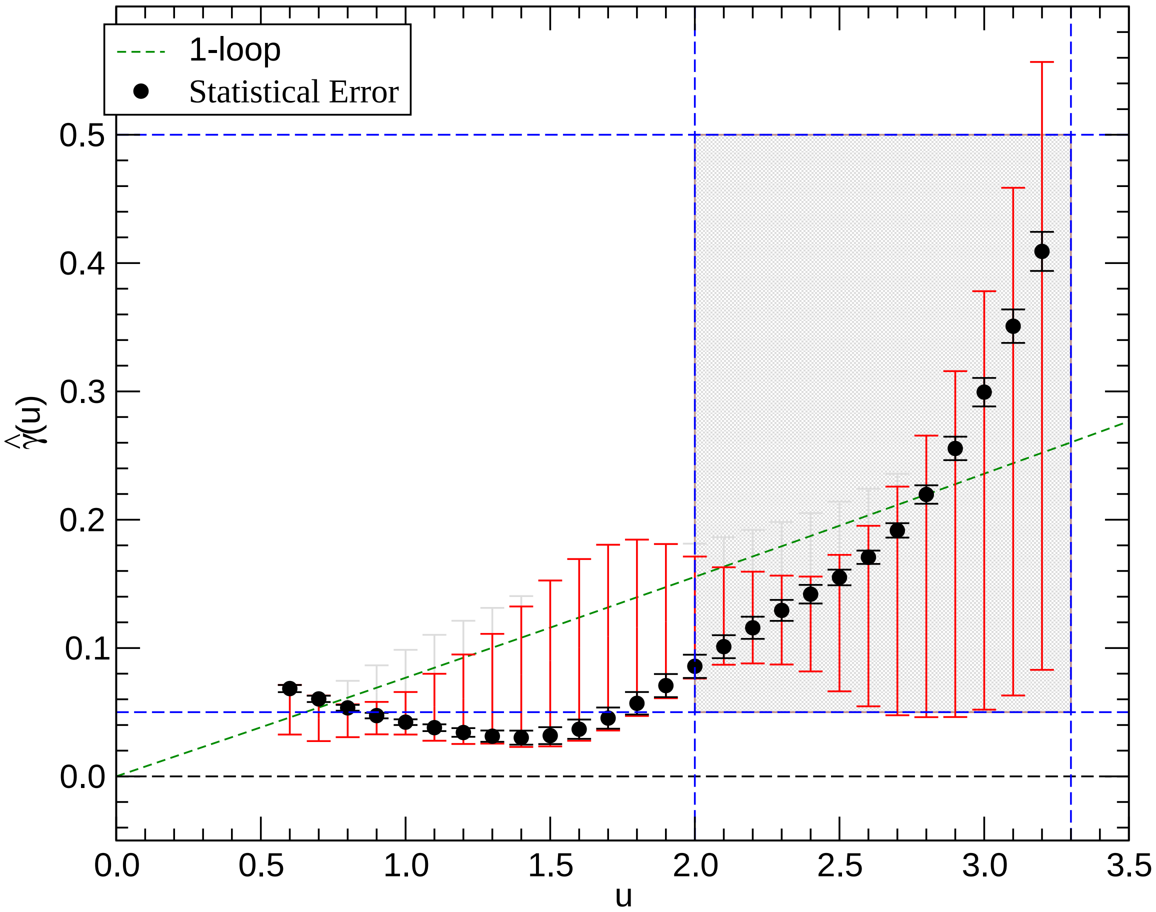


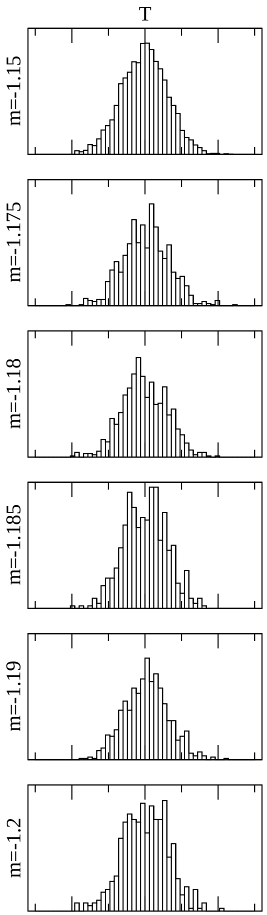
$$\gamma(u) = \frac{\log|\sigma_P(u)|}{\log|s|}, \quad \gamma = 0.2 \Longrightarrow 1/(1+\gamma) = 0.85$$

Conclusions

- importance of reaching the small mass regime!!
- control over the algorithmic performance, finite-size effects
- **identifying an IRFP numerically is subtle - analytic approach**
- several numerical aspects need to be combined

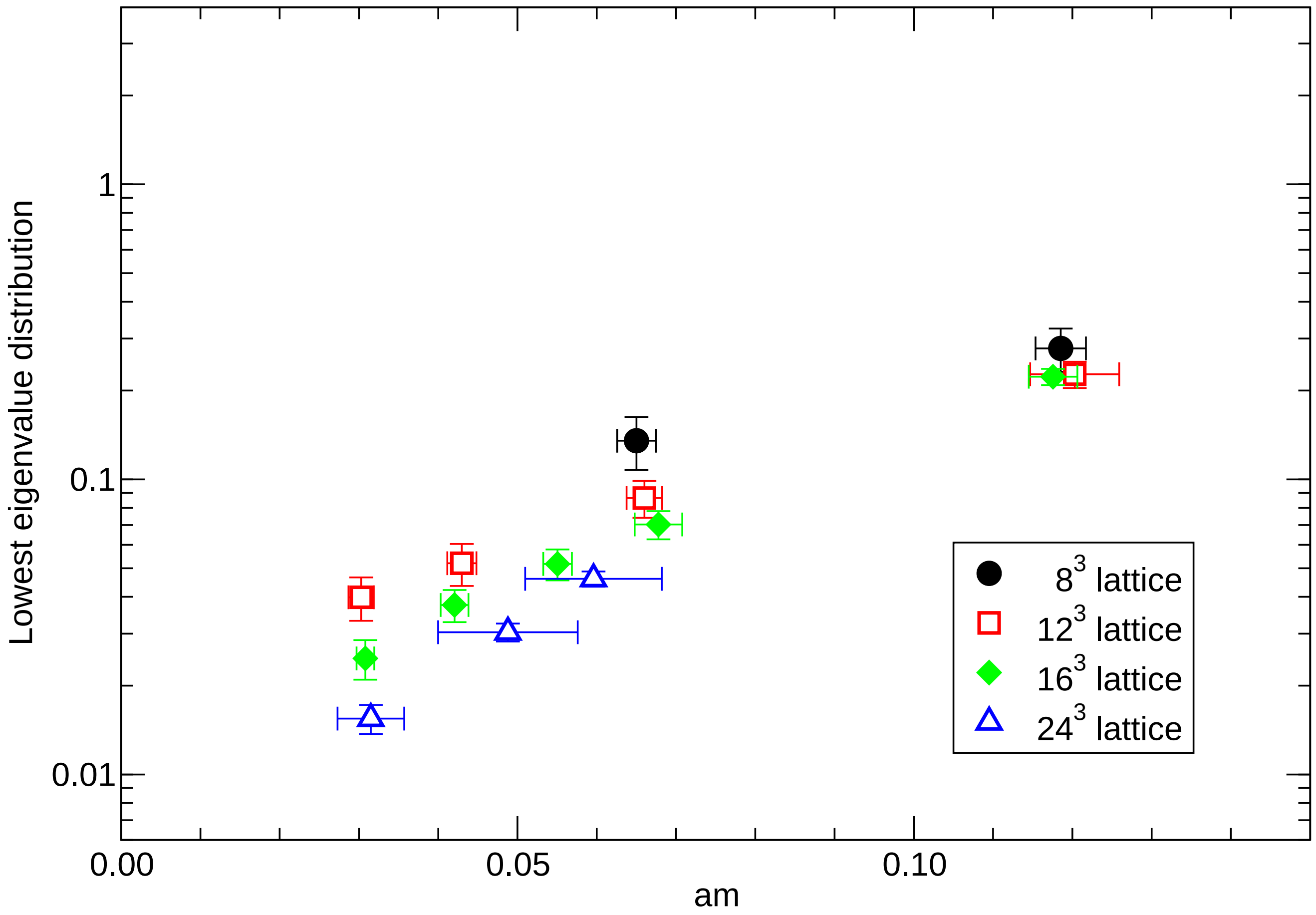
- first preliminary results $\longrightarrow$ larger volumes, smaller masses, scaling to control systematics
- improved spectroscopy $\longrightarrow$ smearing, scalar
- NP walking behaviour: Schrödinger functional
  Wilson loop, other schemes
- **signals for a non QCD-like behaviour**
- can we do some phenomenology?

Polyakov distributions



Lowest eigenvalue for SU(2) adj

[ldd, patella, pica 08]



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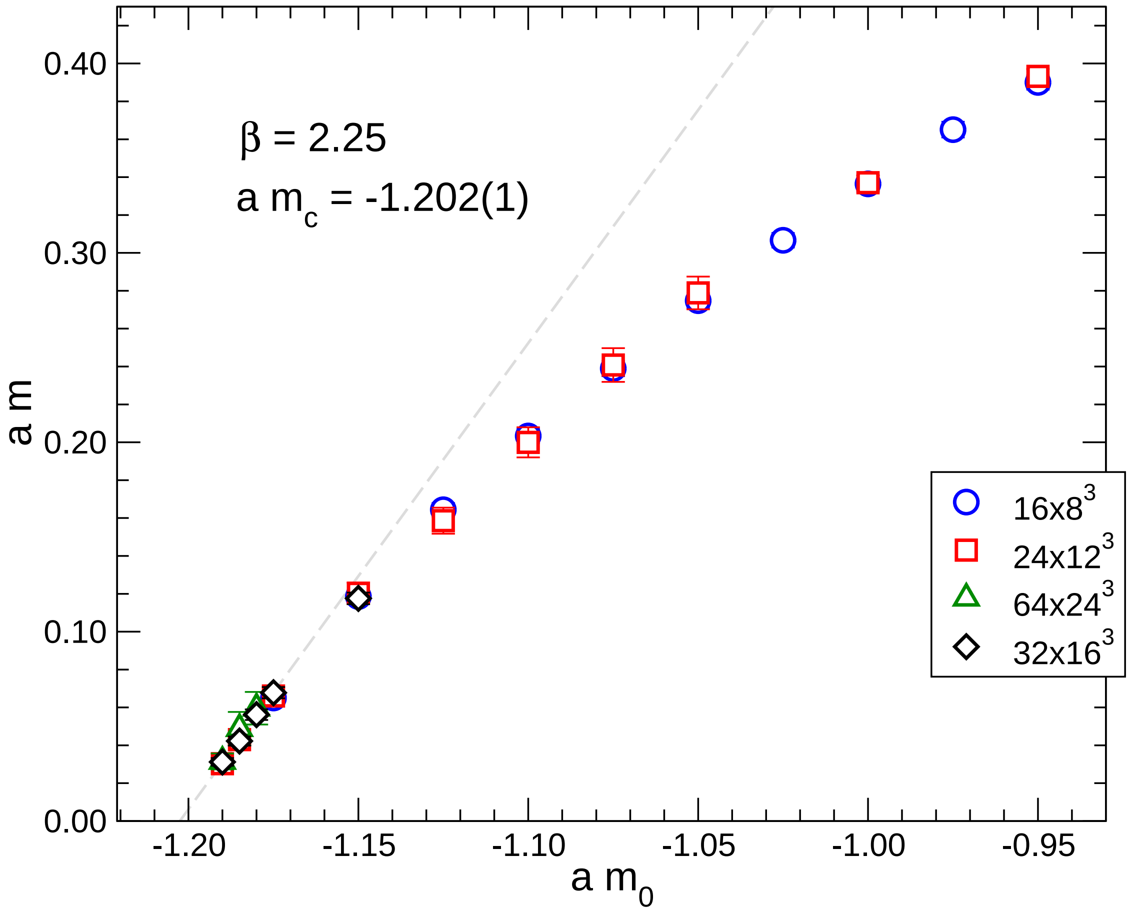
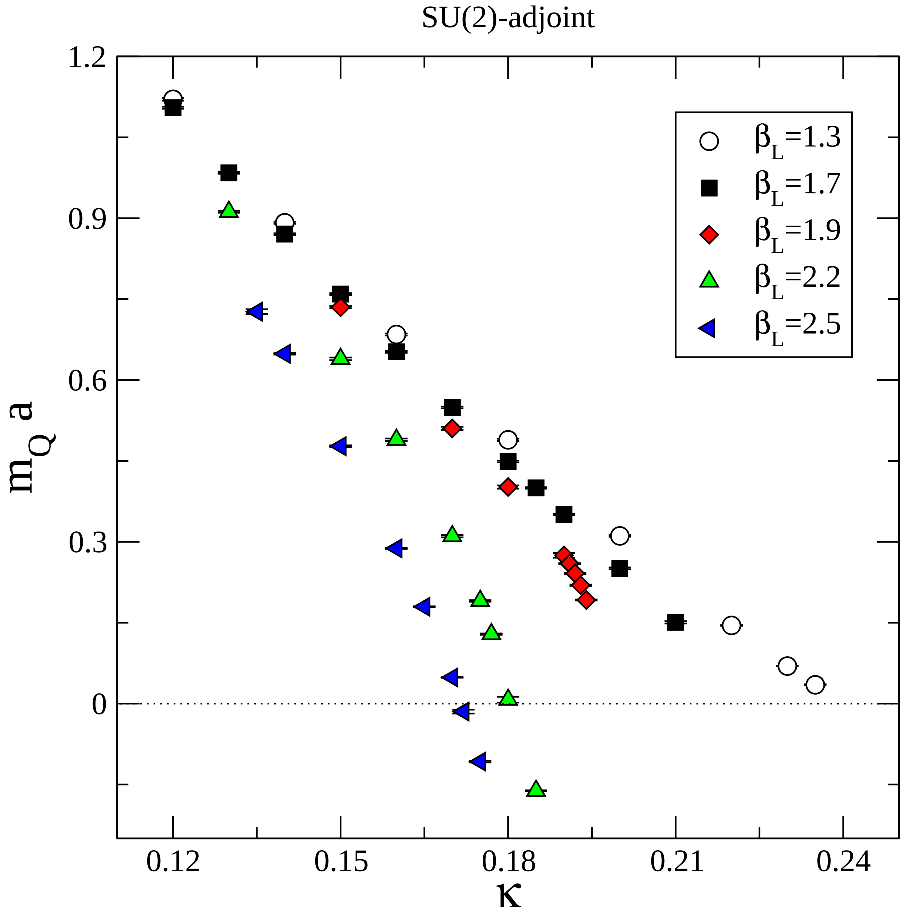
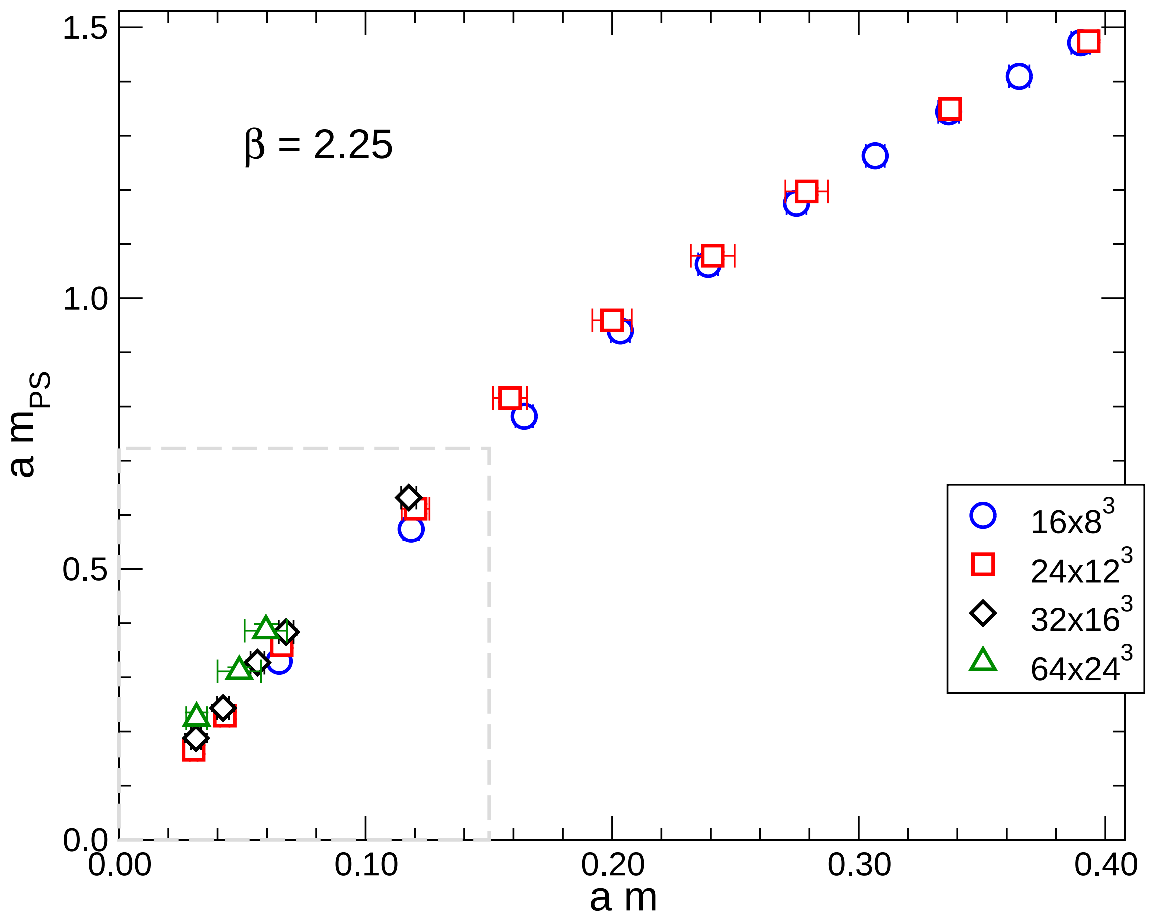


Comparison with other studies



[rummukainen et al 08]

Pseudoscalar mass (1)



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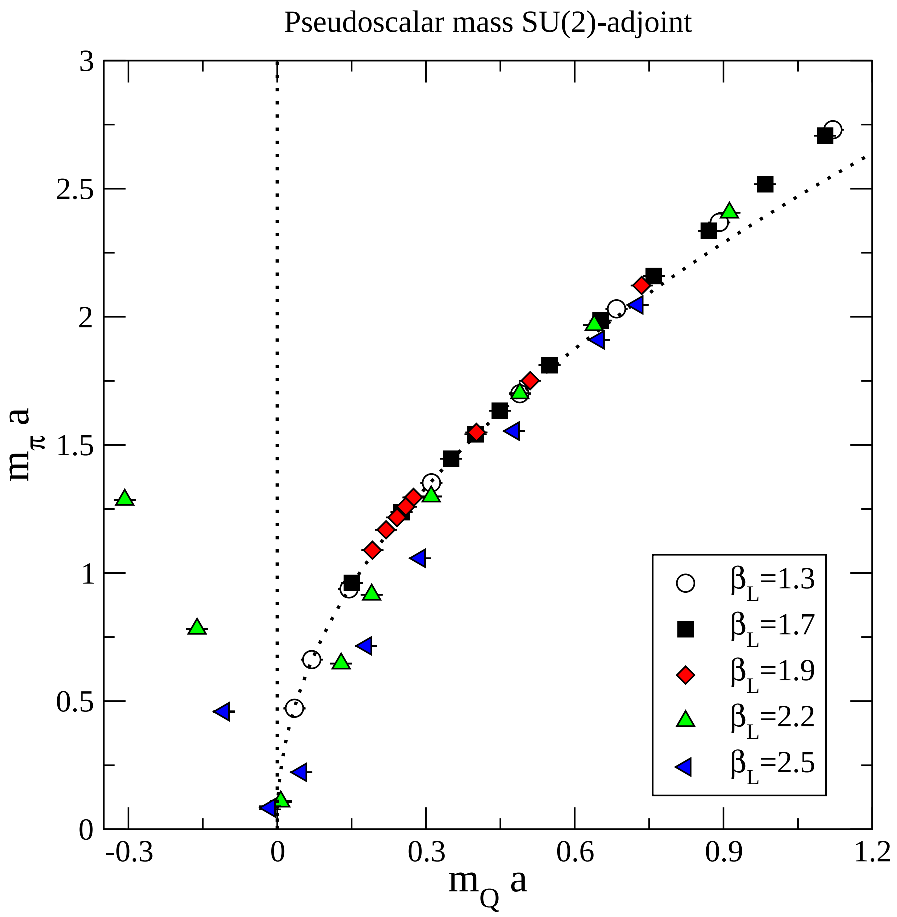

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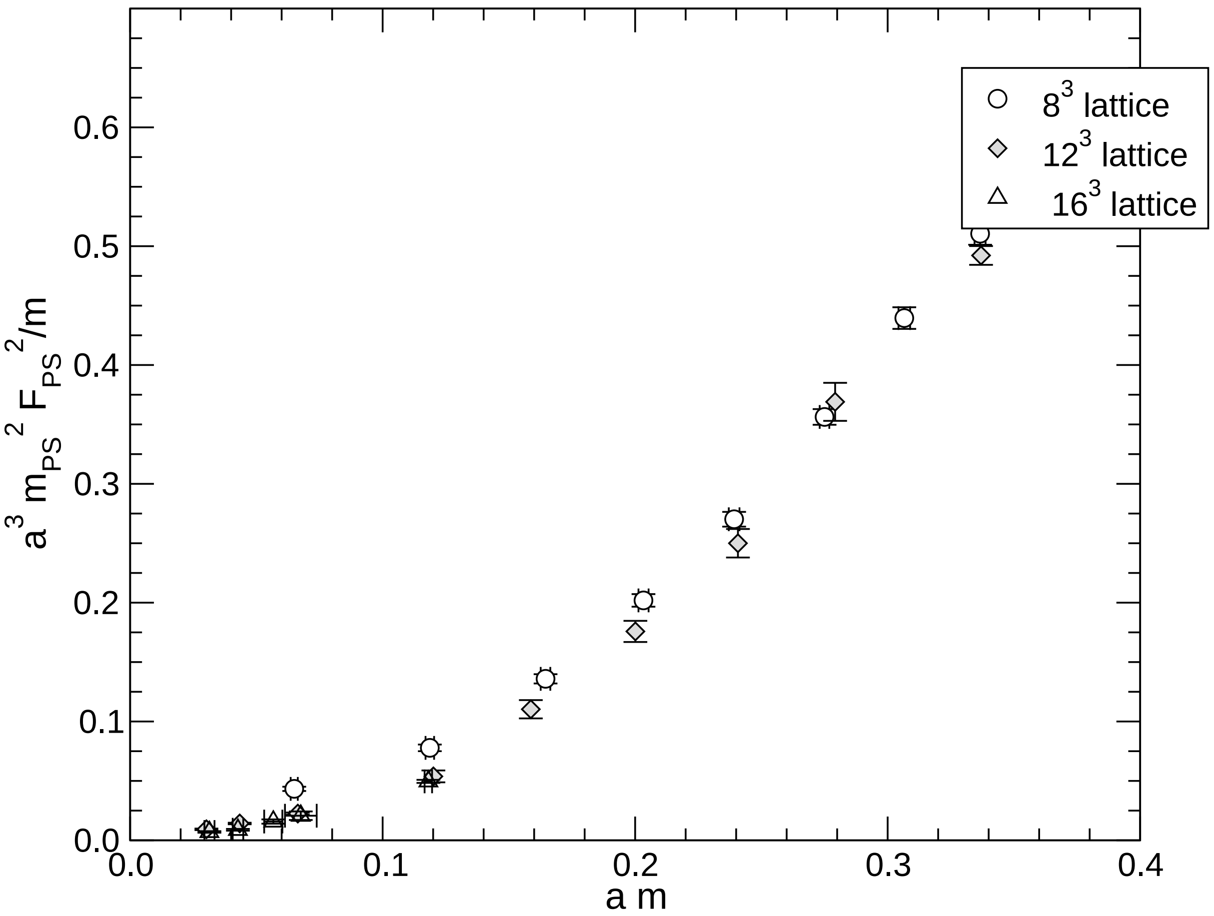

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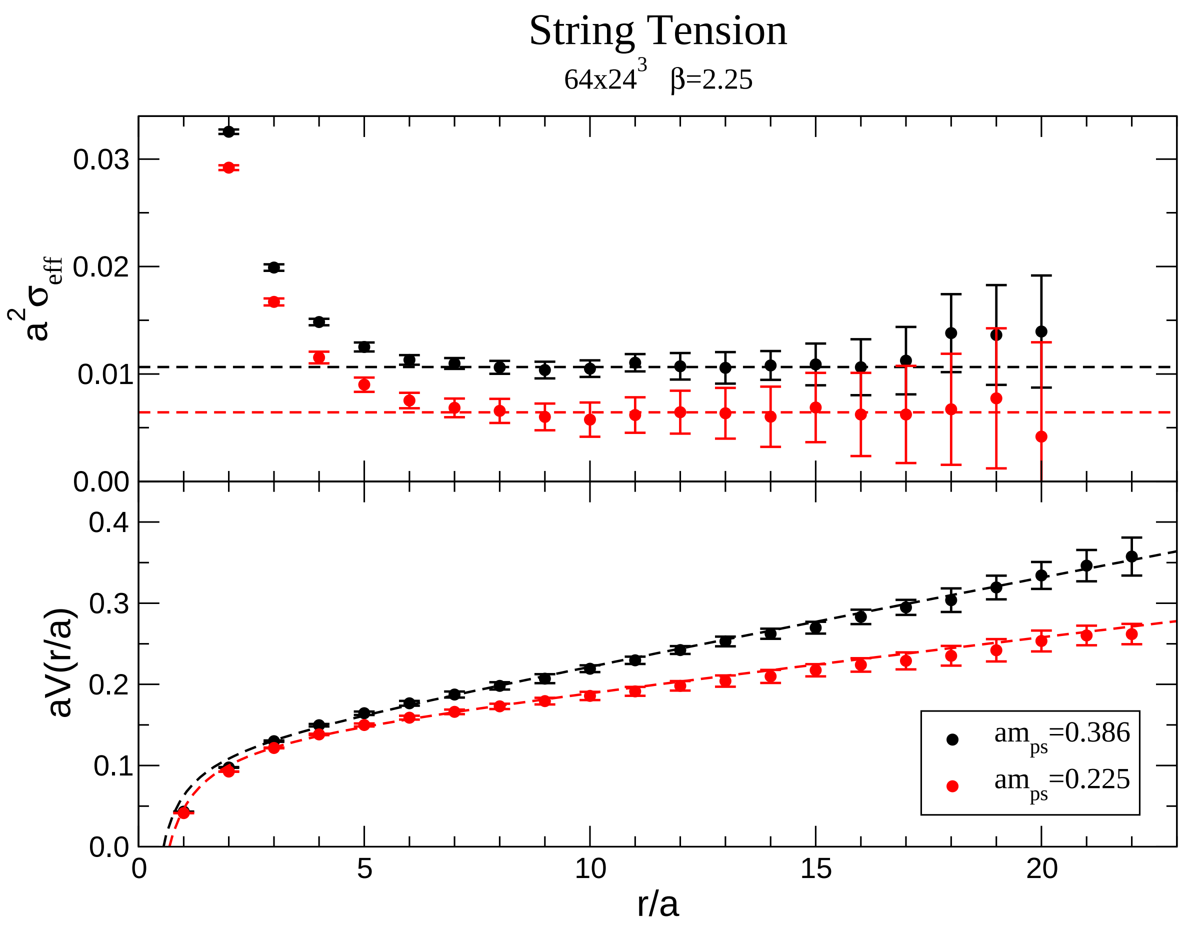


Chiral condensate

chiral condensate computed from the ev distribution [giusti & luscher 08]

Running coupling

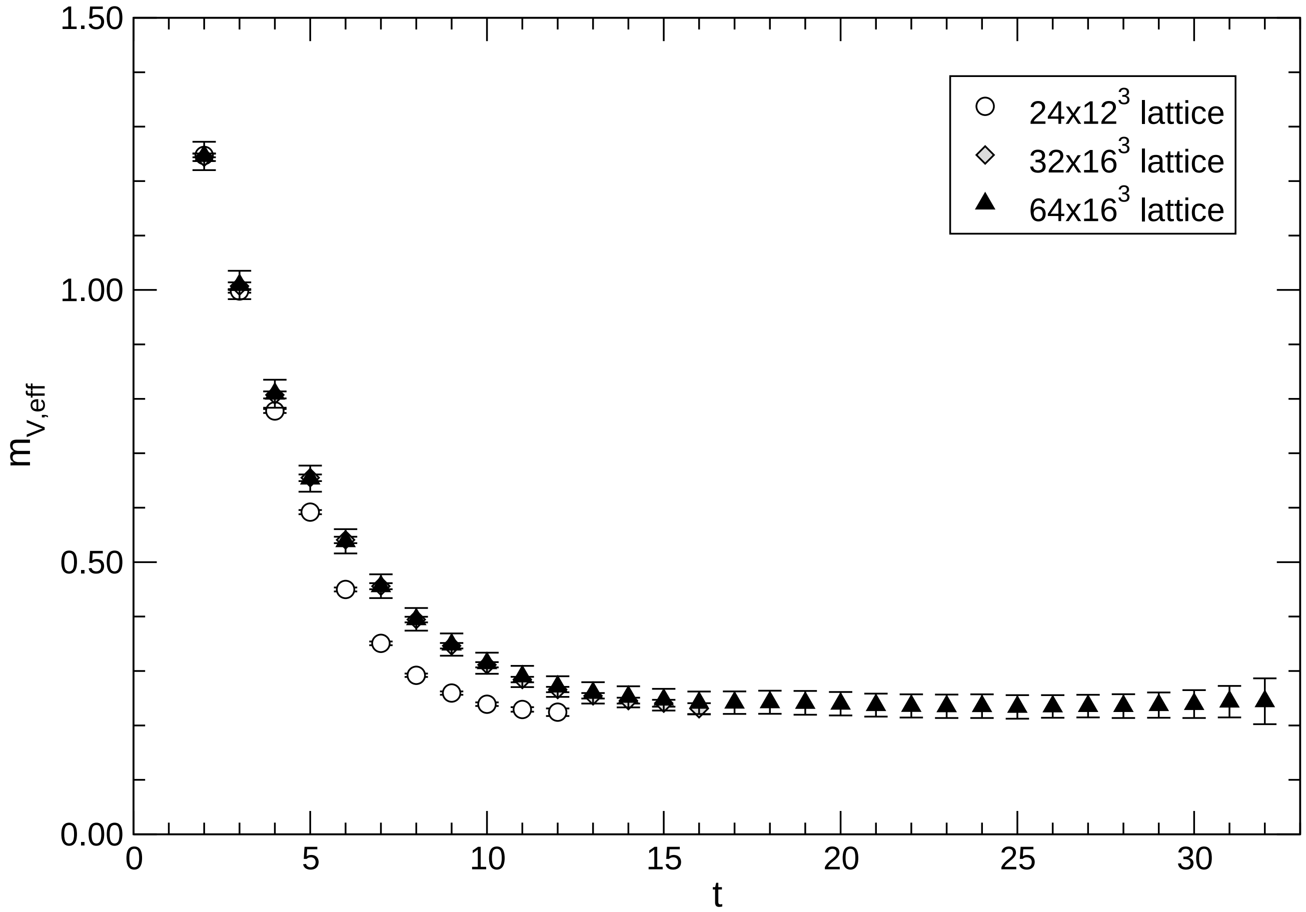
[appelquist et al 07, degrand et al 08, rummukainen et al 09]

Running coupling

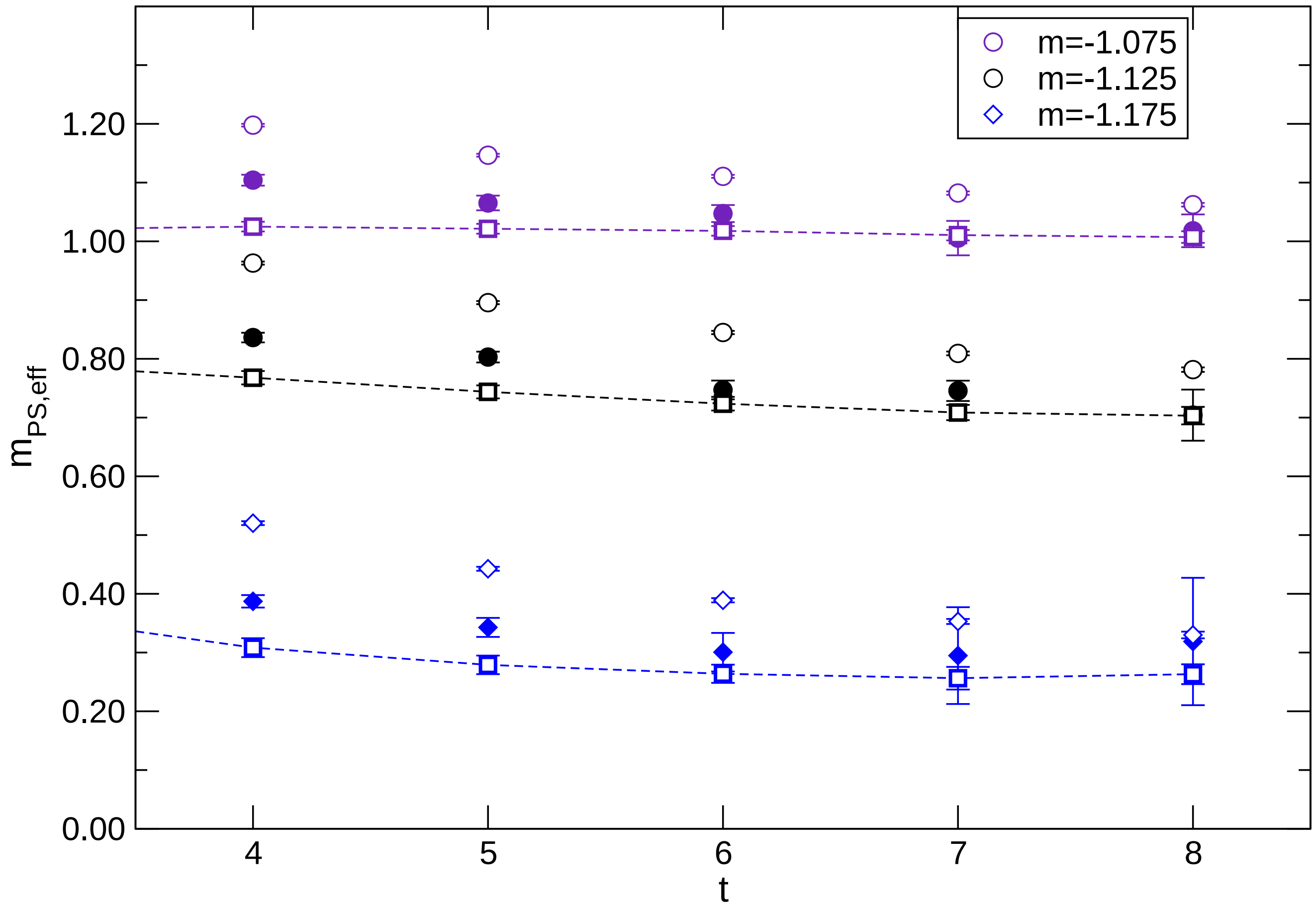
Wilson loop scheme [bilgici et al 09]

All-order beta function?

Systematic errors



Systematic errors



data from 8^3 lattice – preliminary study [kerrane 09]