Introduction to lattice supersymmetry and its applications

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Goals, Methods

DWF for $\mathcal{N} = 1$

Super QCD

Exact lattice SUSY

$\mathcal{N} = 4$ SYM and AdS/CFT

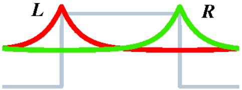
Lattice SUSY

- SUSY plays a key role in many theories of BSM physics
  - Understanding soft SUSY breaking in MSSM in terms of strongly coupled high scale SQCD
  - Extra dims models
  - SUSY technicolor
  - AdS/CFT duals, strings, black holes
- Many aspects are inherently non-perturbative eg dynamical SUSY breaking, gaugino condensation – lattice natural tool.

Problems, solutions

- SUSY broken. Large amount fine tuning in general to take $a \longrightarrow 0$.
- But, some cases evaded/reduced:
  - 4D $\mathcal{N}=1$ SYM: chiral symmetry prohibits gaugino mass. Hence DWF/overlap good approach.
  - 4D $\mathcal{N}=4$ SYM: supersymmetric lattice action exists. (new ideas twisting, orbifolding, Kähler-Dirac fermions).
  - Stepping stones to other theories: super QCD, deformations of $\mathcal{N}=4$ with eg mass terms, AdS/CFT duals

Why DWF good for $\mathcal{N} = 1$



- 5D domain wall: 4d chiral, massless mode localized to wall.
- Can be gauged and subject to Majorana condition – real, positive definite Pfaffian (c.f Wilson)
- Only relevant SUSY breaking op. is gaugino mass (Veneziano et al)
- Good chiral properties of DWF - SUSY broken but no fine tuning $a \to 0$ ...

But more expensive ... See M. Endres talk

History

- Fleming, Kogut, Vranas in 2000: $8^4 \times L_s$ lattices, $\beta = 2.3$, $L_s = 12 - 24$
- Recently 2008-9:
  - S.C,Brower,Fleming,Giedt,Vranas arXiv:0807.2032, arXiv:0810.5746 ($16^3 \times 32 \times L_s$ with $L_s = 16 - 64$, $\beta = 2.3, 2.4$), (1Tflopyear)
  - M. Endres arXiv:0902.4267, ($8^4/16^3 \times 32$ $L_s = 16 - 28$ $\beta = 2.3, 2.35, 2.4$)
- Both groups use hacked CPS code for SU(2). Run on BG/L

Results so far

Broad agreement between 2 recent calcs. Show:

- Confining as expected - static potential, string tension.
- Finite volume effects under control.
- Estimate cut-off effects now.
- Much better extrapolations to chiral limit - measure residual mass - but large ...
- Strong evidence for nonzero $< \bar{\lambda}\lambda >$ as $a \to 0$.

Future

- Large $m_{\rm res} = \frac{\rho(0)}{L_s} + \ldots$ – Need larger L_s. Or better DWFs (möbius ?,..)
- Non-perturbative renormalization to give physical condensate.
- Spectrum ? Interpolating operators known. But 1 Majorana fermion – disconnected diagrams. Hard. cf η' in QCD.
- Eg. $32^3 \times 64$ with $L_s = 64 - 128$ at 3 β's would need at least 100 Tflopyear.

Super QCD - I

- MSSM weakly coupled. But soft SUSY breaking ops typically generated by strongly coupled SQCD like sector at high scales.
- Ingredients: add chiral multiplets (scalar+fermion in fundamental)
- DWF eliminates need to tune fermion masses (Giedt et al. arXiv:0806.0013)
- Fine tuning Yukawas - swap for scalar kinetic terms. Flavor symmetries imply single m_ϕ^2
- Quartic ops still an issue - $n = (2 + \text{few})$ such ops. depending on N_f, N_c.

Super QCD - II

- Find SUSY pt by requiring $< \partial_\mu S_\mu(x)\Theta^i(0) >= 0$ for set of set ops Θ^i with $i \geq n$ number of ops.
- If we want to use multicanonical reweighting techniques need say 10 simulation pts for each fine tuned coupling.
- Implies need $10^{2+\text{few}}$ more CPU than current $\mathcal{N} = 1$ DWF simulations.
- Note: use for $\mathcal{N} = 4$ SYM independent of supersymmetric lattice formulations

SUSY lattices

- Recent work: possible to discretize theories with $\mathcal{Q} \geq 2^D$ supercharges while preserving one or more SUSYs exactly (S.C, David Kaplan, Mithat Ünsal, Phys. Rep, arXiv:0903.4881)
- Simplest way to understand: discretizations of topologically twisted versions of target SYM theory
- In flat space: these twisted theories completely equivalent to usual theory.
- In particular lead to unique lattice formulation of $\mathcal{N}=4$ SYM with one exact SUSY.

Exact SUSY for $\mathcal{N}=4$: pros and cons

- Exact SUSY ensures boson/fermion spectrum paired and $E_{\rm vac}=0$.
- Reduces number of potential counterterms needed to take $a\to 0$.
- Action is local, free of doublers, gauge invariant and supersymmetric.
- Price (mostly technical):
  - Most natural lattice is not hypercubic – A_4^*
  - 16 Fermion fields distributed over links of lattice – fermion action non-standard (Kähler-Dirac).
  - Bosons – 5 Complex gauge links $\mathcal{U}_a = e^{(A_a+iB_a)}$
  - Sign problem ?

Lattice $\mathcal{N}=4$

Action:

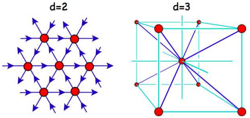
$$S = \sum_{\mathbf{x}} \mathrm{Tr}\left(\mathcal{F}^{\dagger}_{mn}\mathcal{F}_{mn} + \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_m \mathcal{U}_m\right)^2 - \left(\eta \overline{\mathcal{D}}^{(-)}_m \psi_m + \chi_{mn}\mathcal{D}^{(+)}_{[m}\psi_{n]}\right)\right) + S_{\rm closed}$$

Fields defined on A_4^* with

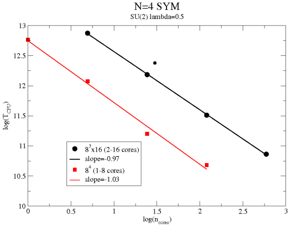
$$\mathcal{F}_{mn} = \mathcal{U}_m(x)\mathcal{U}_n(x+m) - \mathcal{U}_n(x)\mathcal{U}_m(x+n)$$

$$S_{\rm closed} = -\frac{1}{8}\sum_{\mathbf{x}} \mathrm{Tr}\ \epsilon_{mnpqr}\chi_{qr}(\mathbf{x}+\mu_m+\mu_n+\mu_p)\overline{\mathcal{D}}^{(-)}_p \chi_{mn}(\mathbf{x}+\mu_p)$$

Lattices, code



$8^3 \times 16$ 2-16 cores Jpsi (MPI comm using MDP in FermiQCD)



Renormalization

Lattice symmetries:

- Gauge invariance
- $\mathcal{Q}$-symmetry.
- Point group symmetry - eg. S^5 for A_4^*
- Exact fermionic shift symmetry $\eta \to \eta + \epsilon I$

Conclusion:

- Only (marginally) relevant ops that can arise correspond to renormalizations of ops. in bare theory except for SUSY mass term $\mathcal{Q}(\mathcal{U}_a \mathcal{U}_a^\dagger \eta)$
- Examine flows at 1-loop - in progress (with J. Giedt).

  Currently appears that no mass term induced!

Lattice perturbation theory

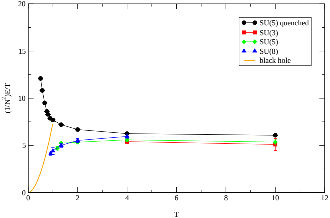
Ingredients:

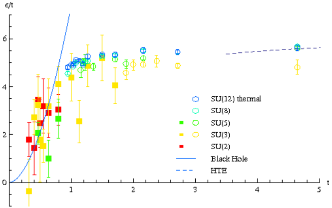
- Bosons: $<\overline{\mathcal{A}}_a(k)\mathcal{A}_b(-k)>=\frac{1}{\hat{k}^2}\delta_{ab}$ with $\hat{k}^2=4\sum_a \sin^2(k_a/2)$
- Fermions: $M_{\rm KD}^{-1}(k)=\frac{1}{\hat{k}^2}M_{\rm KD}(k)$ with $M(k)$ a 16×16 block matrix.
- Vertices: $\psi\eta$, $\psi\chi$ and $\chi\chi$. No lattice tadpoles.
- Small number of Feynman graphs – determine wavefunction renormalization from 1-loop corrections to fermion propagators.

Gravity duals

- Large number of examples of conjectured dualities between (supersymmetric) YM and gravity/string theories.
- YM always strongly coupled –
  New SUSY lattice actions very useful in this respect.
- Simplest example studied so far:
  SYMQM at large N – black hole type IIa SUGRA
  S.C and Wiseman arXiv:0803.4273, Nishimura et al. arXiv:0810.2884
- Current work:
  - $\mathcal{N}=4$ in $D=4$ AdS/CFT, (SUSY)Wilson loops
  - Dimensional reduction to 2D – Is Gregory-LaFlamme black string-black hole transition dual to thermal PT in YM ?

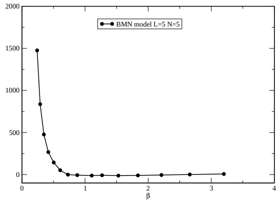
Black holes from YM

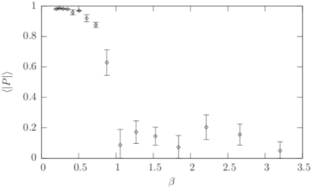




Energy vs temperature for SYMQM system.
Black hole prediction shown (no fit)
Care ... divergence in thermal partition function - needs care ..

Mass deformed SYMQM – BMN model





Thermal phase transition between SUSY confining phase at low T and deconfined broken phase at high T (Relation to Hawking-Page transition in $\mathcal{N} = 4$?)

Summary

Several promising approaches to SUSY on lattice

- Use DWF+reweighting – $\mathcal{N} = 1$ SYM and SQCD. Ultimate goal to understand soft terms in MSSM.
  - Break SUSY but ensure restored as $a \to 0$ with minimum fine tuning
  - Price is (some) reweighting scalar sector and need DWF.
- Formulations with exact SUSY.
  - Reduced fine tuning due to lattice symmetries.
  - Lots of applications to AdS/CFT-like duals, understand quantum black holes, ...
  - Potential sign problem ?