Lattice Supersymmetry and Strong Dynamics

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Plan

Two topics:

- Lattice theories with exact supersymmetry
- Phase diagram of SU(2) with adjoint quarks

Why lattice SUSY ?

- Nonperturbative structure of supersymmetric theories
- Dynamical SUSY breaking
- Definition of SUSY theory outside of perturbation theory
- Tool to explore dualities between gauge theories and gravity

Difficulties

Supersymmetry and lattice appear incompatible

- $\{Q, \overline{Q}\} = \gamma.p$
- Leibniz: $\Delta^+(fg) \neq (\Delta^+ f)g + f(\Delta^+ g)$
- Fermion doubling

Naive discretizations of SUSY theories break SUSY completely
Radiative corrections lead to many SUSY violating operators – must be tuned out in continuum limit. Hard!

Way out ?

- But ...special class of theories exist in which element of SUSY may be retained.
- Extended supersymmetry. In $D = 4$ unique theory: $\mathcal{N} = 4$ SYM
- Lattice theory preserves one SUSY: what happens as $a \to 0$? Residual fine tuning problem ? Full supersymmetry restored ?
- A single SUSY plus rotational invariance guarantees that the theory has at least $\mathcal{N} = 1$ ...
- Maybe such theory would resemble one of the mass deformations of $\mathcal{N} = 4$ discussed by Strassler ?
- For $D < 4$ many models. Dual to string/supergravity theories

Approaches

- Start from twisted/topological form of SUSY theory (Catterall, Sugino, Kawamoto, Giedt, ...)
- Use orbifold/deconstruction techniques from SUSY matrix model (Kaplan, Unsal, Katz, Cohen, Giedt, Damgaard, Matsuura)

SYM case: completely equivalent (Catterall, Unsal, Damgaard et al.)

Example: Twisting in 2D

Simplest theory contains 2 fermions λ^i_α with global symmetry $SO_{\mathrm{Lorenz}}(2) \times SO_{\mathrm{R}}(2)$
Twist: decompose under diagonal subgroup
Consider fermions as matrix

$$\lambda^i_\alpha \longrightarrow \Psi_{\alpha\beta}$$

Natural to expand:

$$\Psi = \frac{\eta}{2} I + \psi_\mu \gamma_\mu + \chi_{12}\gamma_1\gamma_2$$

scalar, vector and tensor (twisted) components!

Supersymmetries

Twisted supercharges (Q, Q_μ, Q_{12}).
Twisted (super)algebra:

$$\begin{aligned} \{Q,Q\} &= \{Q_{12},Q_{12}\} = \{Q,Q_{12}\} = \{Q_\mu,Q_\nu\} = 0 \\ \{Q,Q_\mu\} &= p_\mu \\ \{Q_{12},Q_\mu\} &= \epsilon_{\mu\nu}p_\nu \end{aligned}$$

Notice: Momentum is $Q(\mathrm{something})$ where $Q^2 = 0$.
Plausible that $T_{\mu\nu}$ Q-exact too.
Implies that $S = Q(\mathrm{something})$

Keeping $Q^2_{\mathrm{lattice}} = 0$ all that is needed for exact SUSY of action.

$\mathcal{Q} = 4$ SYM in 2D

In twisted form:

$$S = \frac{1}{g^2}\mathcal{Q}\int \mathrm{Tr}\,\left(\chi_{\mu\nu}\mathcal{F}_{\mu\nu} + \eta[\overline{\mathcal{D}}_\mu, \mathcal{D}_\mu] - \frac{1}{2}\eta d\right)$$

$$\begin{aligned}
\mathcal{Q}\,\mathcal{A}_\mu &= \psi_\mu \\
\mathcal{Q}\,\psi_\mu &= 0 \\
\mathcal{Q}\,\overline{\mathcal{A}}_\mu &= 0 \\
\mathcal{Q}\,\chi_{\mu\nu} &= -\overline{\mathcal{F}}_{\mu\nu} \\
\mathcal{Q}\,\eta &= d \\
\mathcal{Q}\,d &= 0
\end{aligned}$$

Note complexified gauge field $\mathcal{A}_\mu = A_\mu + iB_\mu$

Action

Q-variation, integrate d:

$$S = \frac{1}{g^2} \int \mathrm{Tr}\, \left(-\overline{\mathcal{F}}_{\mu\nu}\mathcal{F}_{\mu\nu} + \frac{1}{2}[\overline{\mathcal{D}}_\mu, \mathcal{D}_\mu]^2 - \chi_{\mu\nu}\mathcal{D}_{[\mu}\psi_{\nu]} - \eta\overline{\mathcal{D}}_\mu\psi_\mu \right)$$

Rewrite as

$$S = \frac{1}{g^2} \int \mathrm{Tr}\, \left(-F_{\mu\nu}^2 + 2B_\mu D_\nu D_\nu B_\mu - [B_\mu, B_\nu]^2 + L_F \right)$$

where

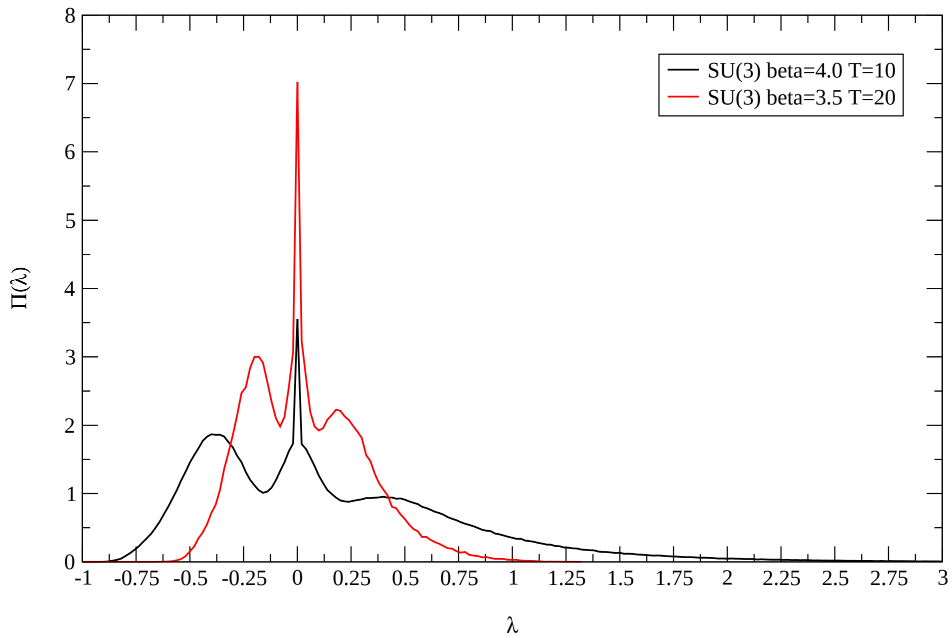
$$L_F = \begin{pmatrix} \chi_{12} & \frac{\eta}{2} \end{pmatrix} \begin{pmatrix} -D_2 - iB_2 & D_1 + iB_1 \\ D_1 - iB_1 & D_2 - iB_2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Lattice ?

- $\mathcal{A}_\mu(x) \to \mathcal{U}_\mu(n)$. Complexified Wilson links.
- Natural fermion assignment – η on sites, ψ_μ links, χ_{12} diagonal links.
- Well-defined prescription for $\mathcal{D}_\mu \to \Delta_\mu^{+/-}$
- Lattice action: real positive def, gauge and $\mathcal{Q}$ invariant.
- Possible to do dynamical Monte Carlo simulations.
  - D=2 periodic and antiperiodic fermion bc.
  - Dimensionally reduce: D=1 theory with 2 exact SUSY's

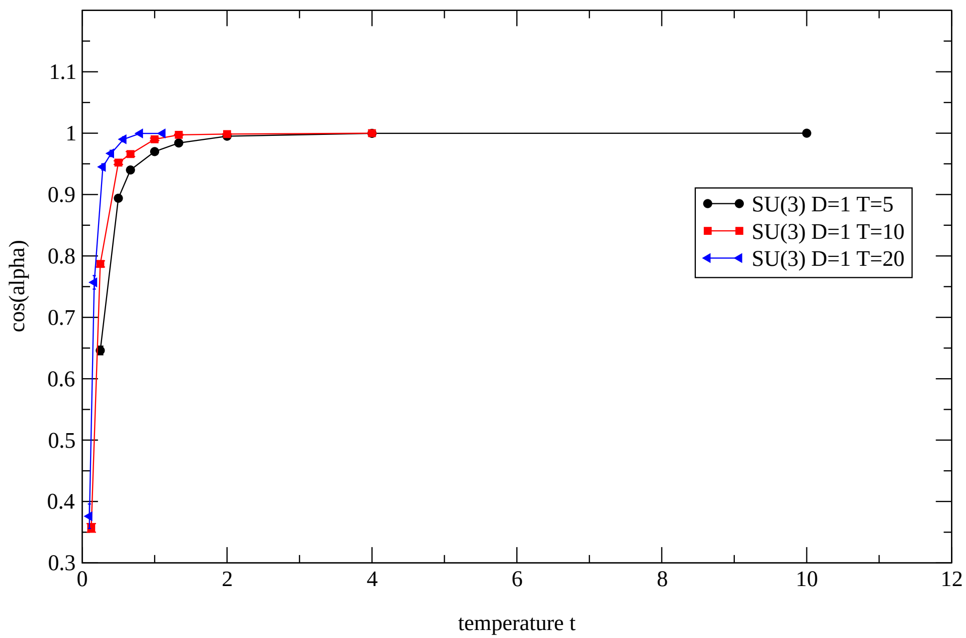
Vacuum ?

Flat directions $[B_1, B_2] = 0$?



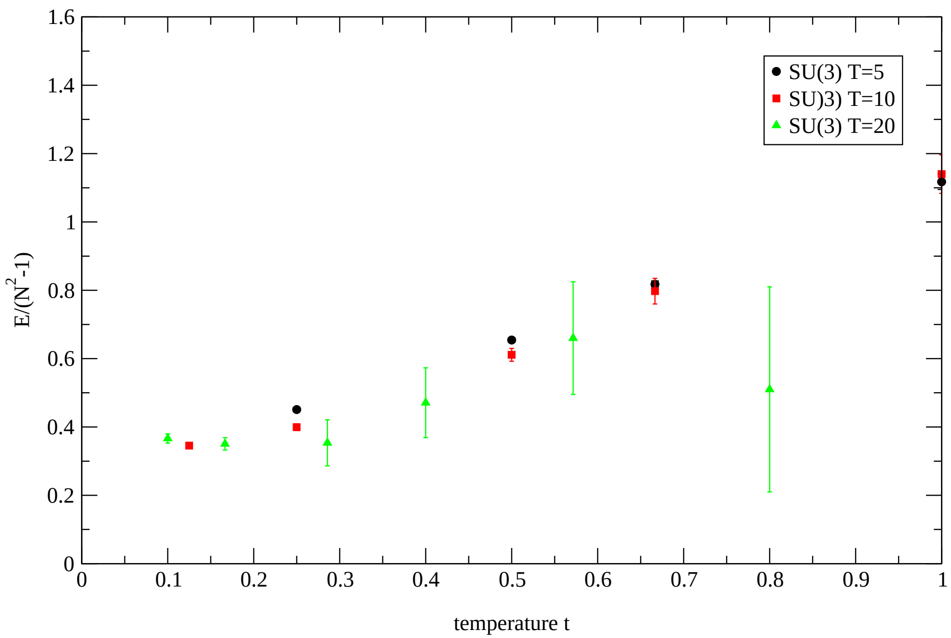
D=1 thermal runs. $e^{2aB_\mu} = \mathcal{U}_\mu^\dagger \mathcal{U}_\mu$
As $a \to 0$ width of scalar distribution finite and determined by continuum (dimensionless) coupling β

Phase of Pfaffian



Write $\mathrm{Pf} = e^{i\alpha}|\mathrm{Pf}|$ Handle by reweighting. Thermal case $\alpha \to 0$ as $a \to 0$.

SUSY breaking in $D = 1$ SYM



In limit $a \to 0$ vacuum expectation value of $E/\lambda^{\frac{1}{3}}$ does not go to zero – SUSY breaking!
Non-trivial since hard to even define Witten index for theory with infinite number of vacua ...

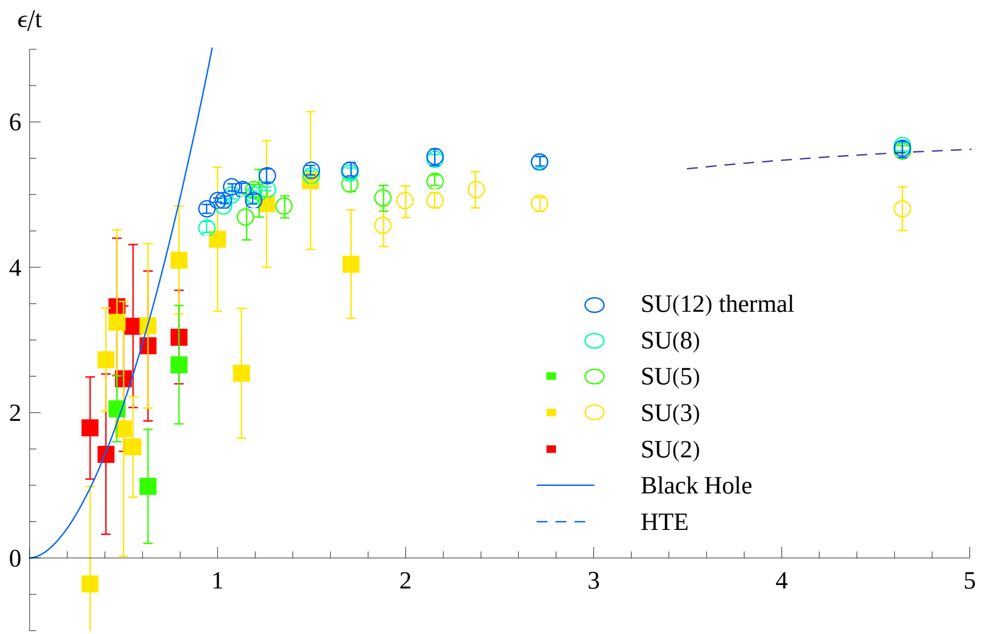
$\mathcal{Q} = 16$ SYM in 4D

- Symmetry:$SO_{\rm Lorenz}(4) \times SO_{\rm R}(4)$
- Again after twisting regard fermions as 4×4 matrix.
- Trace of supercharge matrix yields nilpotent supercharge $\mathcal{Q}$.
- Twisted action contains $\mathcal{Q}$-exact piece formally identical to $D = \mathcal{N} = 2$ case plus a new $\mathcal{Q}$ closed term.
- Lattice discretization follows as for $D = \mathcal{N} = 2$ – $\mathcal{A}_a \to \mathcal{U}_a$.

Details

- Naturally formulated as 5D theory $\mathcal{A}_a$, $(\eta, \psi_a, \chi_{ab})$ where $a, b = 1 \ldots 5$
- After dimensional reduction to 4D – $\mathcal{A}_5$ plus imag parts of $\mathcal{A}_\mu, \mu = 1 \ldots 4$ yield 6 scalars of $\mathcal{N} = 4$
- Fermions: $\chi_{ab} \rightarrow \chi_{\mu\nu} \oplus \overline{\psi}_\mu$, $\psi_a \rightarrow \psi_\mu \oplus \overline{\eta}$
- Twisted action reduces to Marcus twist of $\mathcal{N} = 4$. Equivalent to usual theory in flat space.
- AdSCFT – dual to type II strings with D3-branes. Bigger conjectured duality between (p+1)SYM and type II with Dp-branes.

Supergravity Black Hole from $D = 1$ SYM



Low energy string –> supergravity

Large N –> classical Now $E \to 0$ as $t = T/\lambda^{\frac{1}{3}} \to 0$

Matches energy of black hole solution in type IIa supergravity at low temperature (strong coupling) ! (S.C, T.Wiseman)

Summary so far

- Contrary to folklore can build lattice actions invariant under (some) SUSY.
- Exploit twisted formulations (or orbifold ...) with scalar supercharge and Q-exact actions.
- Reduce/eliminate fine tuning.
- Simulations of $\mathcal{N} = 4$ SYM feasible using modern algorithms from lattice QCD and current hardware.

Dynamics of gauge theories

At fixed N_c and small N_f gauge theories thought to

- Confine and break chiral symmetry
- Exhibit asymptotic freedom in U.V

As N_f raised theoretical arguments suggest

- Critical $N_f = N_f^a$ theory develops I.R stable fixed pt. Conformally invariant. Chiral symmetry restored.
- Increase N_f further find another threshold $N_f = N_f^b$ loss of asymptotic freedom.

$N_f^{a,b}$ depend on repn. of fermions

(Sannino et al)

Why SU(2) with adjoints ?

- Estimates based on resummed perturbation theory indicate $N_f^a \sim 2$ for this theory.
- Need to check with non-perturbative calc.
- If $\beta(g) = 0$ conformal theory may be useful as unparticle sector (Georgi)
- If near conformal $\beta(g) \sim 0$ over range scales – coupling walks, mass scale small, dynamics very different from QCD.
- Since N_f small – walking technicolor models built from this theory may not be ruled out by precision EW data.

Lattice theory

- Use standard Wilson plaquette action for gluons, Wilson fermion action for quarks with replacement

$$U_\mu(x) \rightarrow \mathrm{Tr}(S^a U_\mu(x) S^b U_\mu^\dagger)$$

with

$$\{S^a\}, a = 1, 2, 3$$

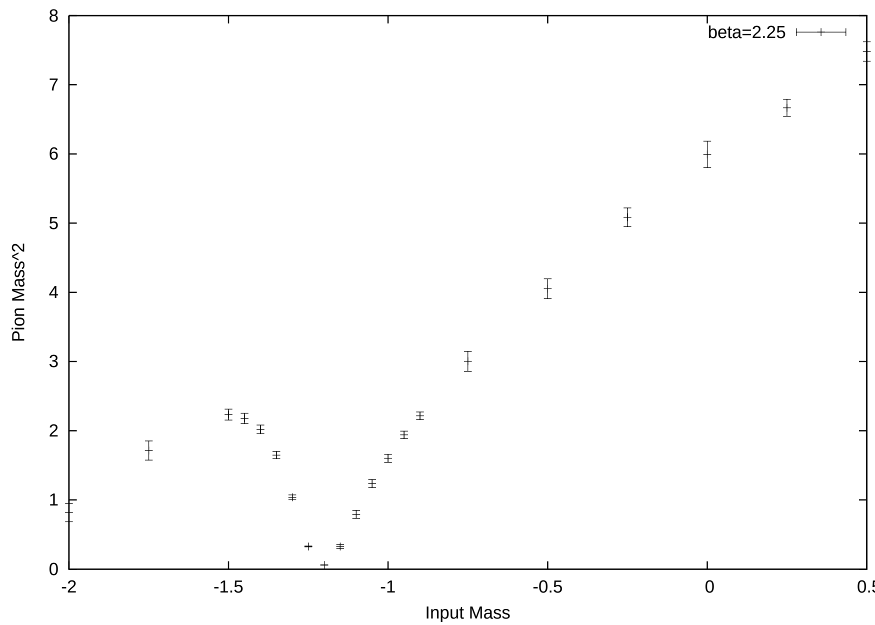
2×2 basis for adjoint.

- First thing – determine 2D phase diagram (β, ma).
- Continuum limits obtained near critical line(s).
- Initial work used small lattices $4^3 \times 8$, sampled phase diagram at coarse resolution contrasting behavior with fundamental quarks (S.C, F. Sannino, Phys. Rev. D76:034504,2007)

New (preliminary) results

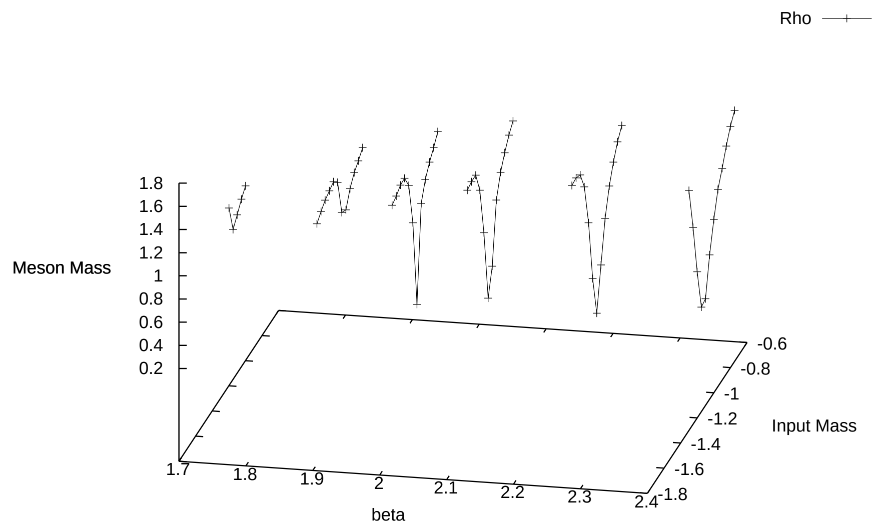
- $8^3 \times 16$ lattices. High resolution of critical region O(100) points in (β, ma) parameter space. Higher statistics.
- Collaboration with Giedt, Sannino and Schneible. Parallel code on IBM BlueGene-L at RPI.
- Examining mesons, string tension and gluonic observables.

Mass scan



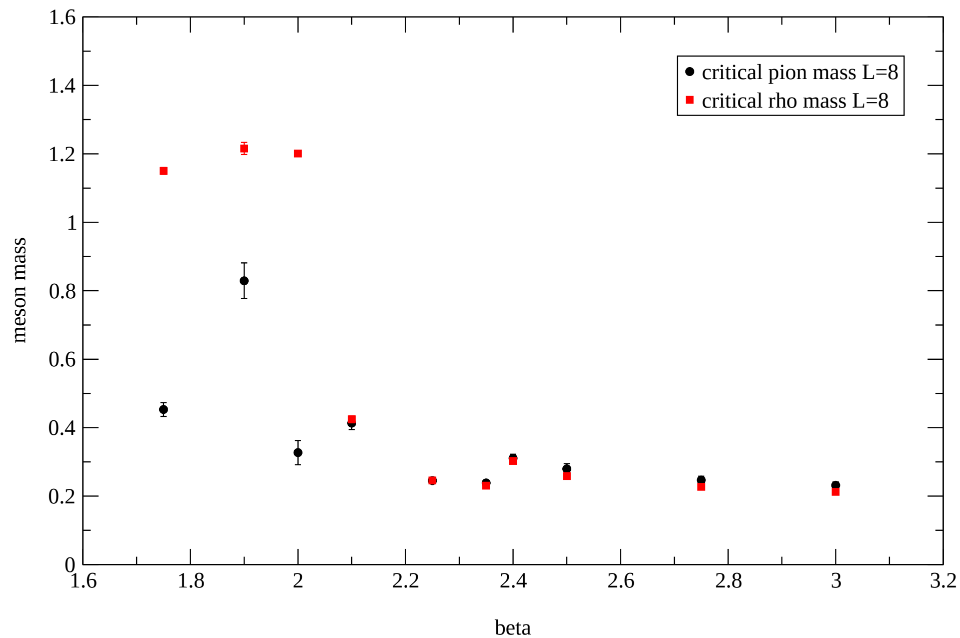
Note: $m_c(\beta)$ determined from minimum pion (rho) mass.
Note: Goldstone behavior ?

3D meson mass plot



Have simulated theory at O(100) points in the (β, ma) parameter space

Meson masses



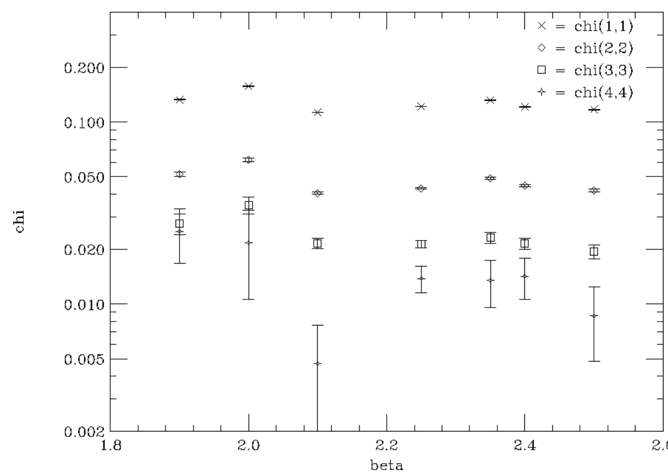
$m_\rho(\beta < 2) \sim 5 m_\rho(\beta > 2)$

m_π degenerate with m_ρ for $\beta > 2$ and independent of bare coupling!

Mass scaling like $1/L$ at weak coupling.

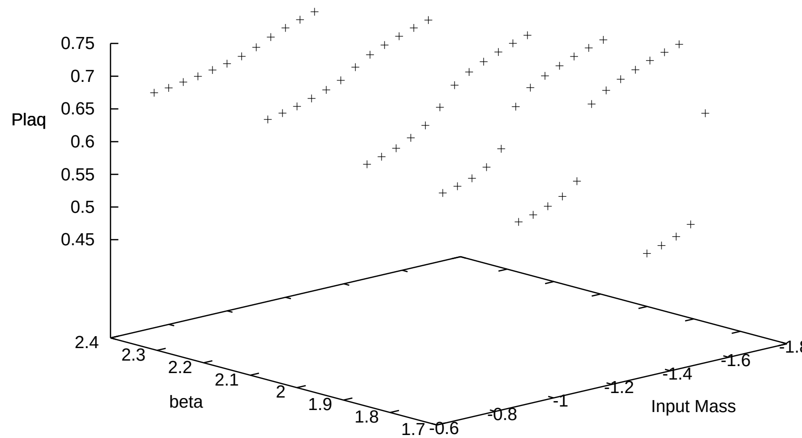
$m_\pi(L = 4)/m_\pi(L = 8) \sim 2$

String tension



Creutz ratios used to estimate σa^2
String tension independent of bare coupling $\beta > 2$
Flows to small values as length scale increased.

Gluon condensate



Discontinuity develops for small β – first order phase transition ?

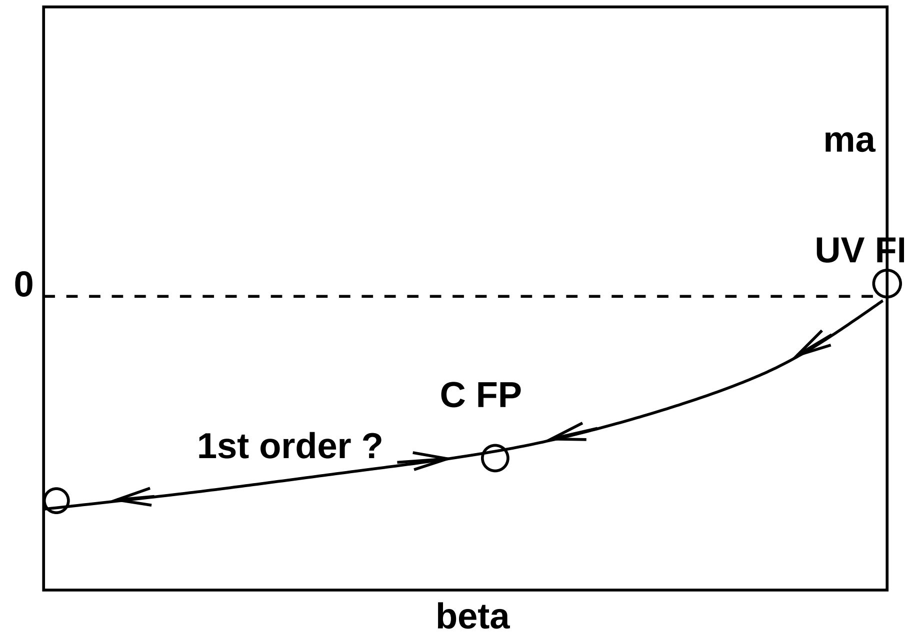
Interpretations(s)

Strong coupling: ρ heavy, first order transition. Lattice artefacts ..
Weak coupling: light masses, small string tension, degenerate π, ρ indep of bare coupling. Explanations:

- Near zero of β-function – finite change in coupling yields very large scale change. Small lattice spacing. Perturbative deconfined regime with small masses.
- Nearby conformal fixed pt – light mass scale independent of bare coupling. Small string tension.

Distinguish ? Larger lattices, asymmetric lattices – use finite size effects to extract anomalous dimension ? Hard!

Cartoon of RG flows



First order line or point ? Strong coupling lattice phase ?

Summary

- Technicolor models with non-standard repns. for fermions may yet offer conservative ways of breaking EW theory.
- Need to check with lattice simulations.
- Current algorithms/hardware make exploratory calcs quite feasible.
- Focused on $SU(2)$ model with $N_f = 2$ adjoint Dirac fermions.
- Mapping phase diagram – seems to be just 1 critical line. Two regimes – strong and weak coupling. First order phase transition between ?
- Behavior of meson masses, string tension difficult to interpret but consistent with near conformal behavior ... Strongly coupled ?