Peskin Takeuchi S-parameter on 2+1 QCD DW lattices†

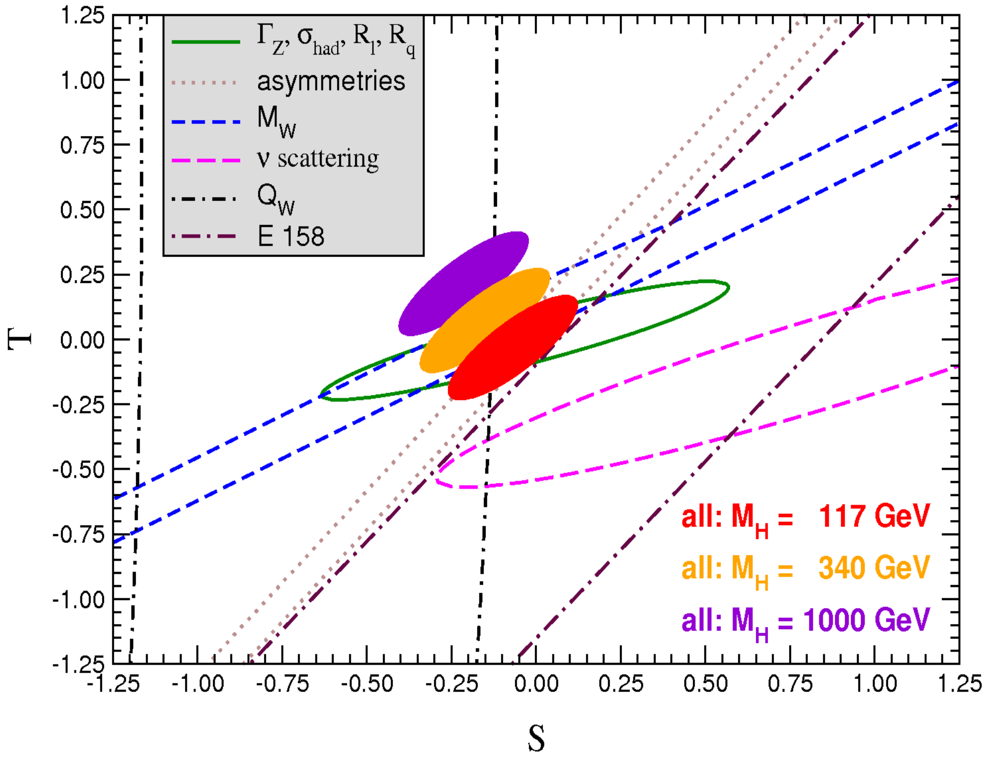
Rich Brower

- Estimates for "technicolor" are base on scaling fromQCD.
- (Modest) goal: Evaluate S for QCD (or minimal technicolor).
  - Establish lattice methods
  - Test phenomenological rationale for S parameter models.
- Other uses of Euclidean correlators: $\Pi(Q^2) =$ FT $<J_\mu(x)\, J_\mu(0)>$
  - Hadronic contribution to g-2
  - Determine "warping" for AdS/CFT models: $Q \simeq 1/z$
  - See Ami Katz (Inverse problems unique at $N_c = \infty$?)
- Comment on Application of Disconnected project to SUSY dark matter
  - Largest uncertainty is Nucleon strange sigma term: $m_s <N|\, s\, s\, |N>/M_N$

† *LSD collaboration using QCDOC lattices*

S-parameter

Precision Electro-weak data



N_c=3,nf =2 Gasser Leutwyler

$$S_{QCD} = \frac{1}{12\pi}[\widetilde{l}_5(\mu) + ln(\frac{\mu^2}{M_H^2}) - \frac{1}{6}]$$

$$-Q^2\Pi(-Q^2) \simeq f_\pi^2 + 4l_5Q^2 - 8C_{87}Q^4 + \cdots$$

$$-Q^2\Pi(-Q^2) \simeq 4\pi\alpha_s\frac{\langle\bar{\psi}\psi\rangle}{Q^2}(1 + O(\alpha_s\log(Q^2)) + \cdots$$

at $m_\pi = 0$,
tree approx.

where

$$i\int dxe^{iqx}\langle J_\mu^R(x)J_\nu^L(0)\rangle = (q_\mu q_\nu - q^2g_{\mu\nu})\Pi(q^2)$$

S parameter[†] = S_{strong} - S_{SM}

$$S(m_{H,ref}) = 4\int_\epsilon^\infty \frac{dq^2}{q^2}\left[Im\Pi_{VV}(q^2) - Im\Pi_{AA}(q^2)\right] - \frac{1}{12\pi}\int_\epsilon^\infty \frac{dq^2}{q^2}\left[(1-(1-m_{H,ref}/q^2)^3\theta(q^2-m^2_{H,ref})\right]$$

Using Dispersion Rel: $$q^2\Pi(q^2) = q^2\int \frac{ds}{\pi}\frac{Im\Pi_{VV}(s) - Im\Pi_{AA}(s)}{s-q^2} - f_\pi^2$$

Naive Scaling Models.

ρ/A_1 (Weinberg sum rules) ==> $$S \simeq 0.25\frac{N_f}{2}\frac{N_c}{3}$$

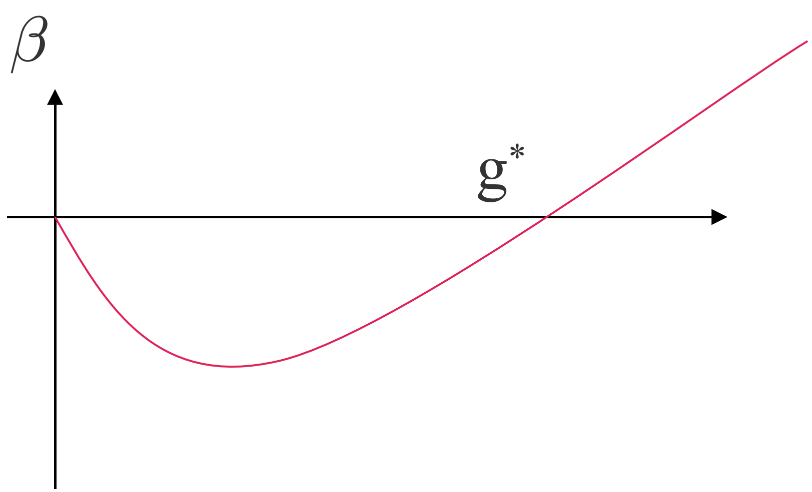
χ PT (leading logs) ==> $$S \simeq \frac{1}{12\pi}[\frac{N_f^2}{4} - 1]\ln(\frac{m_\rho^2}{m_{PGB}^2})$$

† "Estimation of oblique electroweak corrections" Peskin and Takeuchi, PR D46 (1992) 381

But "larger" n_f dependence of $S(N_c,n_f)$ must be non-trivial.

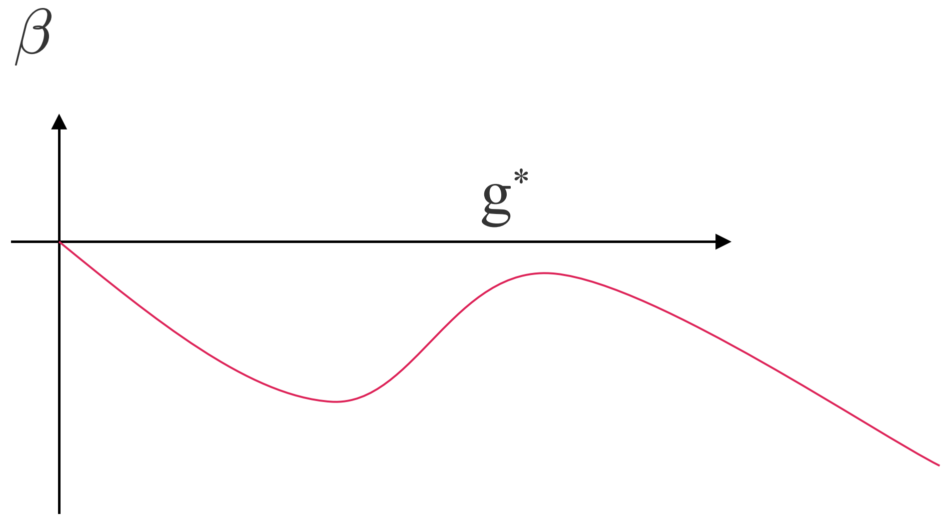
(An IR fixed point at the two-loop β function. Gross and Wilczek, Banks and Zaks, ...)

- Conformal: $n_f > n^*_f$
- Chiral $SU(n_f) \times SU(n_f)$.



Asymptotical free in the UV but no confinement.

- Walking: $n_f < n^*_f$
- Broken $SU(n_f) \times SU(n_f)$

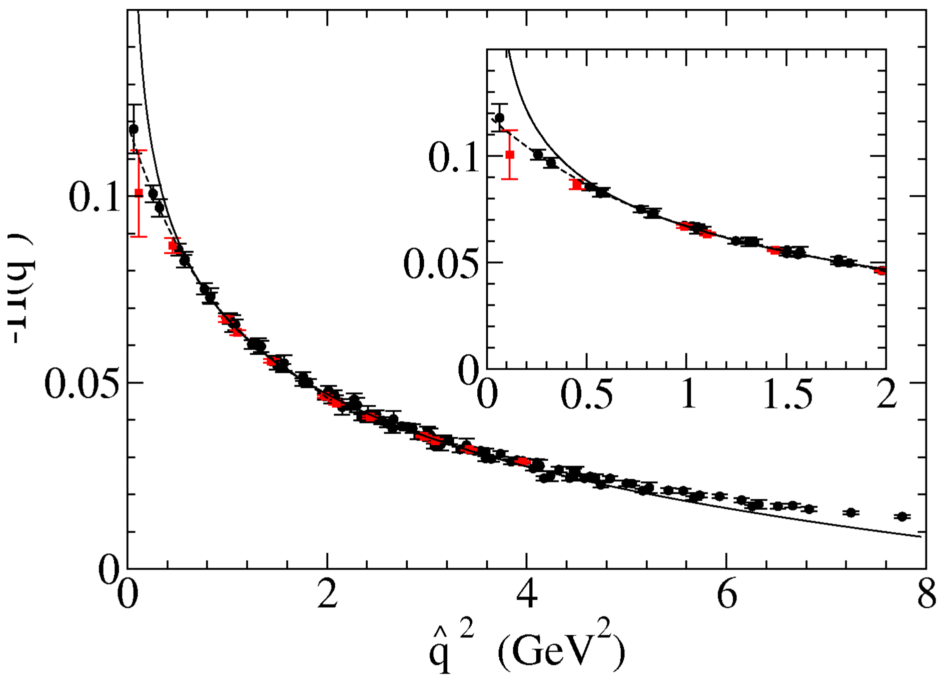


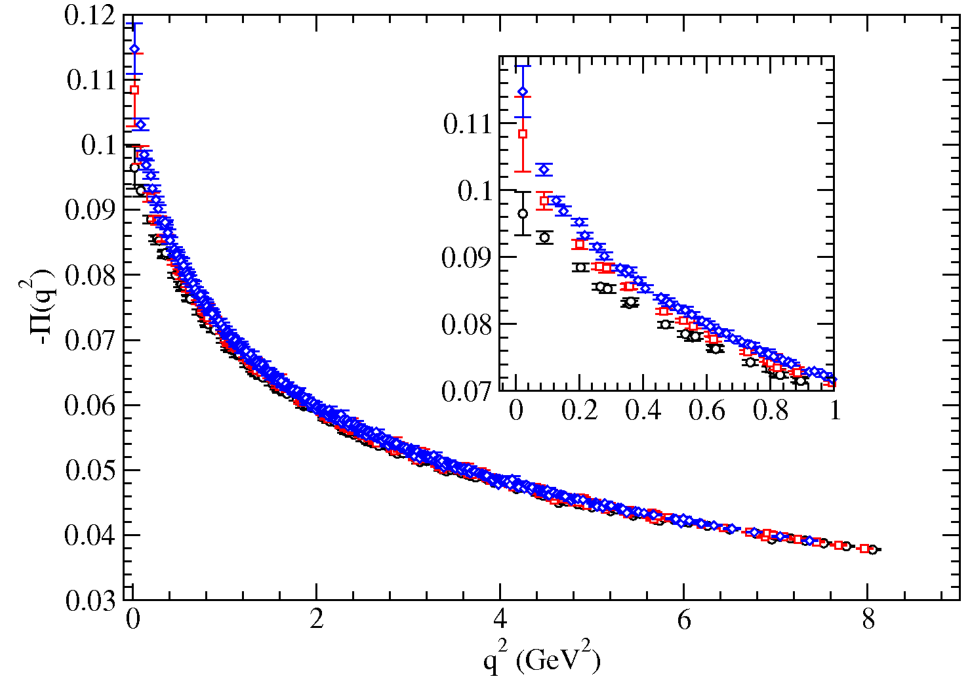
Confinement with new low energy scale.

Pioneering work on $a^{had}_{\mu} = (g-2)/2$

- Quenched DW: T. Blum
  hep-lat/0210018:
  8^3 x24 and 16^3x32
  (L_s =8)

- Staggered:Aubin and Blum
  hep-lat/0608011: 28^3x40 and 40^3x96
  (single lattice spacing, 3 quark
  masses)



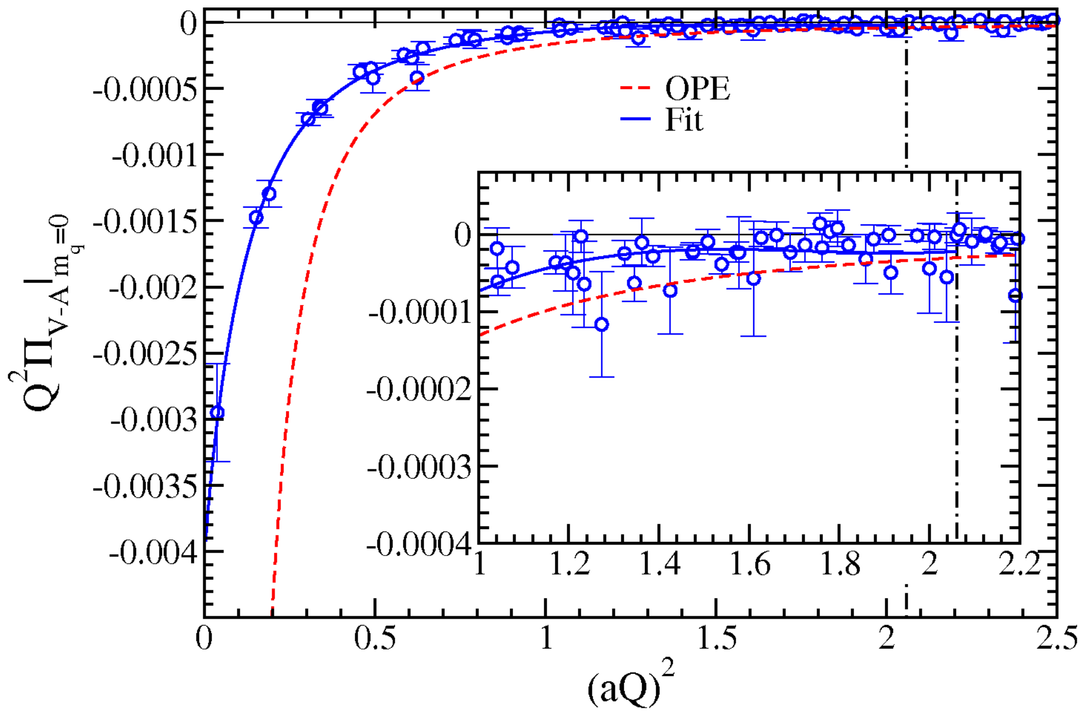


$a^{(had)}_{\mu} = 460(78) \times 10^{-10}$

$a^{(had)}_{\mu} = 713(15) \times 10^{-10}$

JLQCD† calculation of EM $\Delta\, m^2_\pi$ and $Q^2\, \Pi_{VA}$ for S

(Overlap Fermions using non-conserved currents with $Z_A = Z_V = 1.38$)

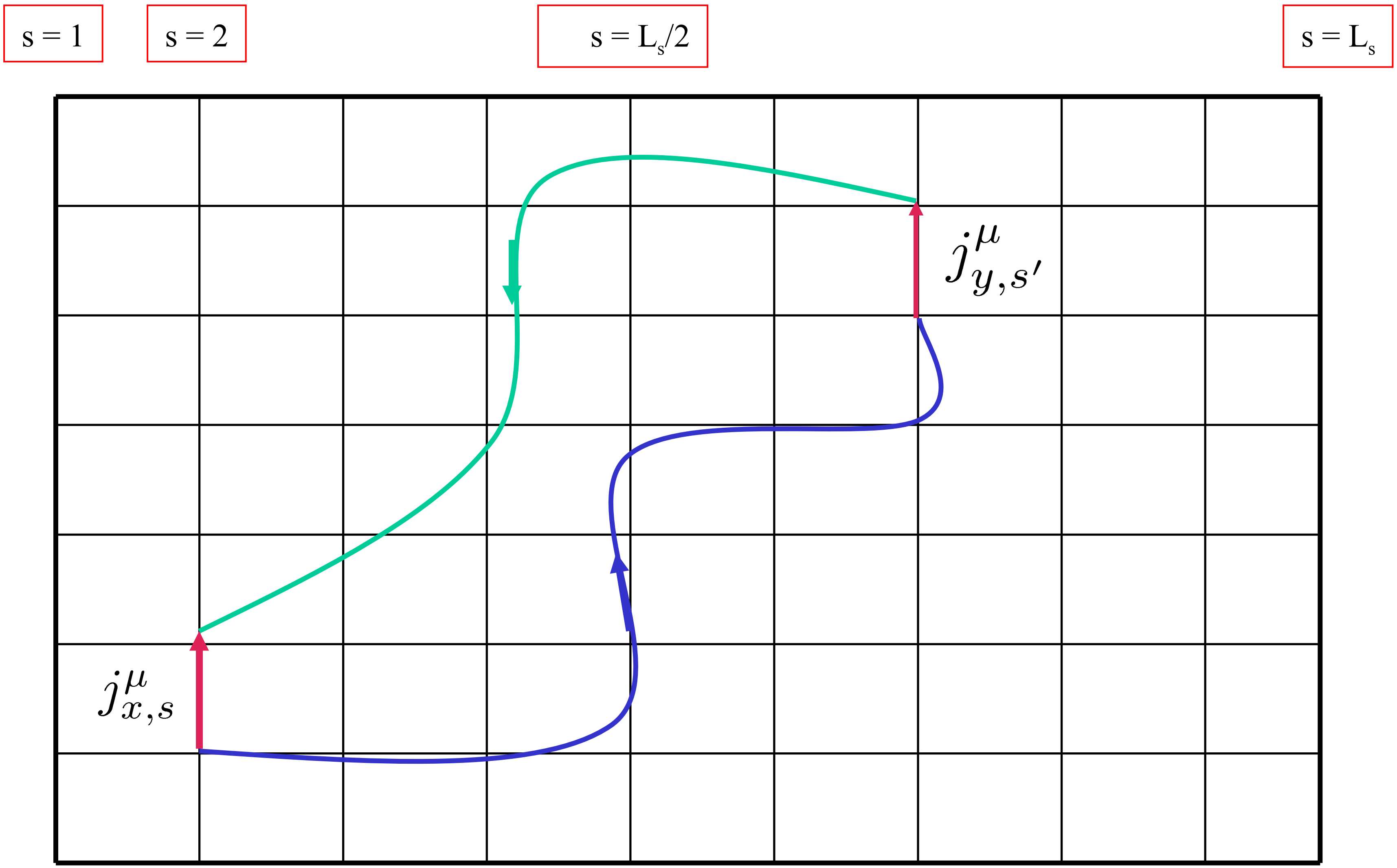


$$L_{10}(\mu) = -0.00474(23) \text{ vs exp } -0.00509(47)$$

200 config. $16^3 \times 32$ lattice., $m^2_\pi = 0.27 - 0.082\ \mathrm{Gev}^2$

† Shintani et al PoS (Lattice 2007) 137 using Dass-Guralnik-Mathur-Low-Young sum rule.

Domain Wall Currents



$$j^{\mu}_{x,s} = \frac{1}{2}[\bar{\Psi}_{x,s}(\gamma_\mu - 1)U_\mu(x)\Psi_{x+\mu,s} + \bar{\Psi}_{x+\mu,s}(\gamma_\mu + 1)U^{\dagger}_{\mu}(x)\Psi_{x,s}]$$

Anti-quark

$$D^{-1}_{x,s;y,s'}(m) = \gamma_5 D^{-1\dagger}_{y,L_s-s';x,L_s-s}(m)\gamma_5$$

Need for Pauli-Villars Subtraction

- Overlap Derivation: $\langle J_\mu(y) J_\mu(x)\rangle \equiv \frac{\delta}{\delta A_\mu(y)} \frac{\delta}{\delta A_\mu(x)} \log[Zov(Ue^{iA})]$ at $A_\mu = 0$

- Overlap ==> DomainWall: $\log[Z_{0v}] = Tr \log[D_{ov}(m)] \equiv$

$$\log[Z_{DW}] = Tr \log[D_{DW}(m)] - Tr \log[D_{DW}(1)]$$

- Equivalent DW expression:

$$\boxed{\langle J_\mu(y) J_\mu(x)\rangle \equiv \sum_{s',s} \langle j^{DW}_\mu(y,s') j^{DW}_\mu(x,s) - j^{PV}_\mu(y,s') j^{PV}_\mu(x,s)\rangle}$$

- With Γ_s = Sign(L_s/2 + 1 -s)

$$\boxed{\langle J^5_\mu(y) J^5_\mu(x)\rangle \equiv \sum_{s',s} \Gamma_s \Gamma_{s'} \langle j^{DW}_\mu(y,s') j^{DW}_\mu(x,s) - j^{PV}_\mu(y,s') j^{PV}_\mu(x,s)\rangle}$$

- BUT cheaper to use local Non-conserved currents on boundary with $Z_V = Z_A + O(m^2_{res})$

SUSY Dark Matter

- Neutralino (SUSY Dark Matter) direct detection
- Scatter of nucleons by t-channel Higgs exchange:
  - Largest uncertainty is strange sigma term:

$$fT = m_s <N|\, s\, s\, |N>/M_N$$

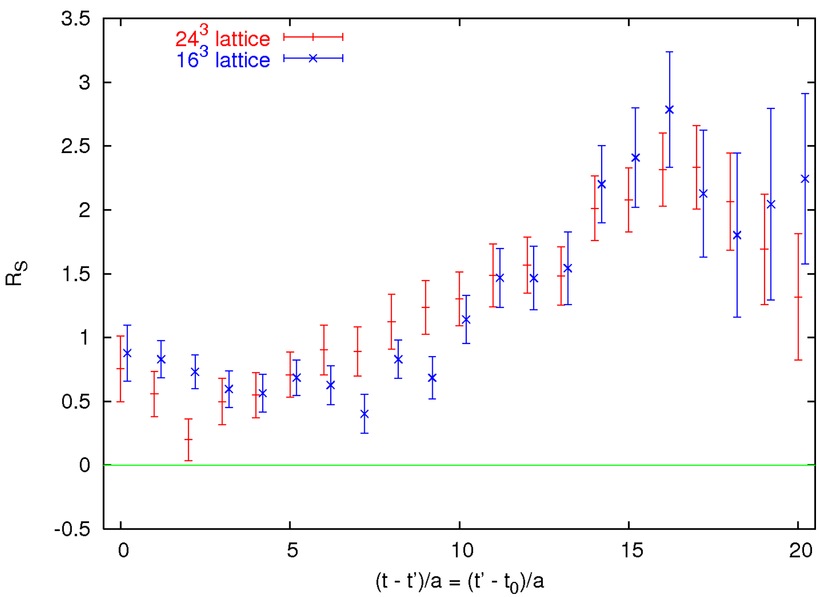
- Need to compute disconnected diagram for strange loop
  - Very expensive but new techniques† (dilution, multi-grid variance reduction, finite volume extrapolation) are working.

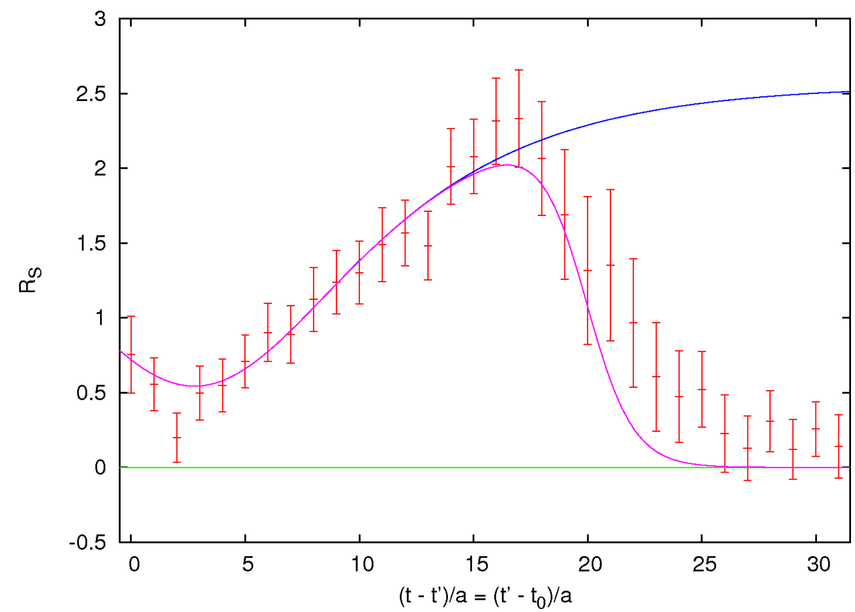
"Determination of Dark Matter Properties at High Energy Colliders" Balitz, Battaglia, Peskin and Wizansky hep-ph./0602187v4

- *"Hadronic Uncertainty in the Elastic Scattering of Supersymmetric Dark Matter" Ellis, Olive and Savage hep-ph/0801.3656v2 "*

† "Strange quark contribution to nucleon form factors", Ronald Babich, Richard Brower, Michael Clark, George Fleming, James Osborn, Claudio Rebbi hep-lat/ 0710.5536

Strange Quark Disconnected Diagram Project





- Strange sigma term in Nucleon $R_s = \langle N(t)\,\mathrm{Tr}[D^{-1}(0)]\,N(-t)\rangle / \langle N(t)\,N(-t)\rangle$

- In the numerator fit to two forward states transition term. In the denominator fit two forward states (blue with $c_B = 0$) plus one backward state (red).

$$R_s = \frac{c_1^2 j_1^2 e^{-2m_1 t} + c_1 c_2 j_{12} e^{-(m_1+m_2)t} + c_2^2 j_2^2 e^{-2m_2 t}}{c_1^2 e^{-2m_1 t} + c_2 e^{-2m_2 t} + c_B^2 e^{-2m_B(L/2-t)}}$$