The Conformal Window in QCD-like Theories

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- "Lattice Study of the Conformal Window in QCD-like Theories" (Thomas Appelquist, George T. Fleming, EN.) PRL 100, 171607 (2008).

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- Theories in the second row of the table lie in the conformal window. Perturbation theory gives some insight, but near N_f^c the fixed point coupling is too strong $\Rightarrow$ non-perturbative study is required.

Estimates of N_f^c

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- However, previous lattice investigation of the conformal window (Iwasaki et al, PRD 69, 014507, 2004) claims the result $6 < N_f^c < 7$.

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Simulate here!

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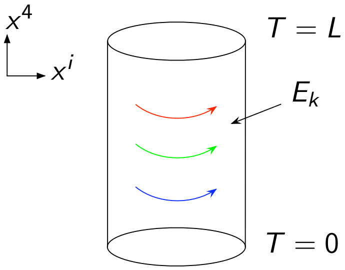
- Define the step-scaling function,

$$\Sigma(2, \overline{g}^2(L), a/L) \equiv \overline{g}^2(2L) + O(a/L)$$

The continuum limit $\sigma(2, u) \equiv \lim_{a \to 0} \Sigma(2, u, a/L)$ is basically a discretized version of the β-function.

The Schrödinger Functional



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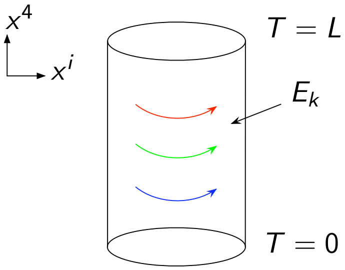

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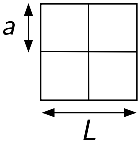
Running coupling

The SF running coupling $\overline{g}^2(L)$ is defined to vary inversely with the response of the action to the strength η of the background field,

$$\frac{dS}{d\eta} = \left.\frac{k}{\overline{g}^2(L)}\right|_{\eta=0}.$$

Step scaling

$u \equiv \overline{g}^2(L)$



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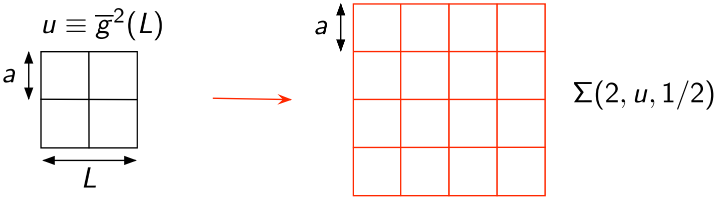

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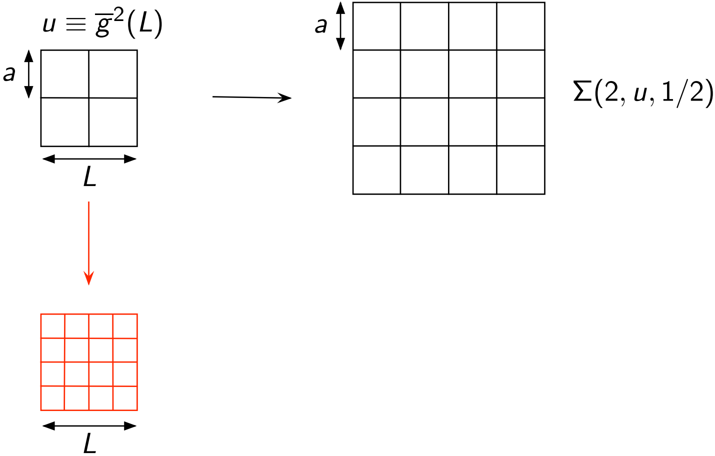

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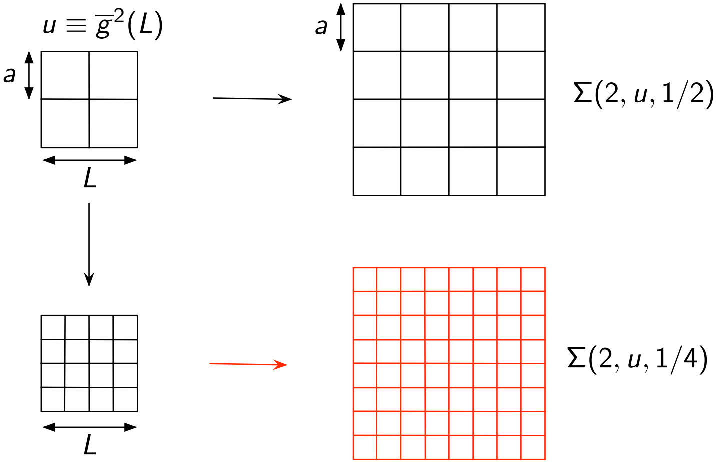
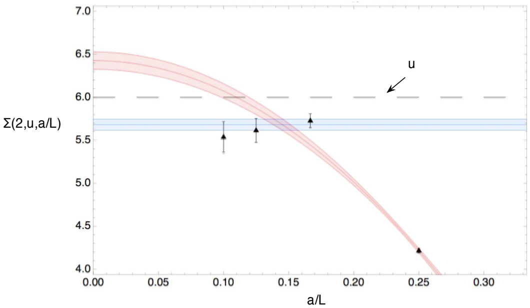


Continuum extrapolation



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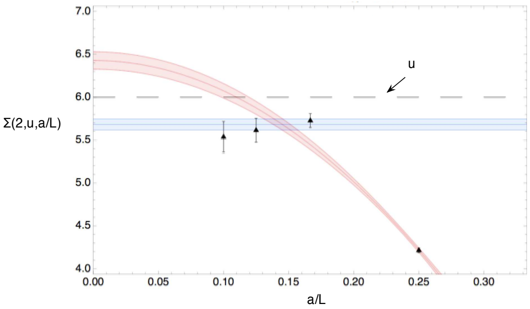

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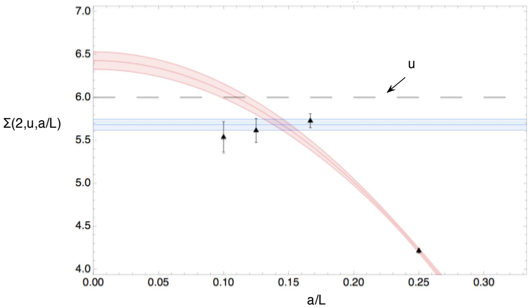

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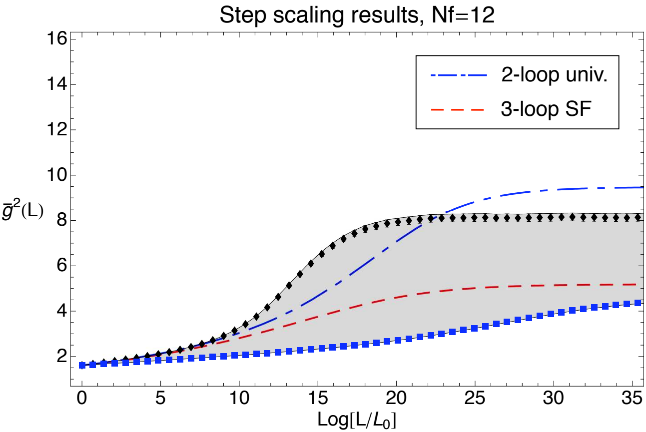
Any reasonable continuum extrapolation should be bounded by the two methods shown above, so we take them to define a systematic error band.

Results, $N_f = 8$ and 12



Apparent IR fixed point.

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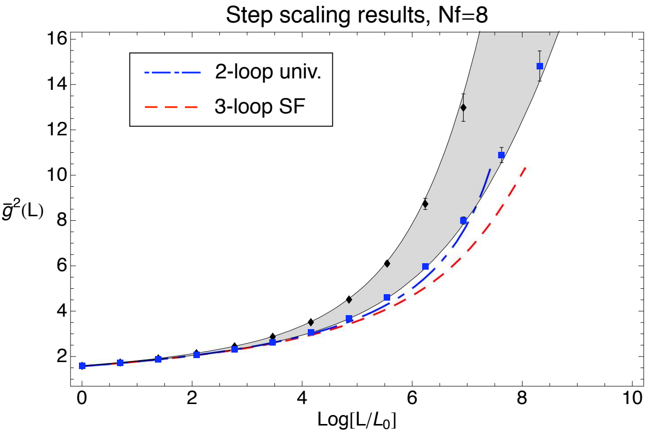


No evidence of a fixed point $\Rightarrow$ $8 < N_f^c < 12$.

Conclusions

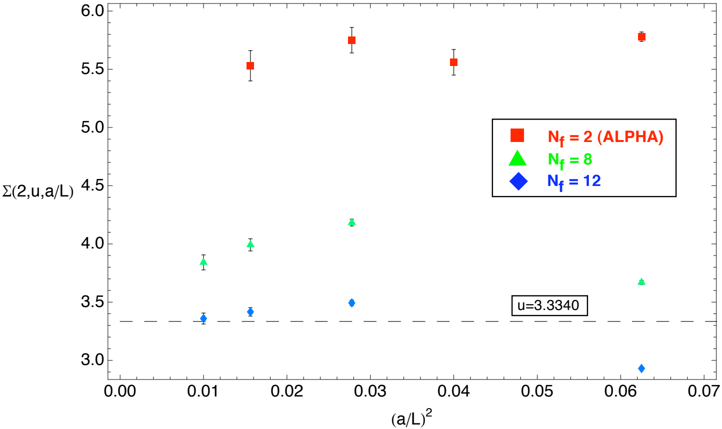
Summary

- We have constrained the lower boundary of the conformal window: $8 < N_f^c < 12$.

Future work

- Continued simulations at 8 and 12 flavors, to reduce systematics.
- Study of running coupling at $N_f = 10$ (allocation granted by USQCD)
- Dynamical simulation at $N_f = 8$, to verify the presence of chiral symmetry breaking.
- Simulation at other values of N_c.

Data comparison with ALPHA



(Ref: Della Morte et. al. (ALPHA), hep-lat/0411025, NPB 713 (2005) p.378.)