Domain Wall Lattice Super-Yang Mills

Joel Giedt

Physics, Applied Physics & Astronomy

Rensselaer Polytecnic Institute

SUSY promo

Weak scale SUSY has a lot going for it: gauge coupl unif. / hierarchy prob. / CDM candidate / logical spacetime symm. extension / flat dir.'s –> baryogensis / perturb. calculable TeV physics. / natural consequence of string/M th. / custodial SU(2) / severely constrained by rare processes / elegant renormalization / light higgs

Rubber meets the road

- Of course it has drawbacks too. SUSY has to be broken somehow.
- This is encoded in the “soft Lagrangian”, introducing a vast parameter space, with generic points absolutely forbidden [EDM’s, FCNC’s].
- The search, for many years, has been for mechanisms of SUSY-breaking that “naturally” lead to a “nice” soft Lagrangian.
- (This is getting progressively harder as experiments nibble away at parameter space in simpler scenarios.)

Strong supersymmetric theories

- The models for SUSY breaking generally involve nonperturbative behavior of supersymmetric gauge theories.
- Thus, to thoroughly study the relevant nonperturbative features $\Rightarrow$ study strong SUSY.
- Aims:
  - Look for unexpected features.
  - Obtain complimentary evidence for continuum results.
  - Develop an alternate computational tool.

Motivations for LSYM

1. Kovner-Shifman metastable state? [Douglas, Shelton, Torroba 07]
2. "No vacuum" problem ($m_Q \to 0$) – runaway.
3. Indep. prf. of gluino cond., ck. $\langle G_{\mu\nu} G_{\mu\nu} \rangle$
4. Quenched SQCD: metastable SUSY-breaking, TeV strong SUSY.
5. Spectrum, EFT, Nonperturb. defn.
6. Working adjoint fermion SU(2) parallel code:
   – simulation code complete;
   – several interesting observables working;
   – some gauge-fixing hacks to wrap up.

Lattice SUSY

- However, lattice SUSY has problems.
- Lattice breaks SUSY: discretization errors.
- Divergences in quantum field theories $\Rightarrow$ errors can be dangerously amplified:

$$\epsilon \times \infty = \infty. \tag{1}$$

- For example:

$$QS = a^2 \mathcal{O}_S, \quad \langle \mathcal{O}_S \mathcal{O}_X \rangle = \mathcal{O}(1/a^2) \tag{2}$$

$\Rightarrow \mathcal{O}(\ln a)$ violation of SUSY.

Various tacks

- For a few years now, I have studied ways that exact lattice symmetries might be used to overcome this problem, with many encouraging results.
- We are making steady progress toward full-scale simulation of realistic models.
- Today I'll tell you about a supersymmetric model that we can study w/o fear, due to lattice chiral symmetries.
- Some emerging results of very-large-scale simulations will be presented.

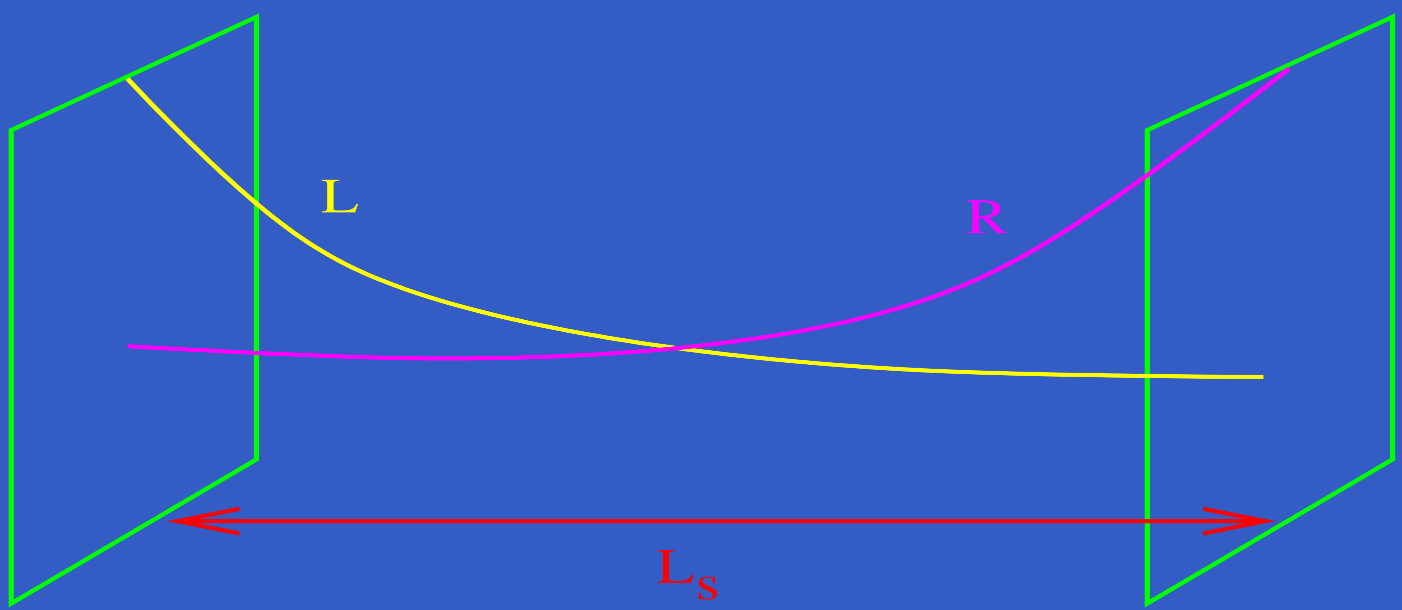
$\mathcal{N} = 1$ 4d SYM w/ chiral fermions

- From [Curci, Veneziano 86] we know that $\mathcal{N} = 1$ 4d SYM with Ginsparg-Wilson fermions require no counterterms.
- Overlap-Dirac was proposed [Narayanan, Neuberger 95] and sketched [Maru, Nishimura 97].
- But LO simulation studies, such as glueball spectra, have yet to be attempted.

$\mathcal{N} = 1$ 4d SYM w/ chiral fermions

Another type of GW fermion is:

- Domain Wall Fermions (DWF) [Kaplan 92] + improvements [Shamir 93].
- Proposed for LSYM [Nishimura 97] [Neuberger 98] [Kaplan, Schmaltz 99]
- Briefly studied [Fleming, Kogut, Vranas 00].



Gluino condensation

- They studied the condensate vs. L_s
- Why is that interesting?
- All the good ideas for spontaneous SUSY-breaking (that I know of) involve gluino condensation:

$$\langle \bar{\lambda}\lambda \rangle \neq 0 \tag{3}$$

Gluino condensation

Essentially 3 types of evidence from continuum techniques:

1. VY/chiral ring
   [Veneziano, Yankielowicz 82;
   Cachazo, Douglas, Seiberg, Witten 02]
2. strong instanton
   [Novikov, Shifman, Vainshtein, Zakharov 83]
3. weak instanton
   [Affleck, Dine, Seiberg 83]

VY/chiral ring

- $S \sim \text{Tr } \mathcal{W}^\alpha \mathcal{W}_\alpha = \text{Tr } \lambda^\alpha \lambda_\alpha + \cdots$.
- $W = S(1 - \ln S)$ unique. [VY]
- $S^{N_c} = \Lambda^{3N_c}$ quantum modified operator relation on ground state. [Cachazo et al.]

Strong instanton [NSVZ 83]

- Saturation of zero-modes of 1-instanton config. using N_c chiral bilinears:

$$\langle \lambda\lambda(x_1) \cdots \lambda\lambda(x_{N_c}) \rangle \neq 0 \tag{4}$$

- Lowest components of SUSY mult.'s ==> x_i indep.
- Cluster decomp. ==> $\langle \bar{\lambda}\lambda \rangle$ nonzero.
- (NB: FKV touched on conjecture of fractional instanton as mechanism, but further study needed.)

Weak instanton

- Starting from super-QCD with $N_f = N_c - 1$, nonperturbative $W_{ADS} \neq 0$ well-established [Affleck, Dine, Seiberg 83].
- By going out on flat dir.'s $Q = \tilde{Q} \neq 0$ to remove all flavors, one can match the unique superpotential at thresholds.
- Find $W = \Lambda^3$, generating func. for $\langle \bar{\lambda} \lambda \rangle$ ==> nonzero.

A 4th pathway

Seems to me, the above approaches are rather indirect.

- We are computing the gluino condensate, directly, by brute force.
- Due to the lattice discretization, it is important to
  - simulate at various a,
  - so that an extrapolation to the physical theory can be made.
- Also important: extrapolation to first order transition point $m_\lambda = 0$.

Chiral critical point

- It is at this point that spontaneous breaking of the Z_{2N_c} symmetry occurs.
- N_c vacua: the theory picks one spontaneously.
- Old-fashioned lattice fermions (Wilson) broke this symmetry to avoid fermion doublers.
- Due to additive renormalization

$$m_{\lambda,R} = \sqrt{Z} m_{\lambda,0} - \delta m_\lambda \tag{5}$$

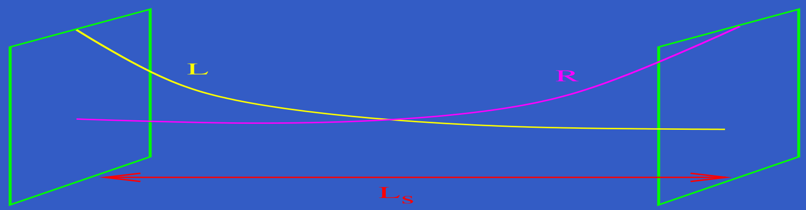
it was impossible to say *a priori* where $m_{\lambda,R} = 0$ really was.

Stumbling in the dark

- Perturbation theory won't tell us $\sqrt{Z}, \delta m_\lambda$.
- The old simulations (Munster-DESY-Roma) tried various masses $m_{\lambda,0}$.
- Very costly (scan, renorm., op. mixing, coding).
- Did not generate enough data to do $a \to 0$ extrapolations. Only 1 lattice spacing.

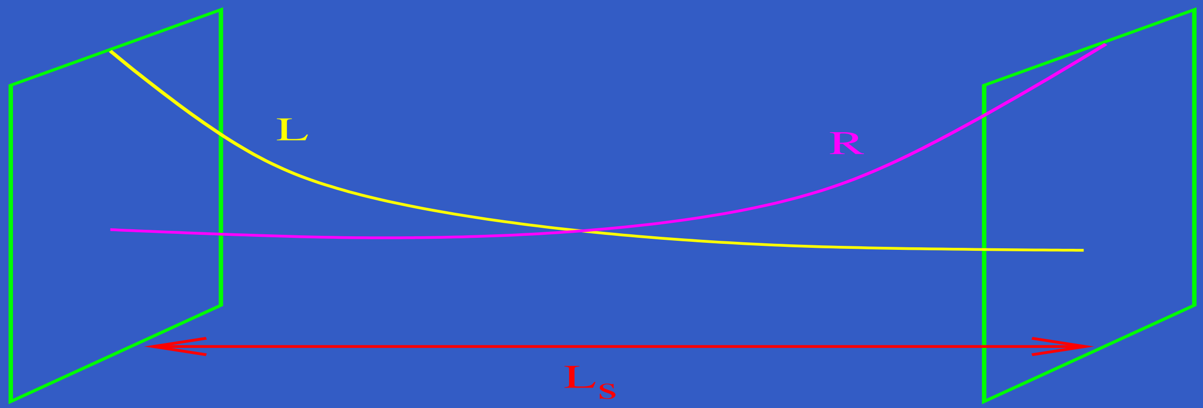
First foray into 5th dimension

- The old DWF simulations [Fleming, Kogut, Vranas 00] avoided fine-tuning. But sim.'s costly ==> small lattices.
- Did not generate enough data to do $a \to 0$ extrapolations. Only 1 lattice spacing.
- Small lattice ==> far from continuum. SUSY?

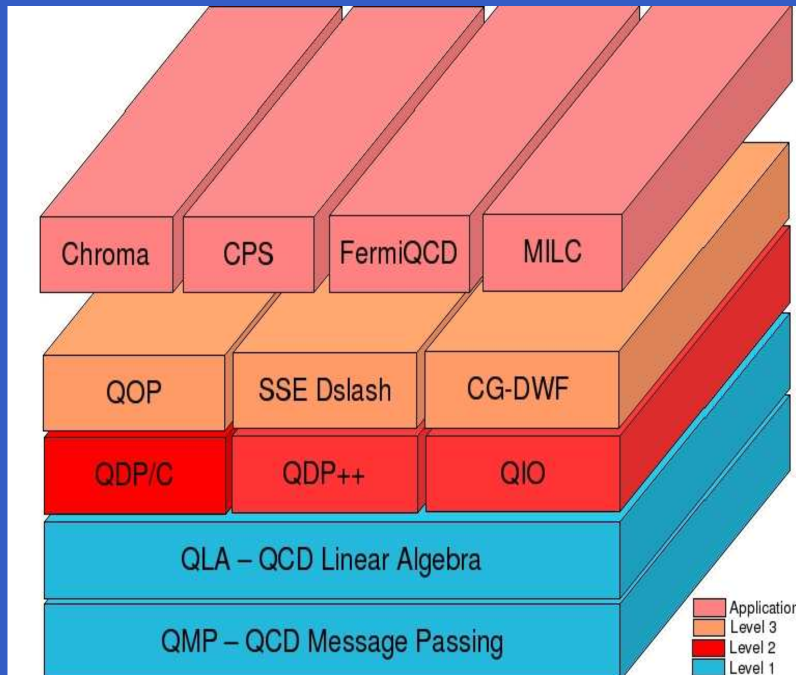


DWF-LSYM @ CCNI

- With collab.'s: Rich Brower (Boston U), Simon Catterall (Syracuse U), George Fleming (Yale U), Pavlos Vranas (LLNL).
- DWF simulations using
  - the best modern code (CPS) and
  - one of the world's fastest computers (CCNI).



A marriage made in heaven: CPS + CCNI



SciDAC Layers and software module arch.

(USQCD, esp. Columbia's CPS)

CCNI BlueGene/L's

(RPI, NYS, IBM)

DWF-LSYM @ CCNI

- Minor hack of CPS code: modify 15 files out of 1800. (CPS = 5MB of C++ code.)
- Currently using 1 to 2 racks: each 5.6 Tflop/s. 9% efficiency ==> 0.75 Tflop/s actual compute rate.
- We will be able to nail the condensate, extrapolate to the continuum, within the year.

Numbers

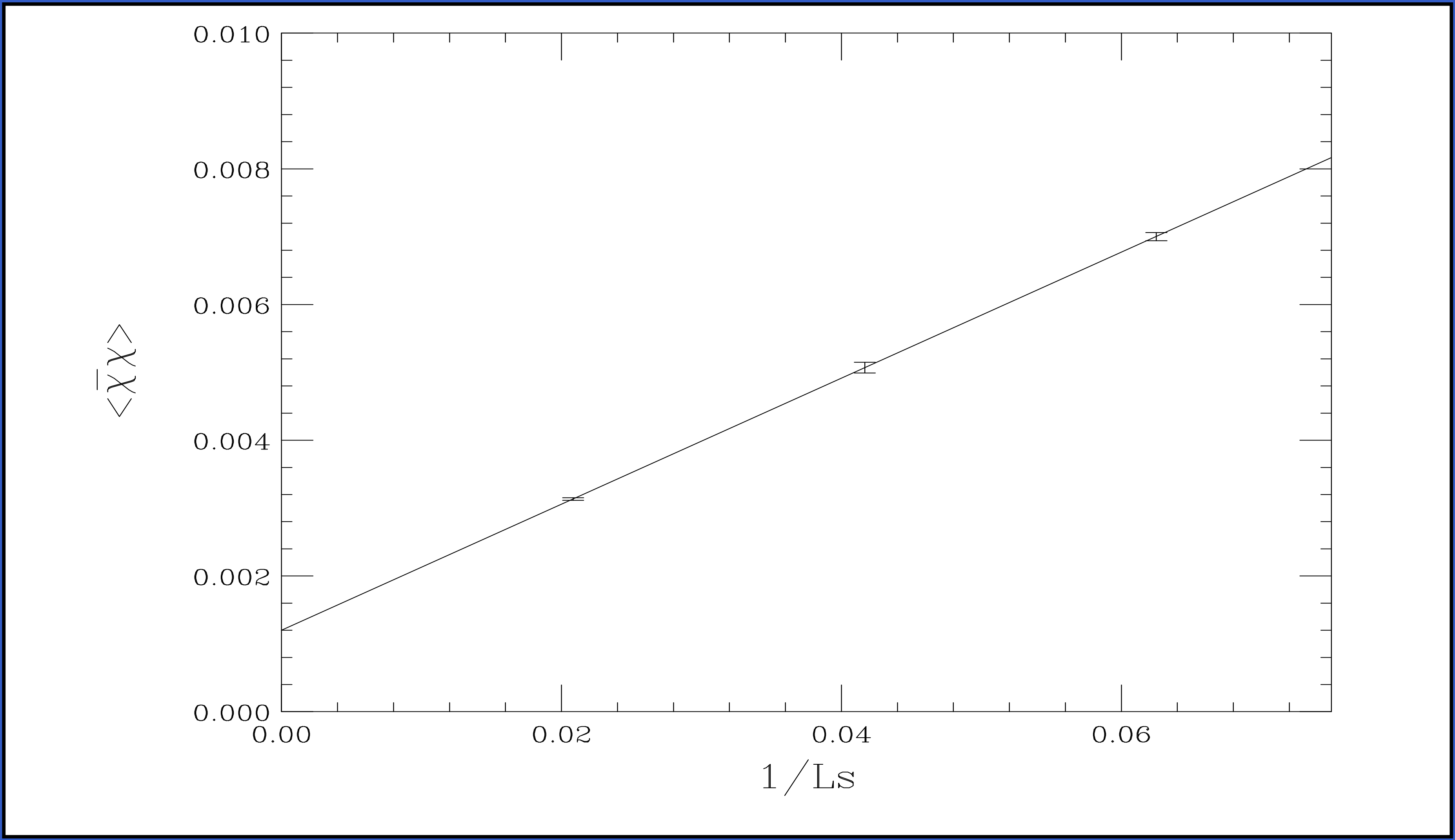
```
Hi Pavlos,

I agree with your old SYM results for 8^4, using BGL.  It took
a few hours of running, which is impressive.  Much more time
was spent with me figuring out what stupid things I was doing...

The total of 600 updates on 8^4 x 16 took about 5 hrs.
using half a rack (512 processors, 1024 nodes). I ran at m_f=0:


size     my condensate    your condensate
--------------------------------------
8^4 x 16   0.00700(6)   0.00694(7)
8^4 x 24   0.00507(8)   0.00516(6)
--------------------------------------


-Joel
```

Early results & comparison

L_s	$\langle\bar{\lambda}\lambda\rangle$(here)	$\langle\bar{\lambda}\lambda\rangle$(FKV)	notes
16	0.00700(6)	0.00694(7)	
24	0.00507(8)	0.00516(6)	
48	0.003134(20)	—	
∞	—	0.00432(22)	method III
∞	0.0012(2)	—	method IV

L_s cases simulated for spacetime volume 8^4. Also shown: $L_s \to \infty$ extrapolations of FKV ($L_s = 12, 16, 20, 24$). Take-away: very large L_s important to $L_s \to \infty$ extrapolation.

The new extrapolation



$\langle\bar{\lambda}\lambda\rangle$ vs. $1/L_s$ for 8^4

Larger lattices (1st ever)

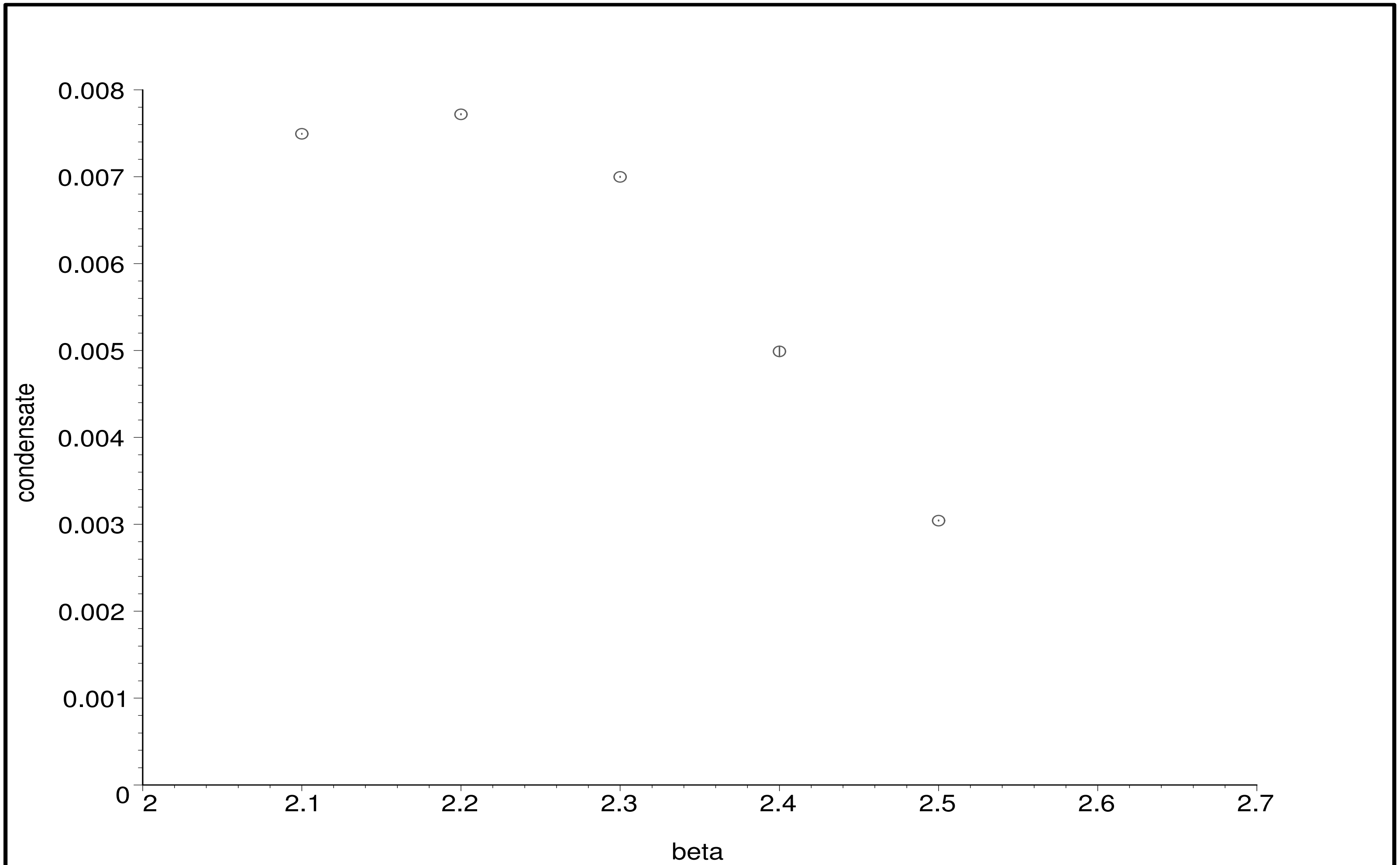


Figure 1: Condensate vs. β for $16^3 \times 32$ lattice with $L_s = 16$.

(Non)renormalization

- Due to nonrenomalization, in continuum:

$$(1/g^2)\mathcal{W}^\alpha \mathcal{W}_\alpha = (1/g_r^2)\mathcal{W}_r^\alpha \mathcal{W}_{r,\alpha} \tag{6}$$

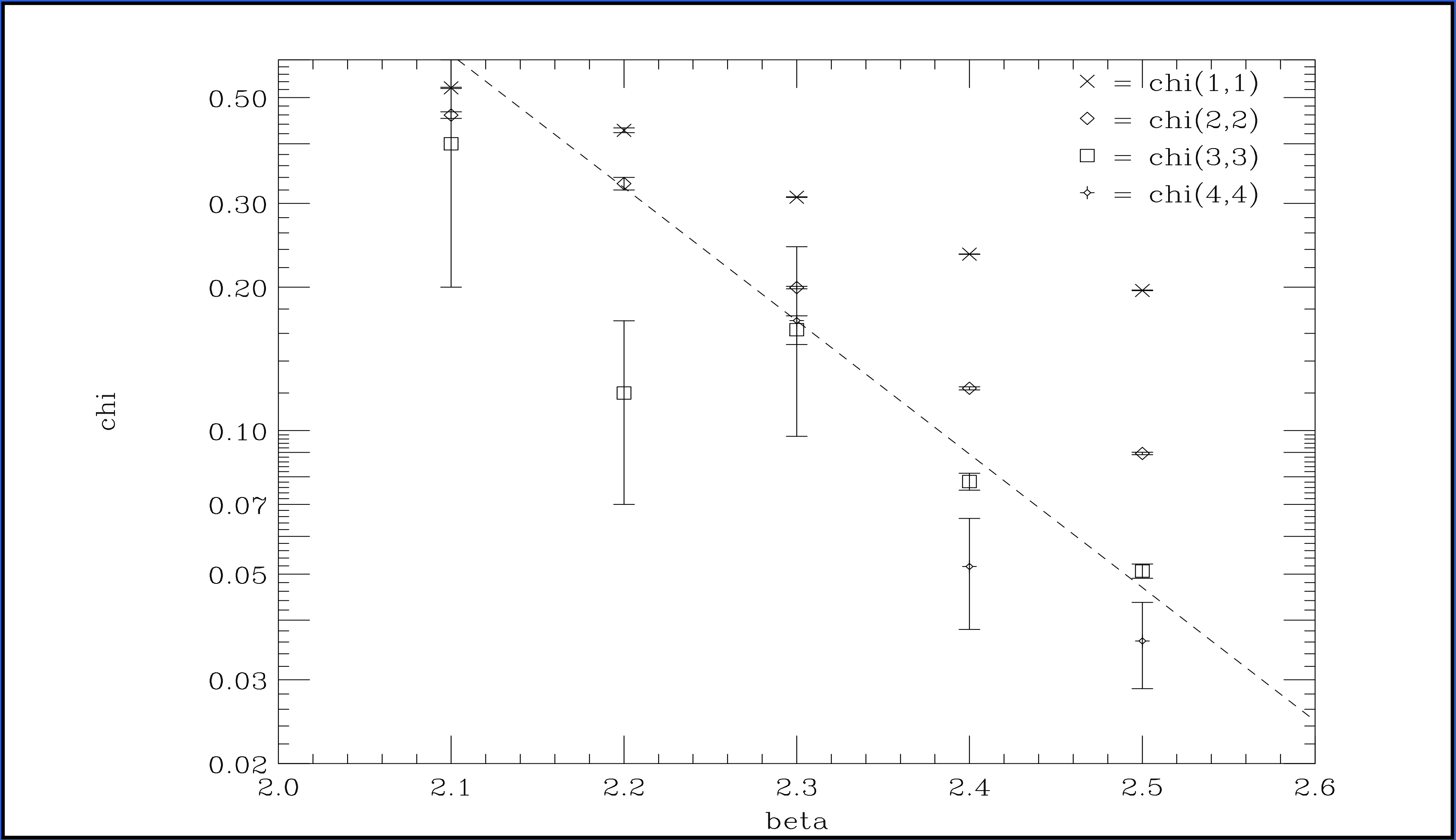
- It follows from this that in the continuum the gluino condensate is not renormalized (absorbing g^2 as usual).
- Since all SUSY violation is short distance, lattice pert. theory would suffice for lattice renormalization of condensate.
- Good check on nonperturb. methods (forthcoming).

Running coupling

- On lattice we usually define $\beta = 4/g^2$. Then 2-loop SUSY RGE's ==>

$$a\Lambda_{SYM} \sim \left(\frac{3}{2\pi^2\beta}\right)^{-1/6} \exp\left(-\frac{\pi^2\beta}{3}\right) \qquad (7)$$

Correct scaling (1st ever)



Creutz ratios for $16^3 \times 32 \times 16$ lattice. The dashed line indicates the 2-loop prediction for the dependence $a^2(\beta)$, obtained from (7).

LSYM Conclusions

- We are well on track to obtain a first ever continuum extrapolation of $\langle\bar{\lambda}\lambda\rangle$ for SYM.
- If we show $\langle\bar{\lambda}\lambda\rangle$ nonzero, it will provide strong evidence by a 4th method.
- Complimentary to VY & Cachazo et al., Affleck-Dine-Seiberg, and the NSVZ strong instanton results.

LSYM Conclusions

- Benchmarks for DWF-LSYM simulation, “phase” diagram of lattice theory.
- Spectrum calculations will follow: continuum limit never obtained before.
- Will attract the attention of HEP community, stimulate strong interest in what is happening at Rensselaer, using CCNI.