Conformal Technicolor

Markus A. Luty
UC Davis

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Strong Electroweak Breaking

Most elegant solution of the hierarchy problem

$$M_W \sim M_0 e^{-8\pi^2/g_0^2} \ll M_0$$

But...

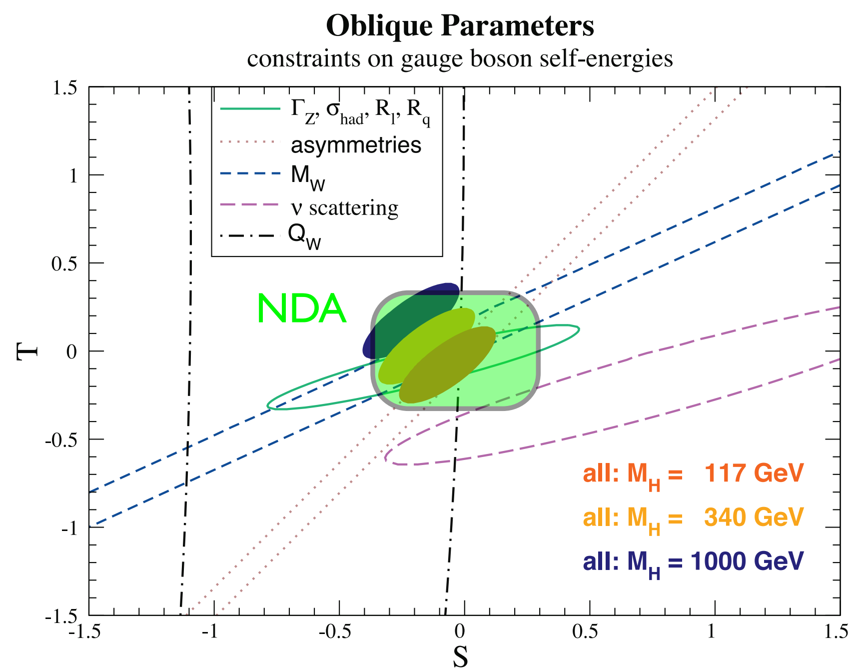
- Precision electroweak data

  May require $S < 0$

- Flavor

$$y\bar{Q}_L u_R H \to \frac{1}{M^{d-1}} \bar{Q}_L u_R \mathcal{O}$$

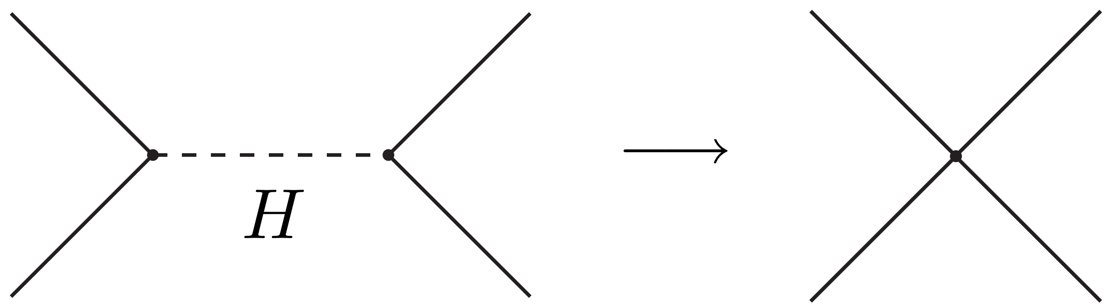
- Why not e.g. $\frac{1}{M^2}(\bar{Q}_L u_R)^2$?   FCNCs...
- $M \sim \text{TeV}$ for top mass



Making it Realistic

- FCNCs

  Just add SUSY! ("Bosonic technicolor" S. Samuel 1990)



  SUSY flavor problems solved for $M_{\mathrm{SUSY}} \gtrsim 10\ \mathrm{TeV}$

- Strong top dynamics

  Requires physics beyond QCD-like technicolor at the weak scale

  Conformal symmetry!

Conformal Technicolor

$\frac{1}{M^{d-1}} \bar{Q}_L t_R \mathcal{O}$ OK if $d \simeq 1$

- QCD-like technicolor: $d = 3$ $\Lambda_t \sim 2 \text{ TeV}$
- "Walking" technicolor: $d = 2$ $\Lambda_t \sim 6 \text{ TeV}$
- Conformal field theory: $d > 1$ $\Lambda_t \sim \text{TeV} \left(\frac{\text{TeV}}{m_t} \right)^{1/(d-1)}$

$d < 2 :$ $\mathcal{O}^\dagger \mathcal{O}$ relevant?

$\dim(\mathcal{O}^\dagger \mathcal{O}) \neq 2d$ unless

- Weak coupling $d \to 1$
- Large N

MAL, T. Okui 2004

Conformal Technicolor

Want e.g. $N \sim 2 \quad d \simeq 1.2 \quad \Rightarrow \quad \Lambda_t \sim 10^4 \text{ TeV}$

Is this possible?

- $d = \frac{6}{5}$ in stringy AdS/CFT construction
- RS constructions for $d \geq 2$

...but $N \gg 1$

Need non-perturbative calculations to check dynamical assumptions

Lattice!

Electroweak Symmetry Breaking

E.g. $SU(2)_{\mathrm{CTC}} \times SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{W}} \times U(1)_{\mathrm{Y}}$

$$\psi_L \sim (2,1,2)_0$$

$$\psi_R \sim (2,1,1)_{\frac{1}{2}}$$

$$\psi'_R \sim (2,1,1)_{-\frac{1}{2}}$$

$$\left.\begin{array}{l} \chi_L \sim (2,1,1)_0 \\ \chi_R \sim (2,1,1)_0 \end{array}\right\} \times K$$

+ explicit mass term $m_\chi \sim \mathrm{TeV}$

Below m_χ expect same universality class as QCD with 2 massless flavors (minimal technicolor)

Dynamics at TeV not QCD-like. $S < 0$?

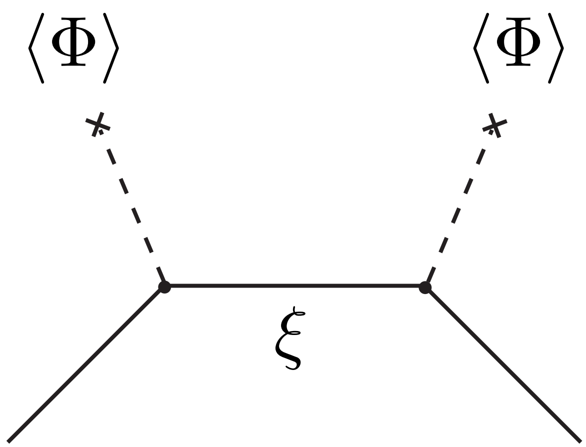
Hierarchy Problem?

$m_\chi \ll M_{\text{Pl}}$ technically natural

Hierarchy problem similar to why $m_\nu \ll M_W$

E.g. see-saw mechanism:

$$\mathcal{L} \sim \Phi \bar{\chi}_L \xi_R + \Phi^\dagger \bar{\chi}_R \xi_L - M \bar{\xi}_L \xi_R + \text{h.c.}$$



$$\Rightarrow \mathcal{L}_{\text{eff}} \sim \frac{\langle\Phi\rangle^2}{M} \bar{\chi}_L \chi_R$$

QCD Conformal Window

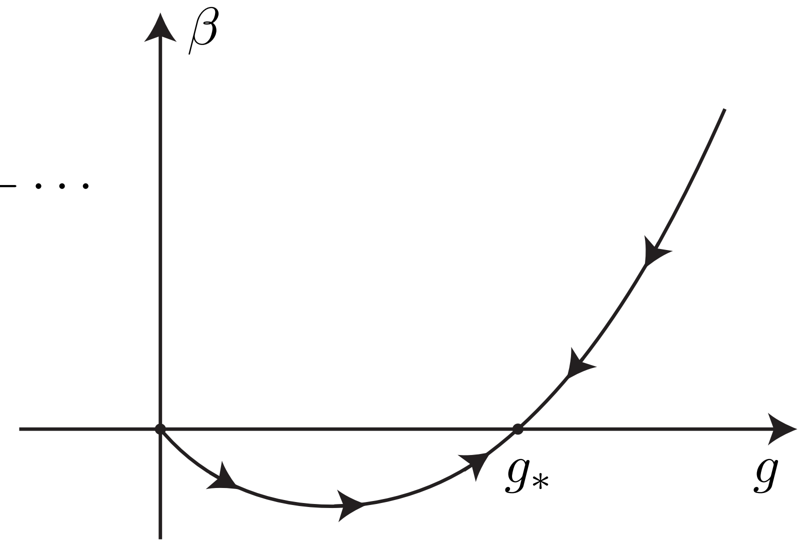
Banks-Zaks fixed point

$$\beta(g) = -\frac{N_c}{16\pi^2} b_1 g^3 + \left(\frac{N_c}{16\pi^2}\right)^2 b_2 g^5 + \cdots$$

$$b_1 = \frac{1}{3}\left(11 - \frac{2N_f}{N_c}\right)$$

$$b_2 > 0$$

$$\frac{N_c g_*^2}{16\pi^2} \ll 1 \quad \text{for} \quad \frac{N_f}{N_c} \simeq 5.5$$



SUSY QCD: generalizes to strong coupling, $N_c \sim 1$

Conformal for $\frac{3}{2} < \frac{N_f}{N_c} < 3 \quad \Leftrightarrow \quad 0 < b_1 < \frac{3}{2}$

Naive generalization: $0 < b_1 < \frac{3}{2} \quad \Rightarrow \; 3.25 < \frac{N_f}{N_c} < 5.5$

Hunting CFTs on the Lattice

Assume lattice can simulate QCD with good separation of scales

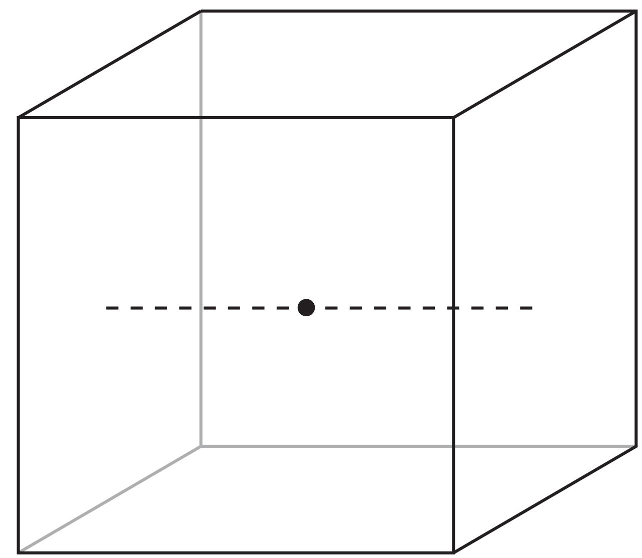
$$L^{-1} \ll m_q \ll \Lambda \ll a^{-1}$$

Look for finite volume effects

$\mathcal{O} = \bar{\psi}\psi$ or $\mathrm{tr}(F^{\mu\nu}F_{\mu\nu})$

$\langle \mathcal{O} \rangle \sim e^{-ML} +$ independent of L

$M =$ mass gap



In both confining and conformal phases, mass gap determined by m_q

Hunting CFTs on the Lattice

Confinement: $M = m_\pi \sim (\Lambda m_q)^{1/2}$

Conformal:

$$\mu \gg \Lambda : \quad \frac{g^2(\mu)}{16\pi^2} \ll 1 \quad \dim(\bar{\psi}\psi) \simeq 3$$

$$\mu \ll \Lambda : \quad g(\mu) \simeq g_* \quad \dim(\bar{\psi}\psi) = 3 - \gamma$$

$$m_q(\mu) \sim \mu \, \frac{m_q}{\Lambda} \left(\frac{\Lambda}{\mu}\right)^{1+\gamma}$$

Gets strong when $m_q(\mu) \sim \mu$

$$M \sim \Lambda \left(\frac{m_q}{\Lambda}\right)^{1/(1+\gamma)}$$

Hunting CFTs on the Lattice

Measure γ from scaling L, m_q

$$\langle \mathcal{O} \rangle \sim e^{-L m_q^{1/(1+\gamma)}}$$

Gives anomalous dimension relevant for top mass

$$\mathcal{L} \sim \frac{1}{\Lambda_t^{2-\gamma}} \bar{t}t \, \bar{\psi}\psi$$

$$m_t \sim \Lambda \left(\frac{\Lambda}{\Lambda_t} \right)^{2-\gamma}$$

$\gamma > 1$ for conformal technicolor

Falsifiable!

Conclusions

- Conformal technicolor is a compelling paradigm for LHC physics
- Lattice calculations can falsify simple models
- Motivates further lattice exploration of CFTs

    Anomalous dimensions

    S parameter

    Hadron spectrum