The Lattice and Holography

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Holographic Map

Old Myth: G.T. at large Nc simplifies ⇒

a weakly coupled string theory.

AdS/CFT: An additional simplification occurs

for a large hierarchy of anomalous dimension of operators:

a few have $\Delta_O \sim 1$, but most have $\Delta_O >> 1$.

Duality: G.T. is dual to an extra-dimensional (ED) theory:

Local primary operators of G.T. $O(x)$.

Fields propagating in ED curved space $\phi(x, z)$.

Δ_O

m_ϕ

Holographic Dictionary

G.T.	ED dual
$\bar{q}q(x)$	$\phi(x,z)$
$J_\mu(x) = \bar{q}\gamma_\mu q(x)$	$A_\mu(x,z)$
$T_{\mu\nu}(x)$	$h_{\mu\nu}(x,z)$
$\bar{q}\gamma_{\mu_1} D_{\mu_2} ... D_{\mu_n} q(x)$	$\phi_{\mu_1\mu_2...\mu_n}(x,z)$

$$\Delta_O >> 1 \quad \Rightarrow \quad m_\phi \sim \Delta_O >> 1$$

Most ED dual fields decouple:

$$J_\mu, T_{\mu\nu} \text{ (+ superpartners)} \quad \Leftrightarrow \quad A_\mu, h_{\mu\nu} \text{ (+ super)}$$

At first: ED duals only for conformal theories.

Then: Non-conformal duals were found.
(similar to Randall-Sundrum ED models)

Lesson:

Lightest states are well described by
the lowest dimension operators.

Warped ED models do good

1. Offer a different perspective on TC models.
2. UV completions for composite Higgs models.
3. Provide mechanisms for addressing FCNC.
4. Generate flavor hierarchies.
5. Have interesting LHC phenomenology.

How likely is LHC strong coupling to be described by a weak ED model?

Wanted:

1. A strongly coupled conformal theory (non-susy).
2. A large hierarchy in anomalous dimensions of operators (rare even for strong coupling).
3. Correlation functions with certain polarization structures suppressed (not due to any symmetry).

LATTICE - HELP!

QCD as a test for ED models

w/ Erlich, Schwartz, Son, and Stephanov.

QCD: $\Delta_O \sim O(1)$ (no hierarchy). ⇒

ED dual contains infinitely many fields.

Conjecture: Lightest states are well described by the lowest dimension operators. ⇒

Might only need to consider a few ED fields.

Evidence for the conjecture:

1. True for G.T. with known ED duals.
2. 2D QCD at Large Nc ('t Hooft model). w/ T. Okui

   nth excited meson PDF: $f_q^n(x) = |\phi_n(x)|^2$

$$\phi_n(x) = \sum_k \langle 0|O_k|n\rangle P_{k-1}(2x-1)$$

$$\langle 0|O_k|n\rangle << 1, \;\; k \gtrsim n$$

   Also seen numerically for 2D QCD with adjoints.

3. Suggested by success of ED models for lightest QCD resonances.
4. LATTICE?

ED toy model for QCD-lite

Chiral Symm breaking sector:

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

$J^V_\mu(x), J^A_\mu(x)$ $\qquad$ $V_\mu(x,z), A_\mu(x,z)$

$\bar{q}q(x)$ $\qquad$ $\phi(x,z)$

Conformal theory:

$$ds^2 = \frac{1}{z^2}(dx_\mu dx^\mu - dz^2)$$

Dilitation: $x^\mu \to \lambda x^\mu,\ z \to \lambda z$

UV: $z << 1,$ $\qquad$ IR: $z >> 1$

$$\int_x e^{iqx}\langle J^a_\mu(x)J^b_\nu(0)\rangle = \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(Q^2)$$

$$\Pi_V(Q^2) = -\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln Q^2 + \frac{1}{24\pi}\cdot\frac{\langle\alpha_s G^2\rangle}{Q^4} - \frac{m_q\langle q\bar{q}\rangle}{Q^4} - \frac{14}{9}\pi\frac{\langle\alpha_s\bar{q}q\rangle^2}{Q^6} + ...$$

$$\begin{aligned} S = \int d^5x\{ \quad & - \quad \frac{1}{4g_5^2 z}(1 + \frac{c_0}{Log(\Lambda z)} + c^{ab}_{4V}z^4 + c^{ab}_{6V}z^6)F^a_V F^b_V \\ & - \quad \frac{1}{4g_5^2 z}(1 + \frac{c_0}{Log(\Lambda z)} + c^{ab}_{4A}z^4 + c^{ab}_{6A}z^6)F^a_A F^b_A + \frac{v(z)^{ab}}{2z^3}D\pi^a D\pi^b\}. \end{aligned}$$

Lattice - Help find condensates at large Nc.

Crude Model

$$c_{nV},\ c_{nA} = 0.$$

$$z_{IR} = 1/(346MeV),\ \langle\bar{q}q\rangle = (308MeV)^3,\ m_q = 2.3MeV,\ m_s = 35MeV.$$

Observable	Measured (MeV)	Model (MeV)	Width (MeV)
m_π	139.6	141	-
m_ρ	775.8	832	146
m_{a_1}	1230±40	1220	250-400
f_π	92.4	84	-
$F_\rho^{1/2}$	345±8	353	-
$F_{a_1}^{1/2}$	433±13	440	-
m_{K^*}	892	897	51
m_ϕ	1020	994	4
m_{K_1}	1272±7	1290	90±2
m_K	498	411	-
f_k	113	117	-
m_{f_2}	1275	1236	185
m_{ω_3}	1667±4	1656	168±10
m_{f_4}	2025±8	2058	225±18
m_{ρ_5}	2330±35	2448	400±100
m_{f_6}	2465±50	2829	255±40
m_η	548	520	-
$m_{\eta'}$	958	867	-
$\Gamma(f_2 \to 2\gamma)$	2.6±.24 KeV	2.71 KeV	-

Conclusions

1. Holographic models capture well certain properties of the low lying states in strongly coupled theories.

2. Provide a natural connection between low-energy parameters and the UV theory (ex. $F_\rho/m_\rho(N_c)$).

3. They are a useful tool for the LHC, where we might access only a few resonances.

4. They work better than they should.

5. LATTICE may help clarify the reach and limitations of the approach.