

MATH224. Homework 9.

1. Find the solutions of the following boundary value problems:

(i)

$$2xu_x - yu_y = u + x; \quad u(s, s^2) = s^3 .$$

(ii)

$$-xyu_x + (1 + y^2)u_y = xu^2; \quad u(s, 0) = s^2 .$$

Check your answers.

2. Suppose $u(x, y)$ satisfies the partial differential equation

$$-4y \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 .$$

Find the characteristic curves for this equation, and name the shape they form.

For each of the following boundary conditions, determine if the boundary value problem for u is well-posed, and if so, find the solution.

(i)

$$u(x, 0) = x$$

(ii)

$$u(0, y) = 2y^3 .$$

3. Find $u(x, t)$ the deviation from equilibrium for a stretched string fixed at its ends $x = 0$ and $x = \pi$, where $u(x, t)$ satisfies the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

and initially the string is stationary, with

$$\begin{aligned} u(x, 0) &= \alpha [\sin(x) + 0.2 \sin(3x)] \\ u_t(x, 0) &= 0 \end{aligned}$$

where α is a constant.