

MATH224. Homework 8.

1. Check whether the following functions are solutions of the given partial differential equations:

(i)

$$c \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0$$
$$u(x, t) = x - ct + \cos(x - ct)$$

(ii)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$
$$u(x, y) = \cosh(2x) \sin(2y - 1)$$

(iii)

$$4 \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$
$$u(x, t) = t^{-1/2} e^{-x^2/t} \quad (t > 0)$$

2. Find the characteristic curves of the partial differential equation

$$(x^2 + 1) \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0, \quad \text{for } y > 0.$$

Hence find the general solution.

Check your solution.

3. Find the characteristic curves of the partial differential equation

$$x \frac{\partial u}{\partial x} + 3y \frac{\partial u}{\partial y} = 0, \quad \text{for } y > 0.$$

Hence find the general solution, and the solution to the boundary value problem $u = x$ when $y = 1$.

Check your solution.