

MATH224. Homework 7.

1. Find the sine (odd) and cosine (even) half range Fourier series of the functions

$$\begin{aligned} \text{(i)} \quad f(x) &= x^2 && \text{for } 0 \leq x \leq L \\ \text{(ii)} \quad f(x) &= 1 && \text{for } 0 \leq x \leq \pi . \end{aligned}$$

For (ii) sketch the graph of the resulting odd function and the graph resulting from the first two terms in the series.

2. Calculate the Fourier series of the function $f(t)$ which is odd, has period 3π and is defined by

$$f(t) = \begin{cases} t & \text{for } 0 \leq t \leq \pi \\ 1 & \text{for } \pi < t < 3\pi/2 . \end{cases}$$

3. Find *particular integrals* for the following linear differential equations:

$$\text{(i)} \quad \frac{d^2y}{dt^2} + 5y = 3\sin t + \sin 4t$$

$$\text{(ii)} \quad \frac{d^2y}{dt^2} + 5y = \sum_{n=1}^{\infty} b_n \sin nt$$

$$\text{(iii)} \quad \frac{d^2y}{dt^2} + 5y = f(t)$$

where f is the periodic function defined by

$$\begin{aligned} f(t) &= t && \text{for } -\pi \leq t < \pi . \\ f(t+2\pi) &= f(t) \end{aligned}$$

Hint: This is the function you looked at in question 2(i) on sheet 6; the series you found there will be useful.