MATH224. Homework 6.

1. Show the following

$$\text{(i)} \qquad \cos A \cos B \;=\; \frac{1}{2}\cos(A+B) \;+\; \frac{1}{2}\cos(A-B)$$

$$\text{(ii)} \qquad \sin A \sin B \;=\; -\frac{1}{2}\cos(A+B) \;+\; \frac{1}{2}\cos(A-B)$$

$$\text{(iii)} \qquad \sin A \cos B \;=\; \frac{1}{2}\sin(A+B) \;+\; \frac{1}{2}\sin(A-B)$$

IMPORTANT: You should learn these as part of your preparation for the examination.

2. The function $f(x)$ is a periodic function with period 2π. Find its Fourier series in the following cases:

$$
\begin{array}{llll}
\text{(i)} & f(x) = x & \text{for} & -\pi \le x < \pi \\
\text{(ii)} & f(x) = |x|/3 & \text{for} & -\pi \le x < \pi \\
\text{(iii)} & f(x) = x^2 & \text{for} & -\pi \le x < \pi.
\end{array}
$$

3. (i) Sketch the graph of the function $g(t)$ where

$$g(t) \;=\; 2 \;-\; 2\left|\cos\left(\frac{t}{2}\right)\right|$$

What is its period?

(ii) Calculate the Fourier series for $g(t)$.