MATH224. Homework 5.

1. Find the general solutions for $x(t)$ and $y(t)$, obeying the system of inhomogeneous differential equations

$$\begin{aligned} \frac{dx}{dt} &= x - 2y + \cos t \\ \frac{dy}{dt} &= 2x - 3y \end{aligned}$$

with initial conditions $x(0) = y(0) = 1$.

2. The free oscillations of the carbon and oxygen atoms in a carbon-monoxide molecule obey the equations

$$\begin{aligned} \frac{d^2x}{dt^2} &= -\frac{A}{m_C}(x - y) \\ \frac{d^2y}{dt^2} &= -\frac{A}{m_O}(y - x) \ . \end{aligned}$$

Here x is the displacement of the carbon atom, m_C is its mass; y is the displacement of the oxygen atom, and m_O is its mass. A is the spring constant of the chemical bond. Either by elimination or by using a matrix method, find the general solution of this system of equations.

How does the quantity $m_C x + m_O y$ depend on time? Verify the solution for $m_C x + m_O y$ by adding the differential equations.

3. Find the general solutions of the differential equations:

(i) $$x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} - 5y = 0$$

(ii) $$x^2y'' - xy' + y = x^2 \ln x$$

(iii) $$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x^2 \ .$$