MATH224. Homework 1.

1. Check whether the solutions listed below satisfy the differential equations, showing your working. (A, B and a are constants.)

<table>
<tr><td></td><td>equation</td><td>solution</td></tr>
<tr><td>(i)</td><td>$\dfrac{dy}{dt} = -4y$</td><td>$y = Ae^{-4t}$</td></tr>
<tr><td>(ii)</td><td>$2x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = 3$</td><td>$y = Ax + \dfrac{B}{\sqrt{x}} - 3$</td></tr>
<tr><td>(iii)</td><td>$y\,y'' + (y' - y)\,y' = 0$</td><td>$y = \sqrt{Ae^x + B}$</td></tr>
<tr><td>(iv)</td><td>$\dfrac{d^2y}{dx^2} + a^2 y = 0$</td><td>$y = A\cos ax + B\sin ax$</td></tr>
</table>

(Remember, it is easy to *check* a solution even if you don't yet know how to solve the differential equation.)

2. Solve the following first-order differential equations

(i) $\qquad \dfrac{dy}{dx} = y\ \cos x$

(ii) $\qquad \dfrac{dy}{dt} = \left(t^2 + 1\right) y^4$

(iii) $\qquad y' = \dfrac{x\left(1 + y^3\right)}{y^2}$

(iv) $\qquad \dfrac{dy}{dx} = \dfrac{x + 2y}{x}$