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String Phenomenology 10 Dimensional Heterotic string models in 4 Dimensions in the Free Fermionic Formulation

Math420 Mathematical Physics Project

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An analysis of a non-supersymmetric string model in the free fermionic formulation.

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Abstract

String phenomenology is the study of string theory while maintaining contact with experimental data. Due to the lack of evidence for supersymmetry, we will construct models that contain a modified S basis vector, $\tilde{S}$. Such string models enlarge the space of possible viable solutions and may offer novel insight into some of the current problems in string phenomenology. In this paper, we discuss a non-supersymmetric model that produces three generations of the standard model; just like the previous supersymmetric models, but this new model also produces tachyonic sectors. It is possible to build models that do not contain tachyonic sectors, and these models may serve as viable models in string phenomenology.

1 Introduction

During the 1970s and the early 1980s, the structure of the standard Model was established experimentally. The model includes matter and interactions: strong, weak, and electromagnetic interactions, but it does not include gravity. The structure of the standard model motivated the studies of grand unified theories and supersymmetry.

The weak force and electromagnetism give the electroweak interaction, and QCD is the study of the strong interaction. The standard model does not include gravity but gravity as described by General relativity and Quantum field theory are incompatible. String theory provides a framework to unify these interactions and matter in one anomaly free theory in ten dimensions.

The free fermionic formulation of heterotic string theory presented by Kawai, Lewellen, and Tye (KLT) and Antoniadas, Bachas, and Kounnas (ABK) [1, 2] allows us to build string models directly in four dimensions and at low experimentally observable energies such as those of the Standard Model. A method to derive a Standard-Like Model in the free fermionic formulation was presented by Faraggi, Nanopoulos and Yuan [3].

In this paper, we will go through the ABK rules for building a string model in the free fermionic formulation. Then we will do the GSO projection for a specific model and derive the massless and tachyonic spectrum of the string model.

2 Standard Model

The standard model is made of the fundamental particles and the interactions, not including gravity. Each particle has its antiparticle with same mass but opposite charge. The Standard model has the gauge group $SU(3) \times SU(2) \times U(1)_Y$. Each group consists of generators and basis vectors. The generators produce the gauge bosons of the and the basis vector correspond to the fermions on which these bosons act. The $SU(3)$ represents the strong interactions; Quantum chromodynamics and couples with eight gluon fields. The $SU(2) \times U(1)$ corresponds to the electroweak interaction. The SU(2) coupling to the three weak fields, and a U(1) coupling with the A field in the anomaly free theory.

The fermions in the standard model come in three generations (matter in the standard model come in three generations). The fermions in the standard model are spins $\frac{1}{2}$, the quarks which form the proton and neutron also have a spin of $\frac{1}{2}$and they also have a colour charge as well. The quarks are never found free but come in pairs of three and form the particle such as a proton (uud) with charge $+\frac{2}{3}$. The electrons have the charge -1. A table of the matter in the standard model particles and interactions is represented below.

Three generations of matter (fermions)			Interactions force carriers	
u	c	t	g	H
d	s	b	γ	
e	μ	τ	Z	
ν_e	ν_μ	ν_τ	W	

Figure 1: Standard model matter and interactions. The particles also have their antiparticles.

2.1 Standard Model embedding in the SO(10)

The Standard Model matter charges of 1 generation can be embedded in the SO(10), spinorial 16 representation of SO(10). We can see how to get the charges out of this SO(10) representation in Figure 2

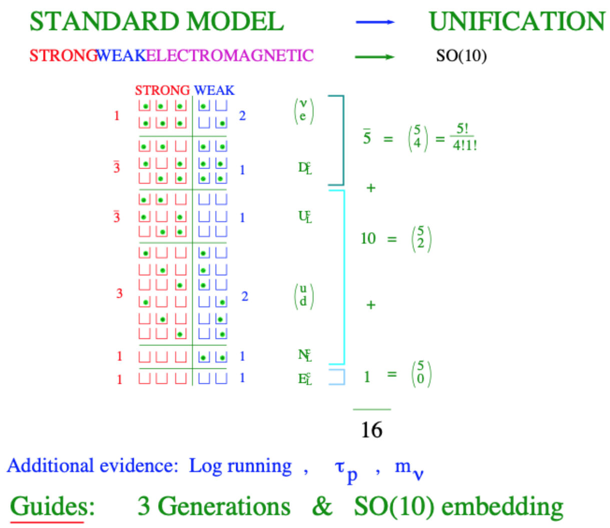

Figure 2: Embedding the Standard Model matter states in SO(10). This hints at GUT structures of the standard model.

Credit: Novel Perspectives in String Phenomenology, Alon E Faraggi [4].

This is how we will produce the 3 generations of the standard model particles from our string model later when we do the GSO projections. These 16 possibilities correspond exactly to the sixteen left-handed states in a single Standard Model matter generation. Three generation quasi-realistic models that possess the SO(10) embedding of the Standard Model matter states were constructed in the free fermionic formulation of the heterotic–string in four dimensions (This is shown in the appendix where we produce the 3 generations of the standard model from the sectors b_1, b_2, and b_3, labelling the generations electron, muon and tau are the finer details of the model, the same details apply for the other particles.)

String theory can potentially provide a structure to build a theory that allows us to unify all the forces; so our theory should contain gravity. String phenomenology is about using string theory to build models to study particle physics. Using the framework of string theory we can build a theory that contains all the particles from the standard model and interactions particles including the spin 2 graviton. We can build supersymmetric heterotic string theory where the free fermionic formulation is built, and our string model will be constructed using the free fermionic formulation. We will now construct the relationship between string theory and modern particle theory.

3 String Theory

String theory could be the theory that can unify all the forces; it is also the quantum theory of gravity. So we ideally want to derive the results from string theory which we observe in particle physics and cosmology but we will not look into cosmology in this paper. The study of gauge theory, heavy-ion physics, and condensed matter fall under the ADS/CFT correspondence. We will not go into the construction of the supersymmetric string theories as these will be very long but we will cover the main concepts from string theory, on the quantisation of the string to produce massless states. A primary reference is *A first course in String theory* by Barton Zwiebach [5].

From Relativity we know that there are spacetime coordinates $x^\mu = (x^0, x^1, x^2, x^3) = (ct, x^1, x^2, x^3)$, we also have the invarient interval

$$-ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

which we can write as

$$\eta^{\mu\nu} dx^\mu dx^\nu$$

where the $\eta^{\mu\nu}$ is the Minkowski matrix, the only difference is that the '-' sign is on the time component instead on the spatial component.

Compact dimensions, one dimensional circle is $x \sim x + 2\pi$ means that the x is equal to $x + 2\pi$ so the space repeats itself every 2π intervals. This can also be seen as a unit circle of circumference 2π. Then the fundamental domain is $x = 0 \leq x < 2\pi R$. Any point in the entire space has a representation on the fundamental domain. Spaces will be built by having a fundamental domain (FD) and adding boundary conditions (B.C) to it. If we again define the FD as $0 \leq x < 2\pi R$ for both x, y we can build a torus. Tours is represented as

$$(x, y) \sim (x + 2\pi R, y),$$

$$(x, y) \sim (x, y + 2\pi R),$$

so the y direction is folded into a circle and the x direction is folded into a circle and joined together, this forms a torus as shown in Figure 3.

Orbifolds are spaces that arise from identifications of fixed points and will include compactifying the space as done before, consider the line parametrised by x, and the identification is $x \sim -x$. The point $x = 0$ is the fixed point of the identification. We can choose the half-line to be the fundamental domain ($x \geq 0$) the $x = 0$ is included in the fundamental domain, and the half-line is the one dimensional manifold. This orbifold is called $\mathbb{R}^1/\mathbb{Z}_2$, the $\mathbb{R}^1$ stands for the one-dimensional real line, and $\mathbb{Z}_2$ describes the identification. This is a space where applying the transformation twice will give back the original coordinate. So this orbifold is a line split in half, where $x = -x$, then we will go from 0 to x then go back to 0 through $-x$, the $x = 0$ is the fixed point.

The strings in string theory are 1 dimensional objects, they have a length parameterised by $\sigma = [0, a]$ and they move in time τ. The string are 1 dimensional objects but when they move in time they will create a 2D surface, they leave a film behind them as they move. The non-relativistic string with mass per unit length μ_0, and tension per unit length T_0 will have the Lagrangian,

$$L = T - V = \frac{1}{2}\mu_0(\frac{\partial y}{\partial t}) - \frac{1}{2}T_0(\frac{\partial y}{\partial x})$$

with,

$$S = \int L\ dt$$

. Varying the action of the Lagrangian δs, y $\to y + \delta y$ and imposing the boundary conditions that at the end points will give the wave equation

$$\frac{\partial^2 y}{\partial x^2} - \frac{\mu_0}{T_0}\frac{\partial^2 y}{\partial t^2} \qquad v = \sqrt{\frac{T_0}{\mu_0}}$$

.

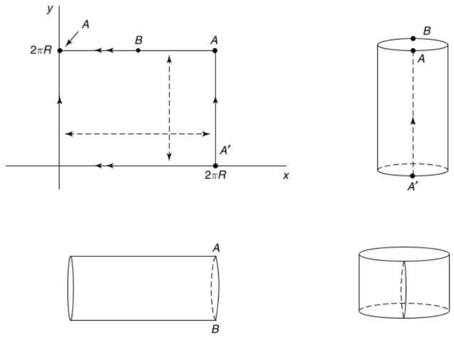

Figure 3: Credit, A first course in String theory [5].

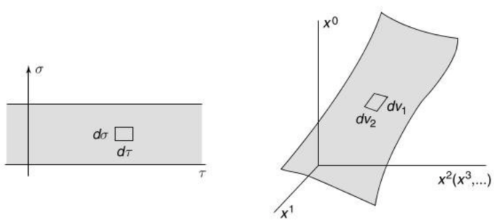

Figure 4: Credit, A first course in String theory [5], Parameterised spacetime worldsheet.

Our parameterised space is defined by τ and σ so we write the spacetime coordinates as $X^\mu = (X^0(\tau,\sigma), X^1(\tau,\sigma), X^2(\tau,\sigma), X^3(\tau,\sigma))$. The area on the worldsheet iwill be given by,

$$dA = \int \sqrt{det(g)}\ d\tau d\sigma$$

.

We can now write the action for the relativistic string Nambu-Goto action,

$$S = -\frac{T_0}{c}\int dtd\sigma\sqrt{(\dot{X}X')^2 - \dot{X}X'} = -\frac{T_0}{c}\int\sqrt{-det(g_{ij})}$$

where the g_{ij} is the induced metric (the metric on the worldsheet spacetime). So we have that the Lagrangian is $L(\dot{X}, X^{'})$ and we can define the following derivatives,

$$P^{\tau}_{\mu} = \frac{\partial L}{\partial \dot{X}} = \frac{(\dot{X}.X^{'})X^{'} - (\dot{X})^2 X^{'}}{\sqrt{(\dot{X}X')^2 - (\dot{X})^2(X')^2}}$$

$$P^{\sigma}_{\mu} = \frac{\partial L}{\partial X^{'}} = \frac{(\dot{X}.X^{'})\dot{X} - (\dot{X})^2 \dot{X}}{\sqrt{(\dot{X}X')^2 - (\dot{X})^2(X')^2}}$$

we again vary the action of the Lagrangian

$$dS = \int d\tau P^{\sigma}_{\mu}\delta X^{\mu}|^{\sigma_1}_0 - \int d\tau d\sigma(\frac{\partial P^{\tau}_{\mu}}{\partial \tau} + \frac{\partial P^{\sigma}_{\mu}}{\partial \sigma})$$

we get equation of motion,

$$\frac{\partial P^{\tau}_{\mu}}{\partial \tau} + \frac{\partial P^{\sigma}_{\mu}}{\partial \sigma} = 0.$$

We want to impose the boundary conditions on the string that $P^{\sigma}_{\mu}(\tau,\sigma)\delta x^{\mu}(\tau,\sigma_*) = 0$, we do not want it summed over μ we want individual components going to zero. Then we have $P^{\sigma}_{0}(\tau,\sigma)\delta x^{0}(\tau,\sigma_*) = 0$, $P^{\sigma}_{0}(\tau,\sigma)$ must be zero as $\delta x^{0}(\tau,\sigma_*)$ is the time component component this can not be zero the string must evolve in time. P^{σ}_{i} or $\partial x^{i}(\tau,\sigma) = 0$, if $\partial x^i = 0$ then δx^i is a constant.

So if we have a D2 brane in the X^1 and X^2 direction but not in the X^3, we will have $P^{\sigma}_{1} = 0$ and $P^{\sigma}_{2} = 0$ on the D brane and $\delta x^3 = 0$ at the end points but not P^{σ}_{3} since the string is not free to move in the x^3 direction at the end points.

When we impose the Dirichlet boundary conditions on the string that P^{σ}_{0} the endpoints of the string are fixed in space, then they are fixed on D branes (however many spatial directions are fixed in), this is an important tool of string phenomenology.

We will work in the static gauge which means $X^0(\sigma,\tau) = c\tau$. Also we will define the velocity of the string as $v_{\perp} = \frac{\partial X}{\partial t} - (\frac{\partial X}{\partial t}.\frac{\partial X}{\partial S})\frac{\partial X}{\partial S}$. Another thing to note is that $P^{\tau\mu}$ and $P^{\sigma\mu}$ are conserved currents on the world-sheet, where $P^{\sigma\mu}$ we to zero on the world-sheet.

Developing the theory further we can also write the Nambu-Goto action as

$$S_{NG} = -\frac{1}{2\pi\alpha^{'}}\int dtd\sigma\sqrt{-det(g)}$$

where α' is the slope parameter, $\alpha = \frac{1}{2\pi T_0}$

For an open string wave equation we can split the solution into left and right parts

$$X^{\mu}(\sigma,\tau) = \frac{1}{2}(F(\tau+\sigma) + G(\tau-\sigma)).$$

These soloutions can then be written in terms of Fourier modes

$$X^{\mu}(\tau,\sigma) = X_0^{\mu} + \sqrt{2\alpha'}\alpha_0^{\mu}\tau + i\sqrt{2\alpha'}\sum n \neq 0\frac{1}{n}\alpha_n^{\mu}e^{-in\tau}cos(n\sigma)$$

where $\alpha_0^{\mu} = \sqrt{2\alpha' p^{\mu}}$.

The last part of the solution is real and solves the wave equation, classically α_n^{μ} have physical interpretations of being amplitudes of the n^th oscillation mode, but in the quantum theory there will become creation and annihilation operators. These operators will define a Hilbert space.

Quantum Field Theory

The soloution to the Klein Gordon equation is the scaler field,

$$\phi(x) = \frac{1}{2\pi^3}\int \frac{1}{\sqrt{2E_{\vec{p}}}}(a_{\vec{p}}e^{-ip.x} + a*_{\vec{p}}\, e^{ip.x})$$

in the quantum field theory we have the field

$$\phi(t,x) = \frac{1}{2\pi^3}\int \frac{1}{\sqrt{2E_P}}(a_{(\overrightarrow{P})}e^{-iE_0t+i\overrightarrow{p}.x} + a^{+}_{(\overrightarrow{P})}e^{iE_pt-i\overrightarrow{p}x})$$

. We also have the commutation relations

$$[a_{\overrightarrow{p}}, a^{\dagger}_{\overrightarrow{p}'}] = (2\pi)^3\delta^{(3)}(p-p')$$

$$[a_p, a_{p'}] = [a^{\dagger}_{\vec{p}}, a^{\dagger}_{\vec{p'}}] = 0$$

the $|0\rangle$ is the vacuum state, acting on the vacuum with the creation operator $\sqrt{2E_p}a_p^{+}|0\rangle = |\vec{p}\rangle$ produces a particle of momentum $\vec{p}$. We can also define an n-particle state

$$|\vec{p_1}, \vec{p_2}, ..., \vec{p_n}\rangle = \sqrt{2E_{\vec{p_1}}, 2E_{\vec{p_2}}, ...2E_{\vec{p_n}}}a^{\dagger}_{\vec{p_1}}a^{\dagger}_{\vec{p_2}}...a^{\dagger}_{\vec{p_n}}|0\rangle$$

. The annihilation operator is $a_{(\vec{p})}$ and has the property that when it acts on the vacuum it will produce 0; $a_{(\vec{p})}|0\rangle = 0$. Further operator the Hamiltonian and momentum are given by

$$H = \int d^3\vec{p}\frac{1}{(2\pi)^3}E_{\vec{p}}\; a^{\dagger}_{\vec{p}}\; a_{\vec{p}}$$

$$P = \int \frac{d^3\vec{p}}{(2\pi)^3}p\; a^{\dagger}_{\vec{p}}\; a_{\vec{p}}$$

these operator act on the vacuum $|0\rangle$.

Light cone coordinates

We will use light cone coordinates to quantise the string where x^0 and x^1 are replaced by x^+ and x^- respectively.

$$x^+ = \frac{1}{\sqrt{2}}(x^0 + x^1) \qquad x^- = \frac{1}{\sqrt{2}}(x^0 - x^1)$$

In the light cone coordinates we have $X^\mu = X^+, X^-, X^2, X^3, ...$, we label the X^+ the time and X^- the space coordinate. The spacetime interval is now

$$-ds^2 = -2dx^+dx^- + (dx^2)^2 + (dx^3)^2 = \hat{\eta}_{\mu\nu}dx^\mu dx^\nu$$

.

$$\hat{\eta}_{\mu\nu} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

any transformation of the form $\sigma^+ \to \tilde{\sigma}^+(\sigma^+)$, $\sigma^- \to \tilde{\sigma}^-(\sigma^-)$.
Also in light cone coordinates we have the exponential factor $e^{ip.x} = e^{-ip^-x^+ - ip^+x^-}$. So p^- is energy and p^+ is spatial.

From $\partial_0 j^0_\mu + \partial_1 j^1_\mu = 0$ we also have the equation of motion $\frac{\partial P^\tau}{\partial \tau} + \frac{\partial P^\sigma_\mu}{\partial \sigma} = 0$. The conserved charge of the string is given by

$$Q_\mu \to P_\mu(\tau) = \int_0^{\sigma_1} d\sigma P^\tau_\mu(\tau, \sigma)$$

P_μ is the conserved momenta of the string.

3.1 Quantising the string

We will define Schrodinger operators as X^I, X^0, P^I, P^+ then the Heisenberg operators are $X^I(\tau), X^0(\tau), P^I(\tau), P^+(\tau)$. We have the hamiltonian which is given by

$$H(\tau) = \frac{P^+(\tau)P^-(\tau)}{m^2} = \frac{1}{2}(P^I P^I + m^2)$$

the P^I are the momenta for the other spatial components.
We have the conformal gauge $\Box X^\mu = 0$. We also have the commutators from quantum mechanics mechanics, $[x^i, p^j] = i\delta^i j$ or $[X^\mu, P^\nu] = i\eta^{\mu\nu}$ and from QTF we have $[\phi(x), \phi(y)] = i\delta(\vec{x} - \vec{y})$. So for the free relativistic string we have $[X^\mu(\tau, \sigma), \Pi^\nu(\tau', \sigma')] = i\eta^{\mu\nu}\delta(\sigma - \sigma')$.The sum over the oscillation modes will be defined is $L_n = \bar{L}_n = 0$

The states of the system will be characterised by the eigenvectors of the operators in our theory.
Going back to the solution of the open string instead of X^μ we have X^I,

$$X^I(\tau,\sigma) = X_0^I + \sqrt{2\alpha'}\alpha_0^I\tau + i\sqrt{2\alpha'}\sum n \neq 0\frac{1}{n}\alpha_n^I e^{-in\tau}cos(n\sigma)$$

the $X^I(\tau,\sigma)^\dagger = X^I(\tau,\sigma)$ and $(\alpha_n^I)^\dagger = \alpha_n^I$ so the oscillators are hermitian. We will define α_n^I and $\alpha_-^I n$ to be annihilation and creation operators.
We define the opererator

$$[\alpha_m^I, \alpha_n^J] = m\delta_{m+n,0}\delta^{IJ}$$

$$[\alpha_m^I, \alpha_{-n}^J] = m\delta_{m,n}\delta^{IJ}$$

$$[\alpha_m^I, {\alpha_n^J}^\dagger] = \delta_{m,n}\delta^{IJ}$$

the operators commute unless $m = -n$ only then we get a non zero component on the right hand side. Also everything from QTF applies, equal time commutations apply.
We have from the classical string

$$2p^+p^- = \frac{1}{\alpha'}L_0^\perp$$

$L_0^\perp$ is defined from

$$(\dot{X} \pm X')^2 = \frac{1}{2\alpha'}\frac{1}{2p^+}2\alpha\sum_{p,q\in R}\alpha_p^I\alpha_q^I e^{-i(p+q)(\tau\pm\sigma)}$$

$$L_0^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha_p^I\alpha_p^I$$

this is what will become Virasoro operators, and that we have the mass defined as

$$M^2 = -p^2 = 2P^+P^- = \frac{1}{\alpha'}L_0^\perp - p^Ip^I$$

$$= \frac{1}{\alpha'}(\frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha_p^I\alpha_p^I) - P^IP^I = \frac{1}{\alpha'}(\frac{1}{2}\alpha_0^I\alpha_0^I + \sum_{p\geq 1}(\alpha_p^I) * \alpha_p^I) - \alpha^I\alpha^I$$

this will give

$$M^2 = \frac{1}{\alpha'}\sum_{p=1}^{\infty}(\alpha_p^I)^*(\alpha_p^I) \geq 0.$$

This is what we had from classical string when we quantise the string[1] we will get,

$$M^2 = \frac{1}{\alpha'}\sum_{p=1}^{\infty}(\alpha_p^I)^\dagger(\alpha_p^I) + a$$

[1]covered in reference [5]

where $a = \frac{1}{2}(D-2)\sum_{p=1}^{\infty} P$, which will give D = 26.

$$(L_n^\perp)^\dagger = L_{-n}^\perp$$

$$[L_m^\perp, L_n^\perp] = (m-n)L_{m+n}^\perp + \frac{1}{2}(D-2)$$

Virasoros' operators generate reparameterisation of the world-sheet, these operators conserve the constraints of the theory, $\partial_\tau X.\partial_\sigma X = 0, \dot{X}.X' = 0, \dot{X}^2 + X'^2 = 0$.

We also find through further calculation that $a = -1$ which will give $2p^+p^- = \frac{1}{\alpha'}(L_0^\perp - 1)$ which gives $H = L_0^\perp - 1$. So mass states will equal

$$M^2|p^\dagger, p_T\rangle = \frac{1}{\alpha'}(N^\perp - 1)|p^\dagger, p_T\rangle = -\frac{1}{\alpha'}$$

where $N^\perp$ (instead of $L^\perp$ we wrote $N^\perp$) is the creation operator for 1 particle, acting on the vacuum. When there are no creation operators acting on the vacuum we get a state with negative mass; a tachyon. Adding more oscillators will create different states, adding just one oscillator will create a massless state, which is of interest to us, since massless states represent particles moving at the speed of light.

Going back to the tachyon, we say that the vacuum is unstable, we will not think of these particles as moving faster than the speed of light. There is an energy density associated with the D branes, and .

3.1.1 Quantising the closed string

Solving the wave equation for the closed strings will give

$$X^\mu = X_L^\mu(\tau+\sigma) + X_R^\mu(\tau-\sigma)$$

where u = $\tau+\sigma$ and v = $\tau-\sigma$, which can be written as

$$X^\mu = X_L^\mu(u) + X_R^\mu(v)$$

and with the boundary condition for the closed string $X^\mu(\tau+\sigma+2\pi) = X^\mu(\tau+\sigma)$ we get the condition,

$$X_L(u+2\pi) - X_L(u) = X_R(v) - X_R(v-2\pi)$$

the sum of the right and left moving waves does not have to be periodic and we think of u and v as completely different functions, than individually the left and right hand side has to be equal to a number, since they are equal and depend on different functions so the only way they will be equal is if they equal a number.The derivatives of the left and right hand side with ∂u and ∂v respectively gives zero.

We can write the solutions for the left and right side of the fields as

$$X_L^\mu(u) = X_0^{L\mu} + \sqrt{\frac{\alpha^\mu}{2}}\bar{\alpha}_0^\mu u + i\sqrt{\frac{\alpha^\mu}{2}}\sum_{n0}\frac{1}{n}\bar{\alpha}_n^\mu e^{-inu}$$

$$X_R^\mu(u) = X_0^{R\mu} + \sqrt{\frac{\alpha^\mu}{2}}\alpha_0^\mu v + i\sqrt{\frac{\alpha^\mu}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-inv}$$

solving the closed string equation we get

$$\sqrt{\frac{\alpha^\mu}{2}}\alpha_0^\mu(2\pi) = \sqrt{\frac{\alpha^\mu}{2}}\bar{\alpha}_0^\mu(2\pi) \rightarrow \alpha_0^\mu = \bar{\alpha}_0^\mu$$

we would solve the equations here like it was done for the open string.

The Virasoro operators acting on the closed strings will give

$$(L_0^\perp - \bar{L}_0^\perp)|\psi\rangle = 0$$

$$(N^\perp - \bar{N}^\perp)|\psi\rangle = 0$$

the mass formula will give

$$M^2 = \frac{2}{\alpha'}(N^\perp + \bar{N}^\perp - 2).$$

Constructing the first state of the closed string $N^\perp = \bar{N}^\perp = 0$ this is the ground state, $M^2 = \frac{4}{\alpha'}$ this is again a tachyon. Next state is $N^\perp = 1$ which gives massless states $M^2 = 0$. This is going to give a graviton, dilation, and Ramond. We will not go into detail here about these states but these things can be studied separately in other textbooks.

3.1.2 Fermionic and Superstring

In the classical theory we have $X^\mu(\tau, \sigma)$, we will now add a function for the fermions ψ_α^μ where $\alpha = 1, 2$, so we have 2 fields (We can think of the action $S = S_{NG}(X) + S(\psi_\alpha^\mu)$). Fermion action

$$S(\psi) = \frac{1}{2\pi}\int d\tau d\sigma[\psi_1^I(\frac{\partial}{\partial\tau} + \frac{\partial}{\partial\sigma})\psi_1^I + \psi_2^I(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial\sigma})\psi_2^I]$$

the $\psi_1^I\psi_1^I$ will equal zero so the action has to be written in this form. Variation of this will give

$$\delta S = \frac{1}{\pi}\int d\tau d\sigma[\delta\psi_1^I(\partial_\tau + \partial_\sigma)\psi_1^I + \delta\psi_2^I(\partial_\tau + \partial_\sigma)\psi_2^I] + \frac{1}{2\pi}\int d\tau[\psi_1^I\delta\psi_1^I - \psi_2^I\delta\psi_2^I]_0^\pi$$

the E.O.M will be given by

$$(\partial_\tau + \partial_\sigma)\psi_1^I = 0 \qquad \psi_1^I = \chi_1^I(\tau - \sigma)$$

this wave moves to the right

$$(\partial_\tau - \partial_\sigma)\psi_2^I = 0 \qquad \psi_2^I = \chi_2^I(\tau + \sigma)$$

moves to the left.

From the boundary conditions and the R.H.S of the variation

$$\psi_1^I \delta\psi_1^I(\tau, \sigma_*) - \psi_2^I \delta\psi_2^I(\tau, \sigma_*) = 0$$

to solve we relate ψ_1 and ψ_2 by $\psi_1^I(\tau, \sigma) = \pm\psi_2^I(\tau, \sigma)$ and the same is true for the $\delta\psi_1^I(\tau, \sigma) = \pm\delta\psi_2^I(\tau, \sigma)$.

We declare that ψ_1 and ψ_2 by

$$\psi_1^I(\tau, 0) = \psi_2^I(\tau, 0)$$

and at the π B.C we can have $\pm$,

$$\begin{aligned} \psi_1^I(\tau, \pi) &= + \pm \psi_2^I(\tau, \pi) \qquad \text{Ramond sector.} \\ &= - \pm \psi_2^I(\tau, \pi) \qquad \text{Neveu-Schwarz sector.} \end{aligned}$$

here the fundamental domain is $\sigma \in [0, 2\pi]$ and

$$\begin{aligned} &\psi_1 \quad \sigma \in [0, \pi] \\ &\psi_2 \quad \sigma \in [-\pi, 0] \end{aligned}$$

what this means that the fermionic function is either periodic or anti-periodic, so at π, $\psi(\tau, \pi) = \psi(\tau, -\pi)$ or $\psi(\tau, \pi) = -\psi(\tau, -\pi)$ relating to R & NS sectors. Another thing to note which has not been derived here is we are now working in D = 10, since there are fermion contributions the critical dimensions reduces to 10 dimensions.

Now for the NS we have the solution

$$\psi(\tau, \sigma) \approx \sum_{r \in \mathbb{Z}+1/2} b_r^I e^{-ir(\tau - \sigma)},$$

and the mass states produced from these states will be

$$\alpha' M^2 = N^\perp - \frac{1}{2}$$

therefore the states produced by acting on the vacuum ($|NS, 0\rangle$) with creation operators will be,

$$\begin{aligned} M^2 &= -\frac{1}{2} & N^\perp &= 0 & &|NS, 0\rangle \\ M^2 &= 0 & N^\perp &= 1/2 & &b^I_{-\frac{1}{2}}|NS, 0\rangle \\ M^2 &= \frac{1}{2} & N^\perp &= 1 & &b^I_{-\frac{1}{2}} b^J_{-\frac{1}{2}}, \alpha^I_{-1}|NS, 0\rangle \end{aligned}$$

states with $N^{\perp}$ integer and half integer are fermionic and bosonic respectively. We have the same process for the R sector but produces different results.

At this point, we have seen that we replace the point particle with strings, then quantising the string can be used to produce states, specifically massless states, and how there is a tachyonic state present which we viewed as relating to an unstable vacuum. This theory needs to be developed further before we can produce the gauge groups that of the standard model, how the ABK formulation is constructed is covered in the references, we will now move on to building our models in the ABK formulation.

4 The Free Fermionic Formulation The ABK Rules and GSO Projections

Our models will be constructed using the free fermionic formulation (FFF), from heterotic string theory. To get to the FFF we start from the bosonic string in $D = 26$ spacetime dimensions, we quantise the string. This is open string theory, and all open string theories contain closed string, and quantum closed strings give rise to graviton states. We then add supersymmetry to the string which will allow both fermionic and bosonic states, it will also reduce the dimensions down to 10. Then quantised fermionic string will result in the Ramond (R) and Nevu-Schwarz (NS) sectors. We will then be able to construct closed superstring theories called type II superstring theories (which also results in two other versions type IIA and IIB).
Supersymmetric heterotic string theory is the theory we will use, which has supersymmetry to introduce world-sheet fermions for every boson. This theory has 10 dimensions but compactifications will produce the 4D spacetime we observe. We want the theory to produce gauge groups that resemble the standard model. One can learn the ABK rules and start to construct models without deep knowledge of how to construct supersymmetric string theories or knowing how to derive the ABK rules, we shall construct these models now.

4.1 ABK rules

A model is defined by specifying two ingredients:
1. A set of boundary conditions basis vectors
2. The one loop phases $C\binom{b_i}{b_j}$ for the intersections of the basis vectors.

These are the explicit rules of the ABK models, this is our starting point.
We have basis vectors $b_1, \ldots, b_n$ or in the model we will go on to analyse $1, S, b_1, b_2, b_3, \alpha, \beta, \gamma$ which will contain the fermions in the following equation with a set of boundary conditions,

$$\begin{gathered}\psi^\mu_{1,2}\{\chi_1 y_1 w_1\}, \{\chi_2 y_2 w_2\}, \{\chi_3 y_3 w_3\}, \{\chi_4 y_4 w_4\}, \{\chi_5 y_5 w_5\}, \{\chi_6 y_6 w_6\}| \\ \bar{y}_1\bar{w}_1, \bar{y}_2\bar{w}_2, \bar{y}_3\bar{w}_3, \bar{y}_4\bar{w}_4, \bar{y}_5\bar{w}_5, \bar{y}_6\bar{w}_6, \underbrace{(\bar{\psi}^1\bar{\psi}^2\bar{\psi}^3\bar{\psi}^4\bar{\psi}^5\bar{\eta}^1\bar{\eta}^2\bar{\eta}^3\bar{\phi}^1\bar{\phi}^2\bar{\phi}^3\bar{\phi}^4\bar{\phi}^5\bar{\phi}^6\bar{\phi}^7\bar{\phi}^8)}\end{gathered} \tag{1}$$

where the bar on top of the letters represents the right movers. It contains 20 real left moving and 44 real right moving fermions; the under-braced are complex fermions. Each individual fermion has a boundary conditions which we specify. The phases defined on these fermions will be either 1 periodic (Ramond) or 0 anti periodic (Nevu Scharwz) or $\frac{1}{2}$.

For example: The basis vector b_i will be defined as follows,

$$b_i = \{\alpha(\psi^\mu_{1,2}), \alpha(\chi^1)\alpha(\chi^2), \alpha(\chi^3)\alpha(\chi^4), \alpha(\chi^5)\alpha(\chi^6)|\alpha(\bar{\phi}^1), \alpha(\bar{\phi}^2), \alpha(\bar{\phi}^3), \alpha(\bar{\phi}^4)\}$$

where each fermion has the a boundary condition associated with it $\alpha(f)$, it is the phase defined by $f \to -e^{i\pi\alpha(f)} f$. The fermions also form an additive group Ξ which are defined by

$$\Xi = \sum_{n=i}^{n} m_i b_i.$$

4.1.1 ABK rules on the basis vectors

1. $\sum m_i b_i = 0$ if and only $\forall m_i = 0 \bmod N_i$
   where N_i is the smallest positive integer where $N_i b_i = 0$
2. $N_{ij} b_i b_j = 0 \bmod 4$
   where N_{ij} is the least common multiplier of b_i and b_j.
3. $N_i b_i b_i = 0 \bmod 8$
4. Number of fermions are even.
5. Basis vector 1 contains all the fermions, hence the boundary conditions on all fermions is 1 periodic (Ramond).

The scalar product between the basis vectors is defined as,

$$b_i.b_j = \left[\left(\sum_{ComplexLeft} + \frac{1}{2} \sum_{RealLeft} \right) - \left(\sum_{ComplexRight} + \frac{1}{2} \sum_{Realright} \right) \right] b_i(f) b_j(f) \tag{2}$$

where $b_i(f)$ and $b_j(f)$ are the fermion numbers.

4.1.2 Rules on one loop phases

1.
$$C\binom{b_i}{b_j} = \delta_{b_i} e^{\frac{2\pi i n_j}{N_j}} = \delta_{b_i} e^{\frac{i\pi b_i b_j}{2}} e^{\frac{i2\pi m_i}{N_i}} \tag{3}$$

2.
$$C\binom{b_i}{b_i} = -e^{\frac{i\pi b_i b_i}{4}} C\binom{b_i}{1} \tag{4}$$

3.
$$C\binom{b_i}{b_j} = e^{\frac{i\pi b_i b_j}{2}} \binom{b_j}{b_i}^* \tag{5}$$

4.
$$C\binom{b_i}{b_j + b_k} = delta_{b_i} C\binom{b_i}{b_j} C\binom{b_i}{b_k} \tag{6}$$

$$\delta_{b_i} = e^{i\pi b_i(\psi^\mu)} = \begin{cases} -1 & b_i(\psi^\mu) = 1 \\ +1 & b_i(\psi^\mu) = 0 \end{cases} \tag{7}$$

4.1.3 Matching Virasoro Condition

For every sector we construct using the basis vectors, we need to find out if it contains massless states or tachyonic using the formula,

$$M_L^2 = -\frac{1}{2} + \frac{\alpha_L.\alpha_L}{8} + N_L = -1 + \frac{\alpha_R.\alpha_R}{8} + N_R = M_R^2, \tag{8}$$

Where the N_L and N_R are the fermionic osscillators that act on the vacume $|0\rangle_\alpha$.
where α_L and α_R are the left moving and right moving part of the basis vector; worked out by equation 2.

For the periodic fermions there is a doubly degenerate vacume $|\pm\rangle$.

$$v_\alpha 0_\alpha \ ; \ |\pm\rangle$$

where v_α is the frequency of the osscillators and is given for a fermion f and its conjugate f^* by:

$$v_f = \frac{1+\alpha(f)}{2} \qquad v_{f^*} = \frac{1-\alpha(f)}{2} \tag{9}$$

The frequencies and the fermionic oscillators are related by

$$N_L = \sum v_L = \sum_{L-osc} v_f + \sum_{L-osc} v_{f^*}$$

$$N_R = \sum v_L = \sum_{R-osc} v_f + \sum_{R-osc} v_{f^*}$$

For the NS vacuum

$$\alpha(f) = 0 \quad \rightarrow \quad v_f = v_f^* = \frac{1}{2}$$

4.2 GSO Projections

$$e^{i\pi b_j.F_\alpha}|s\rangle_\alpha = \delta_\alpha C\begin{pmatrix}\alpha \\ b_j\end{pmatrix}^* |s\rangle_\alpha \tag{10}$$

where b_j is the basis vector and α is the sector and $|s\rangle_\alpha$ is the state being analysed. We calculate $b_j.F_\alpha$ by equation 2.

5 String model

5.0.1 All of the Basis vectors

These are the basis vectors in our model, it is generated by eight basis vectors. Real fermions are paired based on having the same boundary conditions in each sector to form complex fermions,

$$f^n f^m = \frac{1}{\sqrt{2}}(f^n + if^m). \tag{11}$$

Also another thing is that left and left fermions together form a complex fermion i.e (w^2w^4), right and right fermions form a complex fermion i.e ($\bar{w}^2\bar{w}^4$), and left and a right fermion forms a complex fermion i.e ($y^4\bar{y}^4$). These fermions that have a left and right moving fermion can form a doubly degenerate vacuum, but they can not be used in the GSO projection as a complex fermion either in the right or left sector. This becomes important in the GSO projections. The following basis vectors are written as paired fermions.

$$
\begin{aligned}
1 &= \{\psi^\mu_{1,2}, \{\chi^i y^i w^i\}, \{\bar{y}^i\bar{w}^i\}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\dots,8}\} \quad i = 1,..,6 \\
\tilde{S} &= \{\psi^\mu_{1,2}, \chi^{1,2}, \chi^{3,4}, \chi^{5,6}, \bar{\phi}^1, \bar{\phi}^4, \bar{\phi}^5, \bar{\phi}^6\} \\
b_1 &= \{\psi^\mu_{1,2}, \chi^{1,2}, y^3y^6, y^4\bar{y}^4, y^5\bar{y}^5, \bar{y}^3\bar{y}^6, \bar{\psi}^{1,2,3}, \bar{\psi}^{4,5}, \bar{\eta}^1\} \\
b_2 &= \{\psi^\mu_{1,2}, \chi^{3,4}, y^1w^5, y^2\bar{y}^2, w^6\bar{w}^6, \bar{y}^1\bar{w}^5, \bar{\psi}^{1,2,3}, \bar{\psi}^{4,5}, \bar{\eta}^2\} \\
b_3 &= \{\psi^\mu_{1,2}, \chi^{5,6}, w^2w^4, w^1\bar{w}^1, w^3\bar{w}^3, \bar{w}^2\bar{w}^4, \bar{\psi}^{1,2,3}, \bar{\psi}^{4,5}, \bar{\eta}^3\} \\
\alpha &= \{y^1w^5, w^1\bar{w}^1, y^2\bar{y}^2, w^2w^4, y^3y^6, w^3\bar{w}^3, y^4\bar{y}^4, y^5\bar{y}^5, w^6\bar{w}^6, \bar{\psi}^{123}, \bar{\phi}^1, \bar{\phi}^{23}, \bar{\phi}^4\} \\
\beta &= \{y^2\bar{y}^2, y^4\bar{y}^4, w^2w^4, \bar{y}^1\bar{w}^5, \bar{y}^3\bar{y}^6, \bar{\psi}^{1,2,3}, \bar{\phi}^1, \bar{\phi}^{2,3}, \bar{\phi}^4\} \\
\gamma &= \{y^1w^5, w^1\bar{w}^1, y^5\bar{y}^5, \bar{w}^2\bar{w}^4, \bar{y}^3\bar{y}^6, \bar{\psi}^{1,2,3}_{\frac{1}{2}}, \bar{\psi}^{4,5}_{\frac{1}{2}}, \bar{\eta}^1_{\frac{1}{2}}, \bar{\eta}^2_{\frac{1}{2}}, \bar{\eta}^3_{\frac{1}{2}}, \bar{\phi}^{2,3}_{\frac{1}{2}}, \bar{\phi}^4_{\frac{1}{2}}, \bar{\phi}^5_{\frac{1}{2}}\}
\end{aligned}
$$

5.0.2 GSO coefficients

The GSO coefficients for our projections are given in the matrix below.

For example $\delta_S\ C\binom{b_2}{S} = (-1)(-1) = +1$, where δ_S is $e^{i\pi S(\psi^\mu)} = -1$.

$$
\begin{array}{c|cccccccc}
 & 1 & S & b_1 & b_2 & b_3 & \alpha & \beta & \gamma \\
\hline
1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & i \\
\tilde{S} & +1 & -1 & +1 & +1 & +1 & -1 & -1 & i \\
b_1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 \\
b_2 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 \\
b_3 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 \\
\alpha & +1 & +1 & +1 & -1 & +1 & +1 & +1 & +1 \\
\beta & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 \\
\gamma & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1
\end{array}
$$

5.1 Old and New NAHE Set

Free fermionic heterotic string models with three generations were being built since the late eighties [4]. These models have the same underlying structure, the basis vectors spanning these different models had the common NAHE-set, denoted as $\{\mathbf{1}, S, b_1, b_2, b_3\}$. The gauge group at the level of the NAHE-set is $SO(10) \times SO(6)^3 \times E_8$, with 48 multiplets in the spinorial 16 representation of $SO(10)$, obtained from the (observable matter sectors) sectors b_1,b_2 and b_3. The S-vector $\{\psi^\mu, \chi^{1,2}, \chi^{3,4}, \chi^{5,6}\}$ generates the spacetime supersymmetry, which is reduced to $N = 2$ by addition of the basis vector b_1 and further reduced to $N = 1$ by the addition of b_2. The GSO projection of b_3 will either preserve or remove the remaining supersymmetry. The next step is to add basis vectors, typically denoted as $\{\alpha, \beta, \gamma\}$. The addition of these basis vectors breaks the $SO(10)$ gauge symmetry to $SU(3)\times SU(2)\times U(1)^2$ and at the same time reduces the number of generations to three. Where the weak hypercharge is given by the combination

$$U(1)_Y = \frac{1}{2}U(1)_C + \frac{1}{3}U(1)_L.$$

Each of the sectors b_1, b_2 and b_3 produces one generation of the standard model, the 16 multiplets of $SO(10)$.

The basis vector 1 is required by the consistency conditions rules to be included in the model and generates a model with $SO(32)$ gauge group fro the Neveu-Schwarz (NS) sector. The spacetime supersymmetry generator is the basis vector S.

We will construct our model with the modified $\overline{\text{NAHE}}$-set, where the S vector is replaced by the $\tilde{S}$ vector. The $\tilde{S}$ vector will remove supersymmetry from the model. The modified NAHE-set along with the addition of the basis vectors α, β, γ will reproduce the Standard Model gauge group (so we get the 3 generations standard model families). The basis vectors b_1, b_2, b_3 will project out the untwisted tachyons for the NS sector. The states form the sectors $\tilde{S} + b_i$ will not produce massless states and will be too heavy, we are only concerned with producing the lowest energy states (but not tachyonic states as these are unphysical, in general, the $\overline{\text{NAHE}}$-set will produce tachyons, you can choose the GSO in such a way to project all the tachyons out in some cases). The model analysed in this paper will produce tachyonic sectors, hence the tachyons are not projected out.

The $\overline{\text{NAHE}}$-set will produce the same results in some respect to the model with normal NAHE-set, the similarity is that the standard model spectrum is reproduced, despite the fact that the two vacua being very different one is supersymmetric and one is not. The $\overline{\text{NAHE}}$-set will produce the $SO(10)$ gauge group from ψ^{12345}, and then with the addition of basis vectors α, β, γ our model we will break down the gauge group from $SO(10) \to SU(5) \times U(1)$ which then gives $SU(3) \times SU(2) \times U(1)_C\ U(1)_L$.

5.2 Sector analysis

Here we will analyse a sector from this model as an example of how to use the method to analyse the spectrum. I have chosen the sector $b_1+b_3+\alpha+3\gamma$, since this is a sector that contains an oscillator for it to be massless. The left side of this spectrum is massless and right side requires an oscillator, so this example contains all of the method.

The fermions in the sector $b_1+b_3+\alpha+3\gamma$ are:

$$S:\{\chi^{12},\chi^{56},y^2\bar{y}^2,y^5\bar{y}^5,w^1\bar{w}^1,w^6\bar{w}^6,\bar{\phi}^1,\bar{\psi}^{123}_{1/2},\bar{\psi}^{45}_{-1/2},\bar{\eta}^1_{1/2},\bar{\eta}^2_{-1/2},\bar{\eta}^3_{1/2},\bar{\phi}^{234}_1/2,\bar{\phi}^5_{-1/2}\}$$

where I have written the fermions in their complex pairs.

The ABK rules on the basis vectors, are satisfied.

Next step is to check if the sector is massless; or how many oscillator it requires, we use the Virasoro condition for this

$$M_L^2=-\frac{1}{2}+\frac{s_L.s_L}{8}+N_L=-1+\frac{s_R.s_R}{8}+N_R=M_R^2$$

here s_L s_R are the left and right part of the sector s and N_L N_R are the total amount of the left and right oscillators. So the left part of the sector contains

$$\alpha_L.\alpha_L=\{\frac{1}{2}\sum_{real\ left}+\sum_{complex\ left}\}\alpha_L.\alpha_L$$

$$=\chi^{12}\alpha^2+\chi^{56}\alpha^2+\frac{1}{2}(y^2)\alpha^2+\frac{1}{2}(y^5)\alpha^2+\frac{1}{2}(w^1)\alpha^2+\frac{1}{2}(w^6)\alpha^2$$

$$=1+1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=4$$

a $\frac{1}{2}$ is multiplied with real fermions, two real fermions form one complex fermion, which ever way you count you get the same value at the end. We have that the left part of the sector is massless

$$M_L^2=-\frac{1}{2}+\frac{4}{8}=0$$

no oscillator needed here.

For the right moving part we repeat the same procedure,

$$s_R.s_R=\frac{1}{2}(\bar{y}^2)\alpha^2+\frac{1}{2}(\bar{y}^5)\alpha^2,\frac{1}{2}(\bar{w}^1)\alpha^2,\frac{1}{2}(\bar{w}^6)\alpha^2,\bar{\phi}^1)\alpha^2,(\psi^{\bar{1}23}_{1/2})\alpha^2+(\psi^{4\bar{5}}_{-1/2})\alpha^2$$
$$+(\eta^{\bar{1}}_{1/2})\alpha^2,(\eta^{\bar{3}}_{1/2})\alpha^2,\eta^{2^-}_{-1/2})\alpha^2,\phi^{\bar{2}34}_{1/2}\alpha^2,\phi^{5^-}_{-1/2})\alpha^2$$

$$=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+1+\frac{3}{4}+\frac{2}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{3}{4}+\frac{1}{4}=6$$

thus,

$$M_R^2=-1+\frac{6}{8}+N_R=0$$

here $N_R = \frac{1}{4}$.

We can apply one right moving oscillator with boundary condition $\alpha(f) = -1/2$,

$$\nu_f = \frac{1+\alpha(f)}{2} = \frac{1}{4},$$

or a complex conjugate oscillator

$$\nu_f{}^* = \frac{1-\alpha(f)}{2} = \frac{1}{4}.$$

The oscillators that can be applied are $\bar{\psi}^{123}_{1/2}, \bar{\psi}^{45}_{-1/2}, \bar{\eta}^1_{1/2}, \bar{\eta}^2_{-1/2}, \bar{\eta}^3_{1/2}, \bar{\phi}^{234}_1/2, \bar{\phi}^5_{-1/2}$. At this point we also know that the state is

$$\bar{\phi}^a_{1/4}|s\rangle$$

so in this state there is only on right moving oscillator which produce a mass of 1/4.

The next step is to do the GSO projections, we will do GSO projections for all the vectors $1, \tilde{S}, b_1, b_2, b_3, \alpha, \beta, \gamma$. The GSO projections for the first basis vector 1

$$e^{i\pi 1.F_S}|s\rangle = \delta_s C\begin{pmatrix} F_s \\ 1 \end{pmatrix}^* = \delta_{b_1+b_3+\alpha+3\gamma} C\begin{pmatrix} b_1+b_3+\alpha+3\gamma \\ 1 \end{pmatrix}^*.$$

To work this out we need to use the rules on the one loop phases given at the start of the section.

$$\delta_s C\begin{pmatrix} F_s \\ 1 \end{pmatrix}^* = \delta_s \left(e^{i\pi \frac{1.F_s}{2}} C\begin{pmatrix} 1 \\ F_s \end{pmatrix}^* \right)^* = \delta_s e^{-i\pi \frac{1.F_s}{2}} C\begin{pmatrix} 1 \\ F_s \end{pmatrix}$$

$$\delta_s e^{-i\pi \frac{1.F_s}{2}} C\begin{pmatrix} 1 \\ 1+b_3+\alpha+3\gamma \end{pmatrix},$$

$$\delta_s e^{-i\pi \frac{1.F_s}{2}} \delta_1 C\begin{pmatrix} 1 \\ b_1 \end{pmatrix} C\begin{pmatrix} 1 \\ b_3+\alpha+3\gamma \end{pmatrix},$$

$$\delta_s e^{-i\pi \frac{1.F_s}{2}} \delta_1 C\begin{pmatrix} 1 \\ b_1 \end{pmatrix} \delta_1 C\begin{pmatrix} 1 \\ b_3 \end{pmatrix} C\begin{pmatrix} 1 \\ \alpha+3\gamma \end{pmatrix},$$

$$\delta_s e^{-i\pi \frac{1.F_s}{2}} \delta_1 C\begin{pmatrix} 1 \\ b_1 \end{pmatrix} \delta_1 C\begin{pmatrix} 1 \\ b_3 \end{pmatrix} \delta_1 C\begin{pmatrix} 1 \\ \alpha \end{pmatrix} C\begin{pmatrix} 1 \\ 3\gamma \end{pmatrix},$$

and the 3γ can be broken in a similar way, the $\delta_1 = -1$ can be grouped to give $(\delta_1)^3$ and the $\delta_s = +1$ from the definition since it does not have a ψ^μ,

$$= (i).(-1)^3.(-1).(-1).(+1).(-i) = -1$$

.

We now know that the GSO projection of the basis vector 1 have to give -1. Each fermion that is periodic can occupy one of the two states $|\pm\rangle$,

$$|+\rangle = e^{i\pi 1.\alpha(f)} = +1 \qquad |-\rangle = e^{i\pi 1.\alpha(f)} = -1.$$

The notation we will use is $\binom{i}{j}$, i is the total number of periodic fermions states and j is the number of $|-\rangle$ states, for example

$$\binom{2}{1} = |+\rangle|-\rangle,$$

$$\binom{2}{0} = |+\rangle|+\rangle,$$

and

$$\binom{2}{\text{even}} = |+\rangle|+\rangle \text{ or } |-\rangle|-\rangle.$$

Let us begin the GSO projections.

1 Ramond Sector Projection

GSO projections is

$$e^{i\pi 1.F_s} e^{i\pi 1.\alpha(\bar{\phi})}|s\rangle = -|s\rangle$$

in the 1 projection $e^{i\pi 1.\alpha(\bar{\phi})} = -1$ for all oscillators ($1.\alpha(\bar{\phi}^a)$ means that what are the boundary conditions for this fermion in the basis vector 1; which is 1 since all fermion have periodic B.C = 1 on the basis vector 1), hence we require that the number of negative periodic fermion states is even, so $e^{i\pi 1.F_s} = e^{i\pi 1.\#|-\rangle}$ has to give a positive '+' contribution. The states that survive the GSO projections are

$$\binom{7}{\text{even}}_{\chi^{12},\chi 56, y^2\bar{y}^2, y^5\bar{y}^5, w^1\bar{w}^1, w^6\bar{w}^6, \bar{\phi}^1} \bar{\psi}^{123}_{1/2}, \bar{\psi}^{45}_{-1/2}, \bar{\eta}^1_{1/2}, \bar{\eta}^2_{-1/2}, \bar{\eta}^3_{1/2}, \bar{\phi}^{234}_{1/2}, \bar{\phi}^5_{-1/2}.$$

$\tilde{S}$ Projection

The GSO for the $\tilde{S}$ vector is

$$e^{i\pi\tilde{S}.F_s} e^{i\pi\tilde{S}\alpha(\bar{\phi})}|s\rangle = -|s\rangle.$$

There are two different possibilities here, the first is we act on oscillators which are present in the $\tilde{S}$, these are $\bar{\phi}^4_{1/2}, \bar{\phi}^5_{-1/2}$ which will produce

$$e^{i\pi\tilde{S}.\alpha(\bar{\phi}^a)} = e^{i\pi.1} = -1$$

and the other is we act on fermionic oscillators which do not exist in the $\tilde{S}$, $e^{i\pi\tilde{S}\alpha(\bar{\phi}^a)} = e^{i\pi.0} = +1$.

The states are

$$\left[\binom{3}{\text{even}}\right]_{\chi^{12},\chi^{56},\bar{\phi}^1} \left[\binom{4}{\text{even}}\right]_{y^2\bar{y}^2, y^5\bar{y}^5, w^1\bar{w}^1, w^6\bar{w}^6} \{\bar{\phi}^4_{1/2}, \bar{\phi}^5_{-1/2}\},$$

$$\left[\begin{pmatrix}3\\ \text{odd}\end{pmatrix}\right]_{\chi^{12},\chi^{56},\bar{\phi}^1} \left[\begin{pmatrix}4\\ \text{odd}\end{pmatrix}\right]_{y^2\bar{y}^2,y^5\bar{y}^5,w^1\bar{w}^1,w^6\bar{w}^6} \{\bar{\psi}^{123}_{1/2},\bar{\psi}^{45}_{-1/2},\bar{\eta}^1_{1/2},\bar{\eta}^2_{-1/2},\bar{\eta}^3_{1/2},\bar{\phi}^{23}_{1/2}\}.$$

We will carry on the projections all the way, of the 2 state only for brevity, since this is an example of applying the method to derive the spectrum. Once we know the rules and how to do the projections, we can always go back and derive the full spectrum.

b_1 Projection

$$e^{i\pi b_1.F_s}e^{i\pi b_1\alpha(\bar{\phi}^a)}|s\rangle = +|s\rangle.$$

In b_1 sector there are $\bar{\psi}^{123},\bar{\psi}^{45},\bar{\eta}^1$ oscillators present. The following states are not all the different states produced at this point, but only the ones which contain $\bar{\psi}^{123},\bar{\psi}^{45},\bar{\eta}^1$ as these are the states that will survive till the end,

$$\left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}2\\1\end{pmatrix}\right]_{\chi^{56},\bar{\phi}^1} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}3\\ \text{even}\end{pmatrix}\right]_{y^2\bar{y}^2,w^1\bar{w}^1,w^6\bar{w}^6} \{\bar{\psi}^{123}_{1/2},\bar{\psi}^{45}_{-1/2},\bar{\eta}^1_{1/2}\},$$

$$\left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}2\\ \text{even}\end{pmatrix}\right]_{\chi^{56},\bar{\phi}^1} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}3\\ \text{even}\end{pmatrix}\right]_{y^2\bar{y}^2,w^1\bar{w}^1,w^6\bar{w}^6} \{\bar{\psi}^{123}_{1/2},\bar{\psi}^{45}_{-1/2},\bar{\eta}^1_{1/2}\}.$$

b_2 Projection

$$e^{i\pi b_2.F_s}e^{i\pi b_2\alpha(\bar{\phi}^a)}|s\rangle = -|s\rangle.$$

The states break further

$$\left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}2\\1\end{pmatrix}\right]_{\chi^{56},\bar{\phi}^1} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}2\\ \text{even}\end{pmatrix}\right]_{y^2\bar{y}^2,w^6\bar{w}^6} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\psi}^{123}_{1/2},\bar{\psi}^{45}_{-1/2}\},$$

$$\left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}2\\ \text{even}\end{pmatrix}\right]_{\chi^{56},\bar{\phi}^1} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}2\\ \text{even}\end{pmatrix}\right]_{y^2\bar{y}^2,w^6\bar{w}^6} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\psi}^{123}_{1/2},\bar{\psi}^{45}_{-1/2}\},$$

again these were not all the states that are produced but are the ones that will survive to the end.

b_3 Projection

The GSO projection for this sector is

$$e^{i\pi b_3.F_s}e^{i\pi b_3\alpha(\bar{\phi}^a)}|s\rangle = +|s\rangle.$$

The states that survive the projection are

$$\left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{56}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\bar{\phi}^1} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}2\\ \text{even}\end{pmatrix}\right]_{y^2\bar{y}^2,w^6\bar{w}^6} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\psi}^{123}_{1/2},\bar{\psi}^{45}_{-1/2}\},$$

$$\left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{56}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\bar{\phi}^1} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}2\\\text{even}\end{pmatrix}\right]_{y^2\bar{y}^2,w^6\bar{w}^6} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\psi}^{123}_{1/2}, \bar{\psi}^{45}_{-1/2}\}.$$

α Projection

The GSO projection for this sector is

$$e^{i\pi\alpha.F_s} e^{i\pi\alpha\alpha(\bar{\phi}^a)}|s\rangle = +|s\rangle.$$

The states that survive the projection are

$$\left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{56}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\bar{\phi}^1} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}2\\\text{even}\end{pmatrix}\right]_{y^2\bar{y}^2,w^6\bar{w}^6} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\psi}^{123}_{1/2}\},$$

$$\left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{56}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\bar{\phi}^1} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}2\\\text{even}\end{pmatrix}\right]_{y^2\bar{y}^2,w^6\bar{w}^6} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\psi}^{123}_{1/2}\}.$$

β Projection

The GSO projection for this sector is

$$e^{i\pi\beta.F_s} e^{i\pi\beta\alpha(\bar{\phi}^a)}|s\rangle = +|s\rangle.$$

The states that survive the projection are

$$\left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{56}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\bar{\phi}^1} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{y^2\bar{y}^2} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{w^6\bar{w}^6} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\psi}^{123}_{1/2}\},$$

$$\left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{56}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\bar{\phi}^1} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{y^2\bar{y}^2} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{w^6\bar{w}^6} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\psi}^{123}_{1/2}\}.$$

γ Projection

The GSO projection for this sector is

$$e^{i\pi\beta.F_s} e^{i\pi\beta\alpha(\bar{\phi}^a)}|s\rangle = -i|s\rangle.$$

All the states from the previous sector survive the GSO projections. For each oscillator there is a separate state, so each of the 2 states splits into 3 states, in total we end up with 6 states.

If we were to go back and finish doing all the projections then we will have additional 2 states,

$$\left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{56}} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\bar{\phi}^1} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{y^2\bar{y}^2} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{w^6\bar{w}^6} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\phi}^{4}_{1/2}\},$$

$$\left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{\chi^{12}} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\chi^{56}} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{\bar{\phi}^1} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{y^5\bar{y}^5} \left[\begin{pmatrix}1\\0\end{pmatrix}\right]_{y^2\bar{y}^2} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{w^6\bar{w}^6} \left[\begin{pmatrix}1\\1\end{pmatrix}\right]_{w^1\bar{w}^1} \{\bar{\phi}^{4}_{1/2}\}.$$

Overall we get 8 states. This concludes the projections for this sector.

6 Tachyonic sector analysis

This is the sector where we can not achieve a massless state so this sector produces tachyons. The sector is defined as : $\tilde{S} + b_1 + b_2 + b_3 + \alpha + \beta + 2\gamma$

$$S : \{y^4\bar{y}^4, y^2\bar{y}^2, w^2w^4, \bar{w}^2\bar{w}^4, \bar{\phi}^1, \bar{\phi}^{23}\bar{\phi}^6\} .$$

The Virasoro condition

$$M_L^2 = -\frac{1}{2} + \frac{2}{8} + N_L = -1 + \frac{6}{8} + N_R = M_R^2,$$

$$M_L^2 = -\frac{1}{4}N_L = -\frac{1}{4} + N_R = M_R^2,$$

the Left and the right part of the equation are the same, so we can get a tachyon state. Here we can not get an oscillator that produces $+1/4$ to make the state massless, since we do not have $\alpha(f) = 1/2$ boundary condition fermions in this sector.

GSO Projections

The projections are done the same way as in the previous sector. All fermions are periodic in this sector and there are no oscillators.

1 Projection

$$e^{i\pi 1.F_s}|s\rangle = -|s\rangle$$

$$\left[\begin{pmatrix} 8 \\ \text{odd} \end{pmatrix}\right]_{y^4\bar{y}^4, y^2\bar{y}^2, w^2w^4, \bar{w}^2\bar{w}^4, \bar{\phi}^1, \bar{\phi}^{23}\bar{\phi}^6} .$$

$\tilde{S}$ Projection

$$e^{i\pi \tilde{S}.F_s}|s\rangle = -|s\rangle$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\bar{\phi}^1} \left[\begin{pmatrix} 7 \\ \text{odd} \end{pmatrix}\right]_{y^4\bar{y}^4, y^2\bar{y}^2, w^2w^4, \bar{w}^2\bar{w}^4, \bar{\phi}^{23}\bar{\phi}^6} .$$

b_1 Projection

$$e^{i\pi b_1.F_s}|s\rangle = -|s\rangle$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\bar{\phi}^1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}_{y^4\bar{y}^4} \left[\begin{pmatrix} 6 \\ \text{even} \end{pmatrix}\right]_{y^2\bar{y}^2, w^2w^4, \bar{w}^2\bar{w}^4, \bar{\phi}^{23}\bar{\phi}^6}$$

b_2 Projection

$$e^{i\pi b_2.F_s}|s\rangle = -|s\rangle$$

$$\binom{1}{0}_{\bar{\phi}^1}\binom{1}{1}_{y^4\bar{y}^4}\binom{1}{0}_{y^2\bar{y}^2}\left[\binom{5}{\text{even}}\right]_{w^2w^4,\bar{w}^2\bar{w}^4,\bar{\phi}^{23}\bar{\phi}^6}$$

b_3 Projection

$$e^{i\pi b_3.F_s}|s\rangle = -|s\rangle$$

$$\binom{1}{0}_{\bar{\phi}^1}\binom{1}{1}_{y^4\bar{y}^4}\binom{1}{0}_{y^2\bar{y}^2}\binom{2}{\text{even}}_{w^2w^4,\bar{w}^2\bar{w}^4}\left[\binom{3}{\text{even}}\right]_{\bar{\phi}^{23}\bar{\phi}^6}$$

α Projection

$$e^{i\pi\alpha.F_s}|s\rangle = -|s\rangle$$

$$\binom{1}{0}_{\bar{\phi}^1}\binom{1}{1}_{y^4\bar{y}^4}\binom{1}{0}_{y^2\bar{y}^2}\binom{2}{2}_{w^2w^4,\bar{w}^2\bar{w}^4}\binom{1}{0}_{\bar{\phi}^6}\binom{2}{\text{even}}_{\bar{\phi}^{23}},$$
$$\binom{1}{0}_{\bar{\phi}^1}\binom{1}{1}_{y^4\bar{y}^4}\binom{1}{0}_{y^2\bar{y}^2}\binom{2}{0}_{w^2w^4,\bar{w}^2\bar{w}^4}\binom{1}{1}_{\bar{\phi}^6}\binom{2}{1}_{\bar{\phi}^{23}}.$$

β Projection

The state is the same after this projection.

$$e^{i\pi\beta.F_s}|s\rangle = -|s\rangle$$

$$\binom{1}{0}_{\bar{\phi}^1}\binom{1}{1}_{y^4\bar{y}^4}\binom{1}{0}_{y^2\bar{y}^2}\binom{2}{2}_{w^2w^4,\bar{w}^2\bar{w}^4}\binom{1}{0}_{\bar{\phi}^6}\binom{2}{\text{even}}_{\bar{\phi}^{23}},$$
$$\binom{1}{0}_{\bar{\phi}^1}\binom{1}{1}_{y^4\bar{y}^4}\binom{1}{0}_{y^2\bar{y}^2}\binom{2}{0}_{w^2w^4,\bar{w}^2\bar{w}^4}\binom{1}{1}_{\bar{\phi}^6}\binom{2}{1}_{\bar{\phi}^{23}}.$$

γ Projection

$$e^{i\pi\gamma.F_s}|s\rangle = -|s\rangle$$

$$\binom{1}{0}_{\bar{\phi}^1}\binom{1}{1}_{y^4\bar{y}^4}\binom{1}{0}_{y^2\bar{y}^2}\binom{1}{1}_{w^2w^4}\binom{1}{1}_{\bar{w}^2\bar{w}^4}\binom{1}{0}_{\bar{\phi}^6}\binom{2}{2}_{\bar{\phi}^{23}},$$
$$\binom{1}{0}_{\bar{\phi}^1}\binom{1}{1}_{y^4\bar{y}^4}\binom{1}{0}_{y^2\bar{y}^2}\binom{1}{1}_{w^2w^4}\binom{1}{1}_{\bar{w}^2\bar{w}^4}\binom{1}{0}_{\bar{\phi}^6}\binom{2}{0}_{\bar{\phi}^{23}}.$$

This concludes the GSO projections for this sector.

6.1 NS sector

The Neveu-Schwarz (NS) sector is where all the fermions have 0 boundary conditions on all fermions, so the NS sector contains no fermions. Once we apply the Virasoro condition to this sector we get,

$$M_L^2 = -\frac{1}{2} + \frac{\alpha_L.\alpha_L}{8} + N_L = -1 + \frac{\alpha_R.\alpha_R}{8} + N_R = M_R^2$$

$\alpha_L.\alpha_L = 0$ and $\alpha_R.\alpha_R = 0$ since there are no fermions in this sector.

$$M_L^2 = -\frac{1}{2} + \frac{0}{8} + N_L = -1 + \frac{0}{8} + N_R = M_R^2$$

To satisfy this equation and produce the massless sector we require $N_L = \frac{1}{2}$ and $N_R = 1$, that it requires one left moving fermionic oscillator and two right moving fermionic oscillator or one right moving bosonic oscillator. You can also get a tachyonic state, by only having one right moving fermionic oscillator. This will produce particles with negative mass.

The 4 masless states from this sector,

$$\psi^{\mu}_{\frac{1}{2}} \delta \bar{X}^{\nu}_{1} |0\rangle_{NS} \tag{12}$$

$$\psi^{\mu}_{\frac{1}{2}} \bar{\phi}^{a}_{\frac{1}{2}} \bar{\phi}^{b}_{\frac{1}{2}} |0\rangle_{NS} \tag{13}$$

$$\{\chi^i y^i w^i\} \bar{\phi}^{a}_{\frac{1}{2}} \bar{\phi}^{b}_{\frac{1}{2}} |0\rangle_{NS} \tag{14}$$

$$\{\chi^i y^i w^i\} \delta \bar{X}^{\nu}_{1} |0\rangle_{NS} \tag{15}$$

and one tachyonic state.

$$\bar{\phi}^{a}_{\frac{1}{2}} |0\rangle_{NS} \tag{16}$$

GSO Projections

$$e^{i\pi b_i F_{NS}} |0\rangle_{NS} = \delta_{NS} \; C\begin{pmatrix} NS \\ b_i \end{pmatrix} = \delta_{b_i}$$

This condition has to be satisfied for each basis vector.

$$\psi^{\mu}_{\frac{1}{2}} \delta \bar{X}^{\nu}_{1} |0\rangle_{NS}$$

Where $\delta \bar{X}^{\nu}_{1}$ is the bosonic creation operator with frequency $\nu_f = 1$, survives the GSO projections and gives spin 2 graviton.

$$\psi^{\mu}_{\frac{1}{2}} \bar{\phi}^{a}_{\frac{1}{2}} \bar{\phi}^{b}_{\frac{1}{2}} |0\rangle_{NS} \qquad \{a, b = 1, ..., 44\}$$

This state will produce the vector gauge bosons, there are observable and hidden groups which are produced as follows.

Observable standard model gauge group $SU(3)$x$SU(2)$x$U(1)_C$x$U(1)_L$.

- $\psi^\mu\psi^{123}\psi^{123*}\psi^\mu\bar{\zeta}\bar{\zeta}^*|0\rangle_{NS}$ will produce $SU(3)_C$x$U(1)_C$.
- $\psi^\mu\psi^{45}\psi^{45*}\psi^\mu\bar{\zeta}\bar{\zeta}^*|0\rangle_{NS}$ will produce $SU(2)_L$x$U(1)_L$.

The $U(1)_C$x$U(1)_L$ symmetry breaks to the weak hypercharge $U(1)_Y$,

$$U(1) = \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L$$

The $\psi^\mu\bar{\zeta}\bar{\zeta}^*|0\rangle_{NS}$ will produce the the observable $U(1)$, where $\bar{\zeta}$ are complex right moving fermions e.g $\bar{\zeta} = \frac{1}{\sqrt{2}}(\bar{y}^3 + i\bar{y}^6)$.

$$\psi^\mu\bar{\eta}^a\bar{\eta}^{a*}|0\rangle_{NS} \qquad (a = 1, 2, 3)$$

{a = 1,2,3} This will also produce the observable $U(1)$.

The vector gauge bosons of the hidden group are:

- $\psi^\mu\bar{\phi}^1\bar{\phi}^1|0\rangle_{NS} \quad U(1)_7$
- $\psi^\mu\bar{\phi}^4\bar{\phi}^{4*}|0\rangle_{NS} \quad U(1)_8$
- $\psi^\mu\bar{\phi}^5\bar{\phi}^{5*}|0\rangle_{NS} \quad U(1)_9$
- $\psi^\mu\bar{\phi}^6\bar{\phi}^6|0\rangle_{NS} \quad U(1)_{10}$
- $\psi^\mu\bar{\phi}^{23}\bar{\phi}^{23*}|0\rangle_{NS} \quad SO(4)_H \Rightarrow SU(2)_HxU(1)_H$
- $\psi^\mu\bar{\phi}^{78}\bar{\phi}^{78}|0\rangle_{NS} \quad SO(4)_H$

$$\{\chi^i y^i w^i\}\bar{\phi}^a_{\frac{1}{2}}\bar{\phi}^b_{\frac{1}{2}}|0\rangle_{NS}$$

This state corresponds to the spacetime scalers which are neuteral under U(1) symmetries, in this model the scalers are:

- $\chi^{1,2}\bar{\psi}^{45}\bar{\eta}^{1*}|0\rangle_{NS}$
- $\chi^{1,2}\bar{\eta}^2\bar{\eta}^{3*}|0\rangle_{NS}$
- $\chi^{3,4}\bar{\eta}^1\bar{\eta}^{3*}|0\rangle_{NS}$
- $\chi^{3,4}\ \bar{\psi}^{45}\bar{\eta}^{2*}|0\rangle_{NS}$
- $\chi^{5,6}\ \bar{\psi}^{45}\bar{\eta}^{3*}|0\rangle_{NS}$
- $\chi^{5,6}\bar{\eta}^1\bar{\eta}^{2*}|0\rangle_{NS}$

These scaler have charges under the observable gauge group the full table for the NS-sector is in the appendix, but we go through one example here of how to get the charges and different scalers from each state.

			$SU(3)_C$	$U(1)_C$	$SU(2)_L$	$U(1)_L$	$U(1)_{\eta^1}$	$U(1)_{\eta^2}$	$U(1)_{\eta^3}$	$\bar{y}^{3,6}$	$\bar{y}^1\bar{w}^5$	$\bar{w}^{2,4}$
$\bar{h}$	$\chi^{56}\ \bar{\psi}^{45}\ \bar{\eta}^{1*}\|0\rangle$	=	1	0	2	+1	-1	0	0	0	0	0
h	$\chi^{56}\ \bar{\psi}^{45*}\ \bar{\eta}^{1}\|0\rangle$	=	1	0	2	-1	+1	0	0	0	0	0

Here 1 under $SU(3)_C$ means a singlet of $SU(3)$, since there is no ψ^{123} present in these states, the charge under $U(1)_C$ is 0. The $\bar{\psi}^{45}$ will form a doublet of $SU(2)$ and have charge ± 1, depending on if it is a complex conjugate or not denoted by the * sign.

$$\{\chi^i y^i w^i\}\delta\bar{X}_1^{\nu}|0\rangle_{NS}$$

This state corresponds to gauge bosons but is projected out during the GSO projections. So this state will produce nothing in this model.

6.1.1 The tachyonic state

The state

$$\bar{\phi}^a_{\frac{1}{2}}|0\rangle_{NS}$$

is projected out. Usually the S vector responsible for supersymmetry will project this tachyon out, but, we had a $\tilde{S}$ (no supersymmetry) which keeps the tachyon since it contains $\bar{\phi}^{1456}$. The b_1, b_2, b_3 will project the tachyon instead, so no tachyons are produced from the NS sector. With this we conclude the projections for this sector.

Since this non-supersymmetric model projects the tachyons out, we can say that non-supersymmetric models are viable for building future models.

6.2 Remaining Massless Sectors

6.2.1 Sectors b_1, b_2, b_3

A full list of states and quantum numbers is given in the Appendix. From these sectors, we have obtained the 3 generations[2] of left moving chiral fermions, the charges and corresponding particles are given in the table 1 Appendix. The particles are SU(2) doublets and the antiparticle are SU(2) singlets.

[2]The 3 generations produced are not going to explicitly state electron, muon, and tau, etc. These specific details are not covered in the model presented here.

6.2.2 Other massless sectors

The remaining massless sector will be spilt into two groups vector representations of the hidden gauge group and sectors that give states which are chiral representations of the hidden gauge group. The appendix contains the massless sectors.

The $\tilde{S}$ basis vector, produces the states $|S\rangle\bar{\phi}^a|S\rangle$ which which are masless.
Sectors that give massless states are: $\beta+\gamma$, $\beta+3\gamma$, $\alpha+\gamma$, $\alpha+3\gamma$, $b_3+2\gamma$, $b_2+2\gamma$,$b_1+b_2+\alpha+2\gamma$,$b_1+2\gamma^3$, $b_1+b_3+\alpha+2\gamma$, $b_1+b_2+\alpha+2\gamma$, $b_1+b_2+b_3+\alpha+2\gamma$, $\tilde{S}+b_3+2\gamma$, $\tilde{S}+b_2+2\gamma$, $\tilde{S}+b_2+b_3+\alpha+2\gamma$, $\tilde{S}+b_1+2\gamma$, $\tilde{S}+b_1+b_3+\alpha+2\gamma$, $\tilde{S}+b_1+b_2+\alpha+2\gamma$, $\tilde{S}+b_1+b_2+\alpha+\beta$, $\tilde{S}+b_1+b_2+b_3+\beta+2\gamma$, $1+b_2+\alpha+\gamma$, $1+b_3+\alpha+3\gamma$, $1+b_2+b_3+2\gamma$, $1+b_2+b_3+2\gamma$, $1+b_1+\beta+\gamma$, $1+b_1+\beta+3\gamma$, $1+b_1+b_3+2\gamma$,$1+b_1+b_2+2\gamma$, $1+\tilde{S}+\gamma$, $1+\tilde{S}+3\gamma$, $1,\tilde{S}+\alpha+\beta+\gamma$, $1+\tilde{S}+\alpha+\beta+3\gamma$, $1+\tilde{S}+b_3+\alpha+\beta$, $1,+\tilde{S}+b_2+\alpha+\gamma$, $1+\tilde{S}+b_2+\alpha+3\gamma$, $1+\tilde{S}+b_1+b_2+2\gamma$, $1+\tilde{S}+b_1+b_3+2\gamma$, $1+\tilde{S}+b_1+b_2+2\gamma$.

Sectors that gave massless states with fermions oscillators acting either on the left or the right hand side, $b_2+b_3+\beta+\gamma$, $b_2+b_3+\beta+3\gamma$, $b_1+b_3+\alpha+\gamma$, $b_1+b_3+\alpha+3\gamma$, $b_1+b_2+\alpha+\beta$, $\tilde{S}+b_2+b_3+\beta+\gamma$, $\tilde{S}+b_2+b_3+\beta+3\gamma$, $\tilde{S}+b_1+b_3+\alpha+\gamma$, $\tilde{S}+b_1+b_3+\alpha+3\gamma$, $\tilde{S}+b_1+b_2+b_3+\alpha+2\gamma$, $1+b_1+b_2+b_3$, $1+\tilde{S}+b_1+b_2+b_3$.

6.3 Tachyonic Sector

The model gives rise to new tachyonic sector as a result of using the $\overline{\text{NAHE}}$-set, these sectors were not produced in previous models (NAHE-set) which includes the supersymmetry. The sector $\tilde{S}+b_1+b_2+b_3+\alpha+\beta+2\gamma$ produces tachyons.

w^2w^4	$y^2\bar{y}^2$	$y^4\bar{y}^4$	$\bar{w}^2\bar{w}^4$	ϕ^1	ϕ^2	ϕ^3	ϕ^6
-1	0	-1	-1	0	0	0	0
0	0	-1	0	-1	-1	-1	-1

Figure 5: The fermions in the sector and the states.

This sector represents that the model is unstable since it gives rise to tachyonic states which are unrealistic, the tachyon corresponds to an unstable vacuum. So this model that we have constructed turned out not to be viable.

[3]Only state up to $b_1+2\gamma$ have been written in the Appendix A.3.

7 Conclusion

We have now analysed the entire massless spectrum of the model. This model has produced features similar to that of the old NAHE-set, the tachyons in the NS sector was projected out without the supersymmetry, and the spectrum of the standard model from the sectors b_1, b_2, b_3 was produced. The NS sector tachyon is projected out, not by $\tilde{S}$ but by the projections of b_1, b_2, b_3. Also as a result of the $\tilde{S}$ the sectors $\tilde{S} + b_i$ where $i = 1, 2, 3$ were too heavy, no massless states can be produced here.

There was a tachyonic sector in this model produced from the basis vectors $\tilde{S} + b_1 + b_2 + b_3 + \alpha + \beta + 2\gamma$, it is possible to project the tachyon sectors out, an example of producing non-supersymmetric models in the FFF that do not produce tachyonic sectors is presented in [6].

This turned out to be a nonviable model but it produced the structures of the standard model and projected the tachyon out from the NS sector. Also, we know that we can fix the problem of the tachyonic sector by appropriate choices of GSOs and or changing the model slightly. These non-supersymmetric models increase the range of models that can be built. In that string theory is relevant and there is no supersymmetry then we want these models in the ABK formulation to still be valid, as shown here that non-supersymmetric models are relevant since they produce the same physics as the supersymmetric models, with a few differences.

A The Spectrum of the Model

A.1 The untwisted Neveu-Schwarz sector

F	Sector	Name	(C,L)	Q_C	Q_L	$Q_{\bar\eta^1}$	$Q_{\bar\eta^2}$	$Q_{\bar\eta^3}$	$Q_{\bar y^{3,6}}$	$Q_{\bar y^1 \bar w^5}$	$Q_{\bar w^{2,4}}$	$SO(4)$x$SU(2)$	Q_H	$Q_{\bar\phi^1}$	$Q_{\bar\phi^4}$	$Q_{\bar\phi^5}$	$Q_{\bar\phi^6}$
b	NS	h_1	(1,2)	0	-1	1	0	0	0	0	0	(1,1)	0	0	0	0	0
		$\bar h_1$	(1,2)	0	1	-1	0	0	0	0	0	(1,1)	0	0	0	0	0
		h_2	(1,2)	0	-1	0	1	0	0	0	0	(1,1)	0	0	0	0	0
		$\bar h_2$	(1,2)	0	1	0	-1	0	0	0	0	(1,1)	0	0	0	0	0
		h_3	(1,2)	0	-1	0	0	1	0	0	0	(1,1)	0	0	0	0	0
		$\bar h_3$	(1,2)	0	1	0	0	-1	0	0	0	(1,1)	0	0	0	0	0
		ϕ_{12}	(1,1)	0	0	1	-1	0	0	0	0	(1,1)	0	0	0	0	0
		$\bar\phi_{12}$	(1,1)	0	0	-1	1	0	0	0	0	(1,1)	0	0	0	0	0
		ϕ_{13}	(1,1)	0	0	1	0	-1	0	0	0	(1,1)	0	0	0	0	0
		$\bar\phi_{13}$	(1,1)	0	0	-1	0	1	0	0	0	(1,1)	0	0	0	0	0
		ϕ_{23}	(1,1)	0	0	0	1	-1	0	0	0	(1,1)	0	0	0	0	0
		$\bar\phi_{23}$	(1,1)	0	0	0	0	1	-1	0	0	(1,1)	0	0	0	0	0

Table 1: The untwisted Neveu-Schwarz scalar states.

A.2 observable matter sectors

F	Sector	Name	(C,L)x$SU(2)_L$	Q_C	Q_L	$Q_{\bar\eta^1}$	$Q_{\bar\eta^2}$	$Q_{\bar\eta^3}$	$Q_{\bar y^{3,6}}$	$Q_{\bar y^1 \bar w^5}$	$Q_{\bar w^{2,4}}$	$SO(4)$x$SU(2)$	Q_H	$Q_{\bar\phi^1}$	$Q_{\bar\phi^4}$	$Q_{\bar\phi^5}$	$Q_{\bar\phi^6}$
f	b_1	Q_1	$(3,2)$	1/2	0	+1/2	0	0	-1/2	0	0	(1,1)	0	0	0	0	0
		L_1	$(1,2)$	-3/2	0	+1/2	0	0	-1/2	0	0	(1,1)	0	0	0	0	0
		u_1^c	$(\bar 3,1)$	-1/2	-1	+1/2	0	0	+1/2	0	0	(1,1)	0	0	0	0	0
		d_1^c	$(\bar 3,1)$	-1/2	1	+1/2	0	0	+1/2	0	0	(1,1)	0	0	0	0	0
		e_1^c	$(1,1)$	3/2	1	+1/2	0	0	+1/2	0	0	(1,1)	0	0	0	0	0
		ν_1^c	$(1,1)$	3/2	-1	+1/2	0	0	+1/2	0	0	(1,1)	0	0	0	0	0
f	b_2	Q_2	$(3,2)$	1/2	0	0	+1/2	0	0	+1/2	0	(1,1)	0	0	0	0	0
		L_2	(1,2)	-3/2	0	0	+1/2	0	0	+1/2	0	(1,1)	0	0	0	0	0
		u_2^c	$(\bar 3,1)$	-1/2	-1	0	+1/2	0	0	-1/2	0	(1,1)	0	0	0	0	0
		d_2^c	$(\bar 3,1)$	-1/2	1	0	+1/2	0	0	-1/2	0	(1,1)	0	0	0	0	0
		e_2^c	(1,1)	3/2	1	0	+1/2	0	0	-1/2	0	(1,1)	0	0	0	0	0
		ν_2^c	(1,1)	3/2	-1	0	+1/2	0	0	-1/2	0	(1,1)	0	0	0	0	0
f	b_3	Q_3	$(3,2)$	1/2	0	0	0	+1/2	0	0	-1/2	(1,1)	0	0	0	0	0
		L_3	(1,2)	-3/2	0	0	0	+1/2	0	0	-1/2	(1,1)	0	0	0	0	0
		u_3^c	$(\bar 3,1)$	-1/2	-1	0	0	+1/2	0	0	+1/2	(1,1)	0	0	0	0	0
		d_3^c	$(\bar 3,1)$	-1/2	1	0	0	+1/2	0	0	+1/2	(1,1)	0	0	0	0	0
		e_3^c	(1,1)	3/2	1	0	0	+1/2	0	0	+1/2	(1,1)	0	0	0	0	0
		ν_3^c	(1,1)	3/2	-1	0	0	+1/2	0	0	+1/2	(1,1)	0	0	0	0	0

Table 2: The observable matter sectors, that produce the Standard model families

A.3 Other Masless sectors

F	Sector	Name	(C,L)	Q_C	Q_L	$Q_{\bar{\eta}^1}$	$Q_{\bar{\eta}^2}$	$Q_{\bar{\eta}^3}$	$Q_{\bar{y}^{3,6}}$	$Q_{\bar{y}^1\bar{w}^5}$	$Q_{\bar{w}^{2,4}}$	$SO(4)$x$SU(2)$	Q_H	$Q_{\bar{\phi}^1}$	$Q_{\bar{\phi}^4}$	$Q_{\bar{\phi}^5}$	$Q_{\bar{\phi}^6}$
f	$\beta+\gamma$		(1,1)	0	0	0	0	0	0	1/2	1/2	(1,1)	0	-1/2	0	0	0
			(1,1)	0	0	0	0	0	0	-1/2	-1/2	(1,1)	0	-1/2	0	0	0
			(1,1)	0	0	0	0	0	0	-1/2	-1/2	(1,1)	0	-1/2	0	0	0
			(1,1)	0	0	0	0	0	0	1/2	1/2	(1,1)	0	-1/2	0	0	0
f	$\beta+3\gamma$		(1,1)	0	0	0	0	0	0	1/2	1/2	(1,1)	0	1/2	0	0	0
			(1,1)	0	0	0	0	0	0	-1/2	-1/2	(1,1)	0	1/2	0	0	0
			(1,1)	0	0	0	0	0	0	-1/2	-1/2	(1,1)	0	1/2	0	0	0
			(1,1)	0	0	0	0	0	0	1/2	1/2	(1,1)	0	1/2	0	0	0
f	$\alpha+\gamma$		(1,1)	0	0	0	0	0	-1/2	0	-1/2	(1,1)	0	-1/2	0	0	0
			(1,1)	0	0	0	0	0	1/2	0	1/2	(1,1)	0	-1/2	0	0	0
			(1,1)	0	0	0	0	0	-1/2	0	-1/2	(1,1)	0	-1/2	0	0	0
			(1,1)	0	0	0	0	0	1/2	0	1/2	(1,1)	0	-1/2	0	0	0
f	$\alpha+3\gamma$		(1,1)	0	0	0	0	0	-1/2	0	-1/2	(1,1)	0	1/2	0	0	0
			(1,1)	0	0	0	0	0	1/2	0	1/2	(1,1)	0	1/2	0	0	0
			(1,1)	0	0	0	0	0	-1/2	0	-1/2	(1,1)	0	1/2	0	0	0
			(1,1)	0	0	0	0	0	1/2	0	1/2	(1,1)	0	1/2	0	0	0
f	$b_3+2\gamma$		(1,1)	0	0	1/2	1/2	0	0	0	1/2	(2,1)	0	0	-1/2	1/2	0
			(1,1)	0	0	1/2	1/2	0	0	0	1/2	(2,1)	0	0	-1/2	1/2	0
			(1,1)	0	0	1/2	1/2	0	0	0	-1/2	(2,1)	0	0	1/2	-1/2	0
			(1,1)	0	0	1/2	1/2	0	0	0	-1/2	(2,1)	0	0	1/2	-1/2	0
			(1,1)	0	0	1/2	1/2	0	0	0	-1/2	(1,1)	+1	0	1/2	1/2	0
			(1,1)	0	0	1/2	1/2	0	0	0	-1/2	(1,1)	-1	0	1/2	1/2	0
			(1,1)	0	0	1/2	1/2	0	0	0	1/2	(1,1)	+1	0	-1/2	-1/2	0
			(1,1)	0	0	1/2	1/2	0	0	0	1/2	(1,1)	-1	0	-1/2	-1/2	0
f	$b_2+2\gamma$		(1,1)	0	0	1/2	0	1/2	0	1/2	0	(2,1)	0	0	1/2	-1/2	0
			(1,1)	0	0	1/2	0	1/2	0	1/2	0	(2,1)	0	0	1/2	-1/2	0
			(1,1)	0	0	1/2	0	1/2	0	-1/2	0	(2,1)	0	0	-1/2	1/2	0
			(1,1)	0	0	1/2	0	1/2	0	-1/2	0	(2,1)	0	0	-1/2	1/2	0
			(1,1)	0	0	1/2	0	1/2	0	-1/2	0	(1,1)	-1	0	-1/2	-1/2	0
			(1,1)	0	0	1/2	0	1/2	0	1/2	0	(1,1)	+1	0	1/2	1/2	0
			(1,1)	0	0	1/2	0	1/2	0	1/2	0	(1,1)	-1	0	1/2	1/2	0
			(1,1)	0	0	1/2	0	1/2	0	-1/2	0	(1,1)	+1	0	-1/2	-1/2	0
f	$b_1+2\gamma$		(1,1)	0	0	0	0	0	-1/2	0	0	(1,1)	0	0	1/2	-1/2	0
			(1,1)	0	0	0	0	0	-1/2	0	0	(1,1)	0	0	1/2	-1/2	0
			(1,1)	0	0	0	0	0	+1/2	0	0	(1,1)	0	0	-1/2	1/2	0
			(1,1)	0	0	0	0	0	+1/2	0	0	(1,1)	0	0	-1/2	1/2	0
			(1,1)	0	0	0	0	0	+1/2	0	0	(2,2)	-1	0	-1/2	-1/2	0
			(1,1)	0	0	0	0	0	-1/2	0	0	(1,1)	+1	0	1/2	1/2	0
			(1,1)	0	0	0	0	0	-1/2	0	0	(1,1)	-1	0	1/2	1/2	0
			(1,1)	0	0	0	0	0	+1/2	0	0	(1,1)	+1	0	-1/2	-1/2	0
f	$b_1+2\gamma$		(1,1)	0	0	0	0	0	-1/2	0	0	(1,1)	0	0	1/2	-1/2	0
			(1,1)	0	0	0	0	0	-1/2	0	0	(1,1)	0	0	1/2	-1/2	0
			(1,1)	0	0	0	0	0	+1/2	0	0	(1,1)	0	0	-1/2	1/2	0
			(1,1)	0	0	0	0	0	+1/2	0	0	(1,1)	0	0	-1/2	1/2	0
			(1,1)	0	0	0	0	0	+1/2	0	0	(2,2)	-1	0	-1/2	-1/2	0
			(1,1)	0	0	0	0	0	-1/2	0	0	(1,1)	+1	0	1/2	1/2	0
			(1,1)	0	0	0	0	0	-1/2	0	0	(1,1)	-1	0	1/2	1/2	0
			(1,1)	0	0	0	0	0	+1/2	0	0	(1,1)	+1	0	-1/2	-1/2	0

Table 3: The massless sectors

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