An Analysis of Non-Supersymmetric Pati-Salam Models in the Free Fermionic Formulation

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Chapter 1

Introduction

The goal of this project is to work towards building a non-supersymmetric Pati-Salam model in the free fermionic formulation. These models will differ from the commonly seen classification literature by replacement of the supersymmetry basis vector and how we will group the basis vectors relating to the internal degrees of freedom of the orbifold. The grouping of this reduces the total number of sectors but actually constricts our moduli space and has some surprising consequences on the phenomenology of this group of models. We will finally end with looking at a model which has been generated by a computer code and analysing the model by hand whilst discussing some of the key features the choices of GGSO phases gives us.

I will try and motivate the construction of these models starting from a recap of the Standard Model, this will mainly be done to further my understanding of the successes and weakness of current models and research topics rather than to fully derive them mathematically. We will try to quickly explain some advantages and disadvantages of physics beyond the Standard Model as we progress towards motivating string constructions and why string theory is a viable candidate of unification.

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Chapter 2

The Standard Model and GUTs

2.1 The Standard Model of particle physics

The Standard Model (SM) of particle physics is our current best description of three of the four known fundamental forces and the known elementary particles contained in three generations of observed matter. The SM combines the framework of quantum mechanics and special relativity into a single relativistic quantum gauge theory.

There are three sectors of the SM, which we will cover in more detail shortly, but an overview [25]:

1. **Interactions** - spin-1 gauge bosons
   The interaction of the strong, weak and electromagnetic forces are governed by the gauge symmetry group:
$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \tag{2.1}$$

2. **Matter** - spin-$\frac{1}{2}$ fermions
   Our matter representations, defined by their quantum numbers under the standard model gauge group are:
$$\mathbf{3}{*}\big[(\mathbf{3},\mathbf{2})_{+\frac{1}{6}} + (\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + (\overline{\mathbf{3}},\mathbf{1})_{+\frac{1}{3}} + (\mathbf{1},\mathbf{2})_{-\frac{1}{2}} + (\mathbf{1},\mathbf{1})_{+\mathbf{1}}\big]. \tag{2.2}$$
   These are all in the left-handed representations and are Q_L, u^c_L, d^c_L, L_L and e^c_L respectively. Note that here the first factor of 3 comes from there being three generations of fermions within the standard model.

3. **Higgs** - spin-0 Higgs boson
   Finally, we have the recently discovered scalar Higgs field represented

by:

$$(\mathbf{1}, \mathbf{2})_{+\frac{1}{2}}. \tag{2.3}$$

The interaction sector of the SM is governed by the gauge group:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \tag{2.4}$$

Here, the subscripts C, L and Y represent the charges of colour, left-handed weak isospin and weak hypercharge respectively. Each associated group has it's own symmetries and corresponding vector boson(s) which is the 'force carrying' point particle of the group. The first gauge group, $SU(3)_C$, represents the strong nuclear force which is mediated by gluons G^a_μ with $a = 1, ..., 8$. The last two are slightly different; at face value $SU(2)_L$ and $U(1)_Y$ correspond to the weak isospin and hypercharge symmetries. We would expect to have three gauge bosons for the weak isospin W^i_μ with $i = 1, 2, 3$, and one which would mediate hypercharge B_μ. The $SU(2)$ symmetry forms doublets of left-handed fermions in the fundamental representation of the group and singlets of the right-handed fermions which do not feel the weak interaction. Right-handed antiparticles however do interact, this chirality will be explored further in the matter sector. Weak interactions are carried by the following three gauge bosons which are held in the adjoint representation of $SU(2)$ [10]:

$$W^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad W^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad W^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{2.5}$$

Transitions where $T_3 = +\frac{1}{2} \rightarrow T_3 = -\frac{1}{2}$ would be mediated by the W^+ boson and the reverse interaction by the W^-. Weak interactions where T_3 is unchanged, for example neutrino scattering, would emit a W^0. Hypercharge is chosen so that our electric charge Q obeys the Gell-Mann–Nishijima formula [34][49]:

$$Q = T_3 + \frac{1}{2}Y \tag{2.6}$$

where T_3 is the third component of isospin. Any particle with hypercharge interacts by exchange of a B boson. This symmetry acts like the familiar photon in the electromagnetism $U(1)_{e.m}$ but it's strength is proportional to hypercharge Y rather than electric charge Q.

In reality we actually see interactions governed by the weak nuclear force and electromagnetism. This means our gauge group must undergo spontaneous symmetry breaking. This is caused by the Brout-Englert-Higgs mechanism[54] and causes a rotation of the B and W^0 bosons into the observed photon γ of the electromagnetic interaction and Z^0 boson which mediated neutral currents

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B \\ W^0 \end{pmatrix} \tag{2.7}$$

where θ_W is the weak mixing angle also known as the Weinberg angle. This is explained in more detail in the Higgs sector below. Our $W^{\pm}$ are formed through the combination of the W^1 and W^2 bosons:

$$W^{\pm} = \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \tag{2.8}$$

It also gives reasoning to the differing masses of the Z^0 and $W^{\pm}$ bosons in the following way:

$$m_Z = \frac{m_W}{\cos\theta_W} \tag{2.9}$$

Our matter sector contains all the fermions of the standard model, which are characterised by their half integer spin. All observed fermions (at least currently, have spin-$\frac{1}{2}$), are chiral and are governed by the Dirac equation (natural units $c = 1$):

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0 \tag{2.10}$$

where γ^{μ} are the gamma matrices given by

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}, \gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix} \tag{2.11}$$

With $\sigma_{1,2,3}$ the usual Pauli spin matrices. Our fermion fields can be written in a spinor representation

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{2.12}$$

and our left- and right-handed components can be defined using the projectors acting on ψ

$$P_R = \frac{1+\gamma^5}{2} \quad \text{and} \quad P_L = \frac{1-\gamma^5}{2}, \quad \text{with} \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{2.13}$$

This results in our left-handed quarks and fermions forming doublets which can be seen more clearly by our matter representations, written showing the particle content of each individual representation:

$$(\mathbf{3},\mathbf{2})_{+\frac{1}{6}} \leftrightarrow Q_L = \begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \end{pmatrix} \tag{2.14}$$

$$(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} \leftrightarrow u_L^c = \begin{pmatrix} u_r^c & u_g^c & u_b^c \end{pmatrix} \tag{2.15}$$

$$(\mathbf{1},\mathbf{1})_{+\mathbf{1}} \leftrightarrow (e_L^c) \tag{2.16}$$

$$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}} \leftrightarrow L_L = \begin{pmatrix} \nu_l \\ l \end{pmatrix} \tag{2.17}$$

$$(\bar{\mathbf{3}},\mathbf{1})_{+\frac{1}{3}} \leftrightarrow \begin{pmatrix} d_r^c & d_g^c & d_b^c \end{pmatrix} \tag{2.18}$$

Here, the first entry $\mathbf{3}$, $\bar{\mathbf{3}}$ and $\mathbf{1}$ represent the fundamental triplet, conjugate triplet and the singlet representations of the $SU(3)_C$ group. The second entry represents the fundamental doublet and singlet of $SU(2)_L$ for $\mathbf{2}$ and $\mathbf{1}$ respectively [38]. The subscript represents the charge under the $U(1)_Y$ group, this is the hypercharge Y.

Finally, we have our Higgs sector, we've already mentioned one of the roles of the Higgs, which is the spontaneous symmetry breaking of the $SU(2)_L \otimes U(1)_Y$ group into the $U(1)_{e.m}$ by the Brout-Englert-Higgs mechanism. Here we will only add to what we have already mentioned, that is the electro-weak symmetry breaking, but further reading on how the fermions and $W^{\pm}$ and Z^0 obtain mass can be found here [54]. The higgs ϕ is introduced a scalar field in the spinor representation of $SU(2)_L$, a doublet:

$$(\mathbf{1},\mathbf{2})_{+\frac{1}{2}} = \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.19}$$

From this we see that the charge under the $U(1)_Y$ group is $Y(\phi) = +\frac{1}{2}$. We need a theory which is invariant under local transformations, giving us the covariant derivative of the form:

$$D_\mu \phi = \Big(\partial_\mu + igT^i W_\mu^i + i\frac{1}{2}g^i B_\mu\Big)\phi \tag{2.20}$$

where the second and third terms describe the coupling to the $SU(2)_L$ and $U(1)_Y$ groups respectively. Now our Higgs Lagrangian can be defined by

$$\mathcal{L}_{Higgs} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi). \tag{2.21}$$

Renormalisability and invariance under the $SU(2)_L \otimes U(1)_Y$ group requires a potential $V(\phi)$ to be of the form

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \tag{2.22}$$

with the λ term showcasing the quartic self interaction terms of the Higgs scalar field. In order for a stable vacuum for the potential to exist, we require $\lambda > 0$. For symmetry breaking there must exists a non-zero vacuum expectation value (VEV) which can be achieved with choosing $\mu^2 < 0$, if we instead chose $\mu^2 > 0$ then we would get a minimum at $\phi = 0$ and no broken symmetry, this is a circle of minima of radius v.

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \text{with} \quad v = \sqrt{\frac{-\mu^2}{\lambda}}. \tag{2.23}$$

Only a neutral scalar field can acquire a VEV, due to charge conservation [54], so the ϕ^0 component of 2.19 is seen as the neutral component and electromagnetism remains unbroken:

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{e.m} \tag{2.24}$$

More information and detailed explanation of electroweak symmetry breaking and the process of the $W^\pm$ and Z boson obtaining mass can be found in reference [54]. More information on the Standard Model and the mathematics behind it can be found in [14, 44, 45]

2.2 Problems with The Standard Model

Whilst the Standard Model has been an amazingly successful theory and has many predictions confirmed correct, the latest being the discovery of the Higgs boson by two separate research groups ATLAS [1] and CMS [15], there still exists a range of phenomena that the SM cannot explain meaning it cannot be the complete theory of particle physics. The first of these can be attributed to the new discovery of the Higgs; at high energies, the Higgs quartic coupling becomes negative. This causes the vacuum expectation value of the higgs field to be unstable and requires new physics to explain [18].

The Standard Model in it's current framework requires a large number of free parameters, the actual number can vary depending on whether we want to include the newly discovered neutrino masses and oscillations [48]. There is currently no explanation or motivation for many of these free parameters; these are determined empirically through experimental data and have to be inputted into models by hand. Furthering the problem caused by neutrinos, the SM has no real explanation for these oscillations, neutrino masses or even the hierarchy of the neutrino masses. Our current best explanations are either by extending the standard model through the introduction of right-handed neutrinos to the model to allow for a see-saw mechanism [47] or through the use of supersymmetry. More arbitrariness can be asked of the model: why is the standard model and it's interactions governed by the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge groups? Why do we get exactly three generations of chiral matter? Finally why exactly do we get the charge quantisations that we do? Solutions to these problems can be addressed by exploring higher rank gauge groups and the embedding of the Standard Model into these grand unified theories.

Another problem, and one which will be discussed a little further in the grand unified theories section is the unification of gauge couplings. Taking the data of gauge couplings and their running at higher energies allows us to extrapolate these couplings. In the Standard Model when we do this we see that these don't meet at a single unification point. In the next section, we will introduce grand unified theories (GUTS) and then later we will discuss incorporating supersymmetry, which allows a solution for the unification of gauge couplings when these two extensions of the Standard Model are combined [11].

The next set of problems are mainly asymmetry ones. Firstly, the asymmetry between matter and anti-matter this is sometimes called baryon asymmetry. The Standard Model offers no explanation why we see so much more matter than we do anti-matter, the Standard Model predicts we should have had equal quantities of each in the Big Bang. Another question which the physics of the Standard Model can't answer is the one of dark matter and energy. This makes up a large proportion of the observable universe and hence is question that many have attempted to answer through physics beyond the Standard Model. Some solutions to the dark matter problem have been proposed through both supersymmetry and string phenomenology. The latter of these proposes heavy and stable dark matter candidates through a symmetry breaking mechanism in string theory by Wilson lines and further explanation of the mechanism is given in reference [19]. An alternate solution proposed by strings can be that dark energy can lie in the hidden sector of string models called thermal dark energy and our presented in paper [39].

The final and most glaring problems with the Standard Model come from it's incompatibility with Einstein's General Relativity [21] and it's inability to incorporate gravity within it's QFT framework. Gravity as we know it is described by the continuous smooth curvature of space-time and whilst a quantum theory of gravity would not necessarily be one which implies a discreetness of spacetime, which would break the rules Lorentz invariance however finding a quantum theory of gravity is important to answer many questions. Firstly is the cosmological constant problem where the predicted contributions from quantum corrections to the cosmological constant and these are many many orders of magnitude higher than the observed value [62]. Supersymmetry, which we will discuss later when unbroken can't solve this problem either, the vacuum state under supersymmetry implies a vanishing vacuum energy and momentum:

$$\langle 0|P^{\mu}|0\rangle = 0. \tag{2.25}$$

This is due to boson-fermion symmetry, with the bosonic terms cancelling the contributions from the fermionic terms, and we will introduce this concept at the end of this chapter. We will introduce string theory in chapter 3 and we will see that this a possible candidate for unification of the four forces. The theorised quanta of the gravitational force, the graviton, will be shown to appear naturally giving a satisfying motivation to study these extended objects further and how we can get models which reproduce features of the Standard Model by reproducing grand unified gauge groups.

2.3 Grand Unified Theories

In this section we are going to briefly discuss Grand Unified Theories, abbreviated as GUT. The aim of a GUT is to unify the three separate gauge couplings of the Standard Model gauge groups into a singular interaction or gauge. This was motivated by the unification of the weak and electromagnetic interaction, at sufficiently high energy levels, into the single electroweak interaction. GUTs take this a stage further and suggest at even higher energy levels that there will be further unification with the strong force. We must distinguish GUTs from Theories of Everything (TOE), the latter unifies all four fundamental forces and hence would include a quantum description of gravity.

GUTs is a blanket term used for both the embedding of the Standard Model gauge groups into a single higher rank simple group such as $SU(5)$ and $SO(10)$ but also for semi-simple groups such as the Pati-Salam model. These higher rank groups must have a decomposition in which the Standard Model gauge group is contained. In this section we will discuss all three mentioned GUTs and briefly talk about some of the representations and breaking paths, especially in the context of SO(10).

2.3.1 Georgi-Glashow Model

This was the first true unification, proposed by Howard Georgi and Sheldon Glashow in 1974 in their paper titled 'Unity of of All Elementary-Particle Forces' [35]. They unified the three Standard Model gauge groups into a single simple rank 4 gauge group, $SU(5)$. We can see this is the smallest possible embedding in a special unitary group by comparing the rank and dimension of each gauge group [25]. The rank of the group $SU(N)$ is the maximum number of diagonal commuting generators, given by $R = N - 1$.

$$SU(5) : R = 4, \quad D = 24. \tag{2.26}$$

So our Standard model gauge group must be of equal rank and have a dimension of 24 or lower. The dimension and rank of the $U(1)$ group are both equal to 1:

$$SU(3)_c \otimes SU(2)_L \otimes U(1) : R = 4, \quad D = 12. \tag{2.27}$$

What we see here is that the rank of the matrices match but we have an extra 12 dimensions in the $SU(5)$ group compared to our Standard Model. We will see that these correspond to an additional 12 gauge bosons known as

X/Y bosons [58]. The $SU(3)_c$ and $SU(2)_L$ are embedded into the upper-left and lower-right blocks of the traceless 5×5 matrix. Our left-handed fermions in our Standard Model are embedded in the two representations of $\mathbf{10} \oplus \bar{\mathbf{5}}$:

$$\bar{\mathbf{5}} \leftrightarrow \begin{pmatrix} d_r^c \\ d_g^c \\ d_b^c \\ e \\ -\nu_e \end{pmatrix}_L \qquad \mathbf{10} \leftrightarrow \begin{pmatrix} 0 & u_b^c & -u_g^c & u_r & d_r \\ -u_b^c & 0 & u_r^c & u_g & d_g \\ u_g^c & -u_r^c & 0 & u_b & d_b \\ -u_r & -u_g & -u_b & 0 & e^c \\ -d_r & -d_g & -d_b & -e^c & 0 \end{pmatrix}_L \tag{2.28}$$

Here, the superscript of c, i.e. u_b^c, represents the charge conjugate of that particular particle in the same chirality [38]. In terms of the Standard Model gauge group, $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, this can be represented as $\bar{\mathbf{5}} \oplus \mathbf{10}$:

$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1})_{+\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} \tag{2.29}$$

$$\mathbf{10} = (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{3}, \mathbf{2})_{+\frac{1}{6}} \oplus (\mathbf{1}, \mathbf{1})_{+1} \tag{2.30}$$

If we wanted the right-handed fermion generations we would take the conjugate representation, $\mathbf{5} \oplus \overline{\mathbf{10}}$, this can also display in terms of the Standard Model gauge group:

$$\mathbf{5} = (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{+\frac{1}{2}} \tag{2.31}$$

$$\bar{\mathbf{10}} = (\mathbf{3}, \mathbf{1})_{+\frac{2}{3}} \oplus (\bar{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}} \oplus (\mathbf{1}, \mathbf{1})_{-1} \tag{2.32}$$

The interactions, i.e. the gauge bosons, come from the unbroken subgroup which is the **24** adjoint representation, with its decomposition

$$\mathbf{24} = (\mathbf{8}, \mathbf{1})_{\mathbf{0}} \oplus (\mathbf{1}, \mathbf{3})_{\mathbf{0}} \oplus (\mathbf{1}, \mathbf{1})_{\mathbf{0}} + (\mathbf{3}, \mathbf{2})_{-\frac{5}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{2})_{\frac{5}{6}} \tag{2.33}$$

Here, we can see the gauge bosons of each of the Standard Model gauge groups, in order $SU(3)$, $SU(2)$, $U(1)$ and then the final two terms are the new X/Y gauge bosons we briefly mentioned before. In order to break the $SU(5)$ group, we require a scalar field within the adjoint Higgs sector with VEV in the direction of the hypercharge generator $\mathrm{diag}(2, 2, 2, -3, -3)$ and another to cause the electroweak symmetry breaking [38] [57].

However, there are a range of problems with the $SU(5)$ theory which rules it out for a potential candidate for grand unification. Firstly, the addition exotic bosons (which we have called the X/Y bosons) facilitate lepto-quark interactions. These would facilitate rapid proton decay, with the largest branching ratio being

$$p \to e^+ + \pi^0 \tag{2.34}$$

predicting lifetimes shorter than the lower bound recently set at the Super-Kamiokande experiment [51]. With the recent discovery of neutrino oscillations which implies neutrinos has a non-zero mass, a right-handed neutrino would need to be added to the representation by the introduction of an extra singlet meaning a larger representation is required. A candidate for this is $SO(10)$. Further reading on $SU(5)$ can be found in references [35], [57], [10], [56], [38] and [16].

2.3.2 Pati-Salam Model

The Pati-Salam model [53], published in 1973 by Jogesh Pati and Abdus Salam, was the first step towards unification [25]. Whilst not technically a unified group (as it is a combination of three simple groups), we will see that the left- and right-handed quarks and leptons are combined into a single representation. The model has the combined gauge group:

$$SU(4)_C \otimes SU(2)_L \otimes SU(2)_R. \tag{2.35}$$

To quickly check this is a viable candidate to embed the Standard Model, we must check the rank and dimension of the group. The rank and dimension of the Pati-Salam model must be equal or greater than that of the Standard Model gauge group to ensure it's particle content can be fitted. Using that the rank of a $SU(N)$ is the number of maximally commuting generators, given by $N-1$ and the dimension is N^2-1 Our Standard Model has rank and dimension:

$$SU(3)_c \otimes SU(2)_L \otimes U(1) : R = 4, \quad D = 12 \tag{2.36}$$

and our Pati-Salam model has:

$$SU(4)_C \otimes SU(2)_L \otimes SU(2)_R : R = 5, \quad D = 14. \tag{2.37}$$

So we see that the Pati-Salam could be a potential candidate. The 'unification' of this model can be seen by looking at the fermion representations. Our entire generation of left-handed quark and lepton doublets are given in a single representation[38]:

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) \leftrightarrow \begin{pmatrix} u_L^r & u_L^g & u_L^b & \nu_L \\ d_L^r & d_L^g & d_L^b & e_L \end{pmatrix} \tag{2.38}$$

The right-handed fermions can be expressed in the same way

$$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \leftrightarrow \begin{pmatrix} u_R^r & u_R^g & u_R^b & \nu_R \\ d_R^r & d_R^g & d_R^b & e_R \end{pmatrix} \tag{2.39}$$

Here, we can see the extension of the colour charge gauge group $SU(3)_c$ to now include leptons as if they were an additional colour in the larger group of $SU(4)_c$. Since $SU(4)$ and $SU(2) \otimes SU(2)$ are homomorphic to $SO(6)$ and $SO(4)$ respectively, a fermion generation can be unified into a spinorial representation SO(10), shown in [56].

$$\mathbf{16} = \mathbf{(4, 2, 1)} \oplus \mathbf{(4, \bar{1}, 2)} \tag{2.40}$$

The Higgs sector is dependent on breaking path of the gauge group [38]. There are lots of breaking paths and patterns of the Pati-Salam model, one simple way path is:

$$SU(4)c \to SU(3)_c \otimes \ U(1)_{B-L} \tag{2.41}$$

which would give us the overall gauge group

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}. \tag{2.42}$$

Then breaking to the observed Standard Model gauge group:

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y. \tag{2.43}$$

We will be specifically interested in the breaking of a $SO(10)$ group into the Pati-Salam group when we are analysing our model. In the next section we will be quickly explaining the $SO(10)$ group and the possible embedding of other gauge groups in $SO(10)$, specifically in the spinorial representation. More detailed analysis and further reading on the Pati-Salam model can be found at [53], [38], [10], [16] and [56].

2.3.3 SO(10)

In 1975, Harald Frizsch and Peter Minkowski realised that the Pati-Salam group SU(4)xSU(2)xSU(2) is isomorphic to SO(6)xSO(4) with the latter naturally embedded in SO(10) [38]. The benefit of the $SO(10)$ group is that both $SU(5)$ and the Pati-Salam models can be embedded into it. As we will see, this will be the a key feature in our models and how the $SO(10)$ group is broken by specific breaking vectors will be discussed in detail in later sections. As an orthogonal group, it cannot form the complex representations needed for the necessary particles of the Standard Model. A solution to this is to use the spinorial representation and details of the embedding of $SU(N)$ groups into $SO(2N)$ can be found here [57]. The **16** representation of the $SO(10)$

group holds an entire generation of fermions in a spinorial representation [22]:

$$\mathbf{16} \leftrightarrow \begin{pmatrix} u_r^c & u_g^c & u_b^c & \nu_e^c & u_r & u_g & u_b & \nu_e \\ d_r^c & d_g^c & d_b^c & e^+ & d_r & d_g & d_b & e^- \end{pmatrix}_L \tag{2.44}$$

Both our $SU(5)$ and Pati-salam models can be embedded in the larger SO(10) **16** representation, our Pati-Salam model can be embedded directly:

$$\mathbf{16} \rightarrow (\mathbf{4}, \mathbf{2}, \mathbf{1}) \oplus (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \tag{2.45}$$

The $SU(5)$ model can't be embedded into the **16** representation directly but we can embed $SU(5) \otimes U(1)$:

$$\mathbf{16} \rightarrow \mathbf{10} \oplus \bar{\mathbf{5}} \oplus \mathbf{1} \tag{2.46}$$

An extra unitary group is needed to ensure the rank is matched and this fixes the problem of needing to add an additional singlet (the right handed neutrino) for small neutrino masses. The $SO(10)$ gauge bosons are contained in the adjoint **45** representation, this contains all the Standard Model gauge bosons but also an additional 33 [38]. The **45** representation decomposition gives the usual octet of gluons, the electroweak triplet and hypercharge singlet:

$$\mathbf{45} \rightarrow (\mathbf{8}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{0}) + 33 \text{ extra} \tag{2.47}$$

These additional 33 gauge bosons would carry both the colour and the weak charge, allowing for transitions between quarks and leptons [38]. Current research is ongoing to provide experimental bounds on the existence of leptoquarks and further reading can be found in section 115 of the particle data review in reference [58]. What we will be specifically interested in later when we come to our model is the breaking of the $SO(10)$ group to the Pati-Salam group. There is a number of breaking paths for this route to the standard model so we won't list them here, but further reading on the $SO(10)$ group and breaking paths can be found in references [33], [52], [38], [57] [16].

2.4 Supersymmetry

This is going to be an ultra brief introduction of supersymmetry (SUSY). This section is mainly included so when we introduce the superstring that supersymmetry is not an entirely foreign concept. A SUSY transformation is a transformation between a fermion and a boson, and vice versa. Each boson and fermion in the theory has a superpartner which differs by half integer spin. Our superpartners are represented with a tilde above their usual symbol. For example, the superpartner of the electron neutrino ν_e is the electron sneutrino $\tilde{\nu}_e$. A table of particles and their corresponding superpartners is included below.

Particles and Corresponding Superpartners					
Standard Model Particle			**Corresponding Superpartner**		
Name	**Symbol**	**Spin**	**Name**	**Symbol**	**Spin**
Quarks	d, u, s, c, b, t	$\frac{1}{2}$	Squarks	$\tilde{d}, \tilde{u}, \tilde{s}, \tilde{c}, \tilde{b}, \tilde{t}$	0
Leptons	e, μ, τ	$\frac{1}{2}$	Sleptons	$\tilde{e}, \tilde{\mu}, \tilde{\tau}$	0
Neutrinos	ν_e, ν_μ, ν_τ	$\frac{1}{2}$	Sneutrinos	$\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$	0
Photon	γ	1	Photino	$\tilde{\gamma}$	$\frac{1}{2}$
Gluon	g	1	Gluino	$\tilde{g}$	$\frac{1}{2}$
W, Z boson	$W^{\pm}, Z$	1	Wino & Zino	$\tilde{W}^{\pm}, \tilde{Z}$	$\frac{1}{2}$
Higgs	H	0	Higgsino	$\tilde{H}$	$\frac{1}{2}$

We are specifically interested in SUSY GUTs and some of the problems of the standard model which they solve. Supersymmetry could provide the basis for unification of the gauge couplings by contribution from the SUSY partners to the couping above the SUSY mass scale [17]. When supersymmetry is incorporated minimally into the Standard Model, it is often called the Minimally Symmetric Standard Model (MSSM). We can find a unification point of the couplings something which is not possible in the current framework of the Standard Model [17] [20]. In SUSY $SU(5)$ and higher gauge groups containing $SU(5)$ we can solve this for reasonable values of the M_{SUSY}. This is due to the squarks and sleptons being contained in complete $SU(5)$ multiplets and hence not contributing to the running of the coupling constants. These superpartners, specifically the gauginos, higgsino and extra Higgs are cause the improvement over the SM [59].

Another problem of the Standard Model is the reliance of the Higgs fields having a vacuum expectation value of 246 GeV. This VEV is highly unstable when we do quantum loop corrections using a QFT framework and there is a quadratic divergence due to the Higgs self interaction and top quark interaction terms at the high-energy cutoff [59]. This has the consequence of implying the SM can only be an effective field theory when the cutoff is near 1 TeV [59]. At energies near this scale the SM requires additional physics to be able to stabilise the Higgs VEV, SUSY is a possible candidate for this [59]. SUSYs solution occurs due to the introduction of superpartners, specifically new scalar bosons which allow cancellation of the quadratic divergences [59].

A glaring problem of the non-SUSY $SU(5)$ theory was the prediction of rapid proton decay. There have been various methods of solving this using SUSY and further reading on proton decay and review of methods of supressing the decay rate can be found here [50].

Another 'solution' supersymmetry may provide is a possible candidate for dark matter through neutralinos [17], but this currently is all spectulatory and is just included to give an idea of current research and potential theories.

Currently, no SUSY particles have been observed in the LHC or elsewhere. The window for finding these superpartners is growing ever smaller with more and more experimental data and it's looking increasingly unlikely that that supersymmetry is the answer to our questions. This is the motivation of this project and the specific class of models we will be discussing in the free fermionic formulation. The aim of this project moving forward will be to question whether it's possible to construct realistic non-SUSY models and to explore some of the features and constraints of such models.

We will see later that we will actually need to construct models using worldsheet supersymmetry but will not include spacetime supersymmetry or superpartners. It is possible to have a differing number of supersymmetry transformations, e.g. $\mathcal{N} = 2$ or $\mathcal{N} = 4$, which are called extended supersymmetries, but we will only be using models which exhibit $\mathcal{N} = 1$ supersymmetry as this is the only way to construct realistic models. Models with $\mathcal{N} \geq 2$ are unrealistic. We can see this by looking how the multiplets transform. Theories with extended supersymmetry we actually get states with helicity $-\frac{1}{2}$ transforming equivalently (this is a real representation) as states with helicity $\frac{1}{2}$ [63]. We actually need our fermions to transform under a complex representation and hence we will use constructions with $\mathcal{N} = 1$.

Chapter 3

A brief recap of string theory

In the last chapter, we have seen that there have been many attempts to extend the Standard Model through grand unified theories. Many of these attempts have some promising characteristics, but there is a glaring emission with all of them. This emission is the lack of a unified theory which contains gravity within a quantum framework. General Relativity (GR) is infamously incompatible with our accepted quantum field theories (QFT). It is generally accepted to that these two separate branches of physics that have separate purposes; GR to deal with physics on the (cosmological) macro scale and QFT to deal with it on the microscale. Both theories have withstood extensive testing, with both being proved further by the discovery of gravitational waves at LIGO [3] and the elusive Higgs boson at the LHC [2] respectively. String theory attempts to rectify this problem. As we will see gravity is manifest within the spectrum with the theorised graviton, the quanta of the gravitational field appearing naturally.

In this chapter, we will derive and discuss three separate types of string theory. We will start with the most basic which is the bosonic string. This will begin with adapting the familiar relativistic particle action to a classical relativistic string. We will do this first in the Nambu-Goto action but will quickly discover that this representation is difficult to quantise. Upon moving to the Polyakov action, we are given a clear path to progress further to the superstring which allows fermions in the spectrum. We will finally briefly touch on the construction of the heterotic string and the free fermionic formulation, which is the main goal of this project.

3.1 The bosonic string

Starting with the action of a relativistic point particle, the particle will trace out a world-line as it propagates through D-dimensional Minkowski space-time. The action for a relativistic point particle is proportional to the world-line length, using $c = 1$, is:

$$S = m \int d\tau \sqrt{-g_{\mu\nu} \frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau}}. \tag{3.1}$$

This idea can be extended from the 0-dimensional particle to a new 1-dimensional object that we call a string, which has a length but no width. Starting in a D-dimensional Minkowski space-time M, defined by space-time coordinates X^μ, $\mu = 0, 1, ..., D-1$ and the metric $g_{\mu\nu}$. The propagation of this string traces out a curved surface called the world-sheet Σ, this is a two dimensional analogue of the world-line for a relativistic particle. The world-sheet is parametrised by the tangent vectors $\sigma^\alpha = (\tau, \sigma)$ where we consider τ to be a time-like coordinate and σ to be our space-like coordinate. Strings can be either open or closed, and are usually defined by the boundary conditions (BC) at each end. For open strings, these boundary conditions are either Neumann (N), where the string endpoints are free to move or Dirichlet (D) which attach the endpoints to a fixed dynamical object called a brane. Without going into further detail, the brane has to be dynamical to allow momentum to be transferred to and from and to allow the conservation of momentum. The Neumann boundary conditions are given by:

$$\frac{\partial X^\mu}{\partial \sigma}(\sigma = 0, \pi) = 0 \tag{3.2}$$

and the Dirichlet:

$$X^\mu(\sigma = 0, \pi) = \text{constant}. \tag{3.3}$$

We will only be considering closed strings, so we will need σ to be periodic: $\sigma \in [0, 2\pi)$. Defining a map from our worldsheet to the position in space-time, represented by the set of scalar fields $X^\mu(\tau, \sigma)$. As stated, the space-like worldsheet coordinates for a closed string are periodic, hence we obtain the boundary conditions for a closed string

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + 2\pi). \tag{3.4}$$

For the relativistic point particle, our action was proportional to the proper length of the worldline; for the relativistic string, this is similar in that it is proportional to the proper area of the worldsheet

$$dA = d\tau d\sigma \sqrt{-\gamma}. \tag{3.5}$$

The quantity under the square root is the determinant of the worldsheet metric. We will be using this notation where the dropped indices represent the determinant:

$$\gamma = \det(\gamma_{\alpha\beta}) \tag{3.6}$$

The metric on the worldsheet $\gamma_{\alpha\beta}$ is induced by the Minkowski metric $\eta_{\mu\nu}$:

$$\gamma_{\alpha\beta} = \eta_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta} \tag{3.7}$$

A full and easy to follow derivation of the string action can be found in Zweibach [65]. The string is defined by the Nambu-Goto action:

$$S_{NG} = -T\int_\Sigma d^2\sigma\sqrt{-\gamma} \tag{3.8}$$

Here, T is called the string tension, or the mass per unit length. the action is sometimes defined in terms of the Regge slope α' using the relation [60]:

$$T = \frac{1}{2\pi\alpha'}. \tag{3.9}$$

The square root in 3.8 causes difficulty when quantizing the action. We can replace the Nambu-Goto action with the Polyakov action. This is done by replacing the area functional with a corresponding energy functional [65]:

$$S_P = -\frac{T}{2}\int_\Sigma d^2\sigma\sqrt{-h}h^{\alpha\beta}\gamma_{\alpha\beta}. \tag{3.10}$$

This is accomplished by introducing an auxiliary field $h_{\alpha\beta}(\tau,\sigma)$, which is intrinsic on the worldsheet unlike the induced metric $\gamma_{\alpha\beta}$ introduced in 3.7.

We can now discuss the symmetries of the Polyakov action:

1. **Poincaré symmetry**
   This is a global spacetime X^μ symmetry. The action is invariant under Poincaré transformations [60]:

   $$X^\mu(\tau,\sigma) \to \Lambda^\mu{}_\nu X^\nu(\tau,\sigma) + a^\mu \tag{3.11}$$

   This symmetry encompasses both Lorentz transformations and translations.

2. **Reparametrisation invariance (diffeomorphism)**
   This is local to the worldsheet. It allows flexibility in choosing how we define our worldsheet coordinates:

$$\sigma \to \tilde{\sigma}(\sigma) \tag{3.12}$$
$$\tau \to \tilde{\tau}(\tau). \tag{3.13}$$

   The fields $X^\mu(\tau, \sigma)$ transform as scalars, whilst the metric $h_{\alpha\beta}$ transforms as a two-dimensional metric [22]:

$$X^\mu(\sigma) \to \tilde{X}^\mu(\tilde{\sigma}) = X^\mu(\sigma) \tag{3.14}$$
$$h_{\alpha\beta}(\sigma) \to \tilde{h}_{\alpha\beta}(\tilde{\sigma}) = \frac{\partial \sigma^\gamma}{\partial \tilde{\sigma}^\alpha} \frac{\partial \sigma^\delta}{\partial \tilde{\sigma}^\beta} h_{\gamma\delta}(\sigma) \tag{3.15}$$

3. **Weyl Rescaling**
   This is an additional invariance compared to the Nambu-goto action 3.8. This is a local change of scale but also preserves the angles between all lines. The transformation is of form [60]

$$h_{\alpha\beta} \to e^{2\Lambda(\sigma)} h_{\alpha\beta}(\sigma). \tag{3.16}$$

   This symmetry is a local change of scale, but preserves the angles between all lines. This type of invariance is unique to two dimensions and comes from the fact we have introduced an auxiliary field $h_{\alpha\beta}$ on the worldsheet [60].

We can use these symmetries to completely gauge fix our worldsheet metric. This is possible due to our two-dimensional metric only having three independent components. This is first done by choosing a reparametrisation of the worldsheet coordinates such that our metric is locally conformally flat and introducing a function $\Lambda(\sigma)$ onto the worldsheet:

$$h_{\alpha\beta} = e^{2\Lambda(\sigma)} \eta_{\alpha\beta} \tag{3.17}$$

This is what is called our conformal gauge. We can further fix this using our third symmetry, Weyl invariance, effectively setting the function $\Lambda = 0$:

$$h_{\alpha\beta} = \eta_{\alpha\beta} \tag{3.18}$$

leaving the flat metric or Minkowski metric $\eta_{\alpha\beta}$. Using these invariances we are able to reduce the Polyakov action to more simple form:

$$S_P = \frac{T}{2} \int_\Sigma d\tau d\sigma \big(\dot{X}^2 - X'^2\big) \tag{3.19}$$

Here $\dot{X} = \frac{\partial X}{\partial \tau}$ and $X' = \frac{\partial X}{\partial \sigma}$.

Our equations of motion are derived using the variational principle. We can vary our action with respect to two parameters - our coordinates X^μ and our auxiliary field we introduced $h_{\alpha\beta}$. This could've been done earlier from our original Polyakov action, but now we've fixed our metric to be flat our equation of motion from varying X^μ is simply the free wave equation:

$$\partial_\alpha \partial^\alpha X^\mu = 0. \tag{3.20}$$

We obtain the second equation of motion by varying with respect to the metric $h_{\alpha\beta}$. This gives us the stress-energy tensor, in our flat gauge this is once again simplified to:

$$T_{\alpha\beta} = 0. \tag{3.21}$$

This gives us the following two constraints

$$X' \cdot \dot{X} = 0 \tag{3.22}$$

$$\dot{X}^2 + X'^2 = 0. \tag{3.23}$$

Now redefining our worldsheet coordinates to light-cone coordinates:

$$\sigma^\pm = \tau \pm \sigma. \tag{3.24}$$

Our equation of motion becomes:

$$\partial_+ \partial_- X^\mu = 0 \tag{3.25}$$

This also gives us an easier way to present our general solution of our free wave equation 3.20. This is a superposition of left- and right-moving waves

$$X^\mu(\tau, \sigma) = X^\mu_L(\sigma^+) + X^\mu_R(\sigma^-). \tag{3.26}$$

Here, X^μ_L and X^μ_R are two arbitrary functions. As mentioned at the start of this chapter, we have to specify boundary conditions. These are periodic and given in equation 3.4. Applying our boundary conditions, we can express our left- and right-moving periodic solutions in Fourier modes:

$$X^\mu_L(\sigma^+) = \frac{1}{2}x^\mu + \frac{1}{2}\alpha' p^\mu \sigma^+ + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \tilde{\alpha}^\mu_n e^{-in\sigma^+} \tag{3.27}$$

$$X^\mu_R(\sigma^-) = \frac{1}{2}x^\mu + \frac{1}{2}\alpha' p^\mu \sigma^- + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \alpha^\mu_n e^{-in\sigma^-} \tag{3.28}$$

These equations do not independently satisfy our periodic boundary condition 3.4. However, when combined our solution is periodic:

$$X^\mu = \underbrace{x^\mu + \alpha' p^\mu \tau}_{\text{centre of mass motion}} \underbrace{+ i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n}\left(\tilde{\alpha}_n^\mu e^{-in\sigma^+} + \alpha_n^\mu e^{-in\sigma^-}\right)}_{\text{string oscillations}}. \tag{3.29}$$

x^μ and p^μ are the position of the center of mass, and momentum of the string respectively. We require that our spacetime coordinate solution X^μ be real; this implies the coefficients of the Fourier modes

$$\alpha_n^\mu = (\alpha_{-n}^\mu)^* \quad \text{and} \quad \tilde{\alpha}_n^\mu = (\tilde{\alpha}_{-n}^\mu)^*. \tag{3.30}$$

These coefficients classically represent the amplitudes of the nth oscillation mode, are zero modes for the left and right moving parts must match:

$$\tilde{\alpha}_0^\mu = \alpha_0^\mu = \sqrt{\frac{\alpha'}{2}} p^\mu \tag{3.31}$$

Imposing the equation of motion from variation of the metric we obtain the Virasoro constraints

$$(\dot{X} \pm X')^2 = 0. \tag{3.32}$$

To quantise our string, these are promoted to operators with the commutation relations:

$$[x^\mu, p^\nu] = i\eta^{\mu\nu} \quad \text{and} \quad [\alpha_n^\mu, \alpha_m^\nu] = [\tilde{\alpha}_n^\mu, \tilde{\alpha}_m^\nu] = n\eta^{\mu\nu}\delta_{n+m} \tag{3.33}$$

and the rest of our commutation relations are zero. The second of these commutator relations should look familiar, this is just the usual quantum harmonic oscillator creation and annihilation operators. We can see this by redefining them

$$a_n = \frac{\alpha_n}{\sqrt{n}} \quad \text{and} \quad a_n^\dagger = \frac{\alpha_{-n}}{\sqrt{n}} \quad \text{with} \quad n \neq 0. \tag{3.34}$$

To clarify, our left- and right-moving nth modes are $\tilde{\alpha_n}$ and α_n respectively. We differentiate between the creation and annihilation operators by the sign of the subscript; if $n < 0$ then it is a creation operator and $n > 0$ represents an annihilation operator. The easiest way to quantise the bosonic string is in the light-cone gauge, to move to this gauge we introduce spacetime light-cone coordinates:

$$X^\pm = \frac{1}{\sqrt{2}}\left(X^0 \pm X^{D-1}\right) \tag{3.35}$$

This fixes two of our D spacetime coordinates leaving us with $D-2$ remaining. We label these the transverse coordinates [65] and use the notation X^I with $I = 1, ..., D-2$ to represent them. We fix one of our coordinates to:

$$X^+ = \alpha' p^+ \tau \tag{3.36}$$

which fixes our gauge completely. However, this choice removes our manifest Lorentz invariance but this is something we will fix later with solve later by with a critical dimension. This adjusts our commutators slightly [41] [13]:

$$[x^-, p^+] = -i, \qquad [x^i, p^j] = \delta^{ij}, \tag{3.37}$$

$$[\alpha_n^i, \alpha_m^j] = [\tilde{\alpha}_n^i, \tilde{\alpha}_m^j] = n\delta^{ij}\delta_{n+m,0} \qquad [\alpha_n^i, \tilde{\alpha}_m^j] = 0. \tag{3.38}$$

Now we can use these to express our light-cone Hamiltonian of the string, we must be careful with the ordering of our creation and annihilation operators so we introduce the normal ordering constants a and $\tilde{a}$ which account for the uncertainty of the ordering:

$$H = \sum_{i=1}^{D-2} \frac{p_i^2}{2p^+} + \frac{1}{\alpha' p^+}\Big(N + \tilde{N} - a - \tilde{a}\Big) \tag{3.39}$$

where we have used the number operators:

$$N = \sum_i \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i \quad \text{and} \quad \tilde{N} = \sum_i \sum_{n=1}^{\infty} \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i. \tag{3.40}$$

Using this, we can construct our Hilbert space. We do this by first defining our vacuum or ground state $|0, p^\mu\rangle$, this is the state which is annihilated by all annihilation operators

$$\alpha_n^i |0, p^\mu\rangle = \tilde{\alpha}_n^i |0, p^\mu\rangle = 0. \tag{3.41}$$

Our normal ordering constants must satisfy:

$$a = \tilde{a} = \frac{D-2}{24}. \tag{3.42}$$

This actually comes from imposing Lorentz invariance for our first excited state which in turn requires a massless state which gives us $a = \tilde{a} = 1$. This is beyond this quick overview, but we will be covering calculating the critical dimension for the superstring in detail and further reading can be found in both [60] and [65]. From these conditions, it's easy to see that this is a theory which requires $D = 26$ space-time dimensions and this is the critical dimension of the bosonic string. Using our creation operators, we can construct our Hilbert space by acting with successive creation operators on the vacuum. A condition on physical states requires them to be level matched $N = \tilde{N}$, an idea we will expand on when we cover the heterotic string. Using the mass-shell condition (which is a feature of the global Poincaré symmetry) and combining with the Hamiltonian we can obtain an expression for the string mass:

$$M^2 = \frac{2}{\alpha'}(N + \tilde{N} - a - \tilde{a}). \tag{3.43}$$

Analysing the states of the spectrum:

- **Ground state**:

$$|0, p^\mu\rangle \tag{3.44}$$

  We see that the ground state, where $N = \tilde{N} = 0$, will have a negative mass squared term; this is an instability of the model and is a unphysical tachyonic state. This would seem to appear to travel faster than light and hence breaking the laws of special relativity, but really should be considered as a unstable maximum in the field [60]. Once again, we will discuss tachyonic states in more detail in a later chapter as they are of particular importance to remove for our class of models.

- **First excited state**:

$$\sum_{1\leq i,j\leq D-2} R_{ij}\alpha^i_{-1}\tilde{\alpha}^j_{-1}|0, p^\mu\rangle \tag{3.45}$$

  The first excited state, where $N = \tilde{N} = 1$ gives us a massless vector boson. This must be massless which we consider as the photon. Here, R_{ij} is a second rank tensor of the $SO(1, D-1)$ Lorentz group. This will not fit into a $SO(D-1)$ representation but can be fitted into the $\mathbf{24} \otimes \mathbf{24}$ representation of $SO(24)$ [60]. This gauge group can be decomposed into its irreducible representations containing traceless symmetric tensor, anti-symmetric tensor and the trace each with it's own associated field [65][60]. Firstly, the traceless symmetric component generates a massless spin 2 field $G_{\mu\nu}$. The particle associated with this field is the gravition and is the natural appearance of gravity (general relativity) in string theory. The anti-symmetric tensor represents the Kalb-Ramond field $B_{\mu\nu}$, which a rank 2 tensor generalisation of the electromagnetic potential A_μ [60]. The trace representation gives us a scalar field Φ and corresponding particle is the dilaton.

- **Second and higher excited states**:
  Our second excited state is degenerate and can represented in two ways:

$$\alpha^i_{-1}\alpha^j_{-1}\tilde{\alpha}^i_{-1}\tilde{\alpha}^j_{-1}|0,p^\mu\rangle \quad \text{and} \quad \alpha^i_{-2}\tilde{\alpha}^i_{-2}|0,p^\mu\rangle \tag{3.46}$$

  All states with $N = \tilde{N} \geq 2$ are all massive states, that all fit nicely into $SO(D-1)$ representations (unlike our first excited state). This is the symmetric tensor representation [60].

The bosonic string appears to 'solve' the infamous problem of gravity by the appearance of a spin 2 particle naturally within the spectrum. However, it also comes with a lot of it's own unique problems. Firstly, like it's name suggests, it's an entirely a theory of bosons. No fermions appear in the spectrum and hence it cannot be a phenomenologically complete theory. To introduce fermions to spectrum, we must introduce a supersymmetry. This can be done either to the worldsheet as we will introduce in the next section or instead to spacetime itself. One glaring issue we briefly mentioned is the tachyonic ground state. This is an unphysical state and later in our string models we will be taking specific care to ensure removal of these. Finally, the conformal anomaly of the bosonic string is only able to be cancelled in $D = 26$. This gives us an extra 22 unseen dimensions than we observe (the usual 3 with 1 temporal dimension). This is a problem which can be solved by compacting the extra dimensions onto a manifold, usually either a torus or a Calabi-Yau manifold but this is not something we will be exploring further. Further reading on compacting extra dimensions and manifolds can be found in the books [12] and [13].

3.2 The superstring

In the last section, we were left with a theory describing only bosons and with a range of problems such as a tachyon within the spectrum, leaving us with an unrealistic theory and one that can be solved with the introduction of supersymmetry. Here, we can begin to discuss the Ramond-Neveu-Schwarz formalism that we will use (worldsheet SUSY). There is another formalism called the Green-Schwarz formalism which requires space-time SUSY. In the supersymmetry section, we detailed that current experiments, mainly conducted at the LHC, have massively restricted the window for space-time superpartners. Therefore, it is looking increasingly unlikely that space-time supersymmetry exists. Supersymmetry can still be used as an important tool for us. We will, however have to restrict ourselves to worldsheet supersymmetry. As we see later, we will specifically use models which do not include a spacetime supersymmetry generator. Motivated by the lack of observation of these superpartners. We start by introducing worldsheet fermionic superpartners for our bosonic fields $X^\mu(\tau,\sigma)$ [25]:

$$X^\mu(\tau,\sigma) \to X^\mu(\tau,\sigma), \psi^\mu(\tau,\sigma) \tag{3.47}$$

Our superstring action in our conformal gauge is a modification of the Polyakvov action:

$$S = S_B + S_F = -\frac{T}{2}\int_\Sigma d\tau d\sigma (\partial_\alpha X_\mu \partial^\alpha X^\mu + i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu) \tag{3.48}$$

here, ρ^α are the two-dimensional gamma matrices analogs to 2.11:

$$\rho^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \rho^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{3.49}$$

and obey a Dirac algebra $\{\rho^\alpha, \rho^\beta\} = 2\eta^{\alpha\beta}$. These are analagous to the four-dimensional gamma matrices common in quantum field theory. We must mention that $\bar{\psi}$ is our Dirac adjoint and related to the Hermitian adjoint in the following way:

$$\bar{\psi} = \psi^\dagger i\rho^0 \tag{3.50}$$

our worldsheet fermionic fields ψ^μ are two component Majorana spinors (meaning they are real) and are defined by [40]:

$$\psi^\mu = \begin{pmatrix} \psi^\mu_- \\ \psi^\mu_+ \end{pmatrix} \tag{3.51}$$

when we move back to our light-cone coordinates defined as $\sigma^{\pm} = \tau \pm \sigma$. Looking at our action 3.48, we can see that the bosonic term is unchanged from the bosonic string and hence it's equations of motion will be same as derived in the previous section. Now, we will briefly only consider the fermionic contributions and their corresponding boundary conditions and equations of motion. In our light-cone coordinates, our fermionic action is:

$$S_F = iT \int_{\Sigma} d\sigma^+ d\sigma^- (\psi_-^{\mu} \partial_+ \psi_{-\mu} + \psi_+^{\mu} \partial_- \psi_{+\mu}) \tag{3.52}$$

The equation of motion for the two spinor components can be easily seen from our first term and is the Dirac equation $\gamma^{\alpha}\partial_{\alpha}\psi = 0$ or explicitly in components

$$\partial_+ \psi_- = 0 \quad \text{and} \quad \partial_- \psi_+ = 0. \tag{3.53}$$

Here, we have dropped the indices. The first component describes a left-moving wave and the second a right-moving wave. These equations of motion are called the Weyl conditions in two dimensions [12], this shows that there is a decoupling of the left- and right-moving degrees of freedom. Here, we could show that there's a global worldsheet supersymmetry. Varying both the bosonic and fermion fields, for the total action 3.48, showing the mixing of the two sets of fields in the following way:

$$\delta X^{\mu} = \bar{\epsilon}\psi^{\mu} \tag{3.54}$$
$$\delta \psi^{\mu} = \rho^{\alpha}\partial_{\alpha} X^{\mu} \epsilon \tag{3.55}$$

where ϵ is a constant infinitesimal Majorana spinor. Since we already know we have worldsheet supersymmetry, this is not necessary. There is also two conserved currents coming from our two global worldsheet symmetries of the action. These are the energy momentum tensor $T_{\alpha\beta}$, which comes from our translational invariance, and a supercurrent J_A^{μ} from our supersymmetry. These can be derived using the Noether method and details can be found in references [12] [64]. Explicitly, in our light-cone gauge these are:

$$J_{\pm}^{\mu} = \psi_{\pm}^{\mu} \partial_{\pm} X_{\mu} \tag{3.56}$$
$$T_{++} = \partial_+ X_{\mu} \partial_+ X^{\mu} + \frac{i}{2}\psi_+^{\mu}\partial_+\psi_{+\mu} \tag{3.57}$$
$$T_{--} = \partial_- X_{\mu} \partial_- X^{\mu} + \frac{i}{2}\psi_-^{\mu}\partial_-\psi_{-\mu} \tag{3.58}$$
$$T_{-+} = T_{+-} = 0 \tag{3.59}$$

The last line comes from our Weyl invariance. What we find when following the same quantisation steps as the bosonic string is that there are negative norm states within the spectrum, which appear from our time-like boson and fermion and mustn't exist in the physical spectrum. We solved this in the previous chapter by fixing our gauge to the light-cone gauge using conformal invariance. The same procedure can also be done for our superstring, using superconformal invariance, and introducing the RNS Virasoro conditions to ensure these negative norm states are removed:

$$J_+ = J_- = T_{++} = T_{--} = 0. \tag{3.60}$$

We will not actually use this now, but it will be useful for when we want to analyse our spectrum physical states, as it will allow us to create independent physical states by acting upon the vacuum with transverse operators without having to worry about negative norm states. Another piece of the puzzle is to define our boundary conditions, like our bosonic string we will only be discussing closed strings. We have two possible periodicity conditions:

$$\psi^\mu_\pm(\tau,\sigma) = \pm\psi^\mu_\pm(\tau,\sigma+\pi). \tag{3.61}$$

In these conditions, the choice of positive sign on the right means periodic which is called Ramond (R) boundary conditions, whilst the negative sign is anti-periodic called Neveu-Schwarz (NS). This type of boundary conditions will be revisited when we move onto the Free Fermionic Formulation. We can give our mode expansions for our right-movers

$$\psi^\mu_-(\sigma,\tau) = \sum_{n\in\mathbb{Z}} d^\mu_n e^{-2in\sigma^-} \quad \text{and} \quad \psi^\mu_-(\sigma,\tau) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} b^\mu_r e^{-2ir\sigma^-} \tag{3.62}$$

then for the left-movers we have

$$\psi^\mu_+(\tau,\sigma) = \sum_{n\in\mathbb{Z}} \tilde{d}^\mu_n e^{-2in\sigma^+} \quad \text{and} \quad \psi^\mu_+(\tau,\sigma) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} \tilde{b}^\mu_r e^{-2ir\sigma^+}. \tag{3.63}$$

Here, the Fourier coefficients for the fermionic fields will be once again promoted to operators upon quantisation following the same rules as the bosonic string, with $n, r < 0$ the creation and $n, r > 0$ the annihilation operators. The closed string therefore, has four possible sectors coming from the tensor product of the pairings of boundary conditions. States with NS-NS and R-R are space-time bosons, whereas states in NS-R and R-NS are space-time fermions [12]. To quantise, we once again introduce promote our Fourier coefficients to operators. This is canonical quantisation. Our bosonic operators α^μ_m and $\tilde{\alpha}^\mu_m$ still obeying the commutation relations 3.38 but now our fermionic fields obey the equal-time anti-commutator relations

$$\{\psi^\mu_A(\tau,\sigma), \psi^\nu_B(\tau,\sigma')\} = \pi\delta_{AB}\delta(\sigma-\sigma')\eta^{\mu\nu}. \tag{3.64}$$

Inserting into our mode expansions gives us our fermionic commutators

$$\{b_r^\mu, b_s^\nu\} = \{\tilde{b}_r^\mu, \tilde{b}_s^\nu\} = \eta^{\mu\nu}\delta_{r+s,0} \tag{3.65}$$

$$\{d_m^\mu, d_n^\nu\} = \{\tilde{d}_m^\mu, \tilde{d}_n^\nu\} = \eta^{\mu\nu}\delta_{m+n,0} \tag{3.66}$$

and our bosonic commutators are still

$$[x^\mu, p^\nu] = i\eta^{\mu\nu} \quad \text{and} \quad [\alpha_n, \alpha_m] = [\tilde{\alpha}_n, \tilde{\alpha}_m] = m\delta_{m+n}\eta^{\mu\nu}. \tag{3.67}$$

The subscripts n,m take integer values, whereas r,s take half-integer values. Once again, our ground state in our Fock space is defined as the state which is annihilated by all annihilation operators. Our ground state in the Neveu-Schwarz sector

$$\alpha_m^\mu|0, p^\mu\rangle_{NS} = b_r^\mu|0, p^\mu\rangle_{NS} = 0 \quad \text{for} \quad m, r > 0 \tag{3.68}$$

and for the Ramond sector

$$\alpha_m^\mu|0, p^\mu\rangle_R = d_m^\mu|0, p^\mu\rangle_R = 0 \quad \text{for} \quad m > 0. \tag{3.69}$$

There are exactly similar expressions for the left-moving ground states. The Neveu-Schwarz ground state is unique and is a spin-0 spacetime scalar whereas the Ramond ground state is degenerate[12]. All excited states in the NS sector are space-time bosons (spin-0) and all our bosons have half-integer energy spacing as the operator b_r^μ changes the energy level by a half integer unit [40]. The Ramond sector corresponds to fermions with integer energy spacing coming from the d_n^μ operator and this asymmetry in the energy spacing of fermions and bosons must be solved with a GSO projection [40], which we will introduce shortly and will be of vital importance for our free fermionic models. The reason for this degeneracy in the R sector is that the d_0^μ operator doesn't affect the mass of a state and hence $d_0^\mu|0\rangle_R$ has the same mass as $|0\rangle_R$ [40] [61]. This degeneracy can be seen by look at the zero mode Ramond fermionic operator which satisfy a Clifford algebra:

$$\{d_0^\mu, d_0^\nu\} = \eta^{\mu\nu}. \tag{3.70}$$

This allows us to identify the zeroth mode operator in terms of the higher dimension (later we will show this is $D = 10$ through the critical dimension) gamma matrices Γ^μ. These are analogous to the four dimension gamma matrices we introduced in equation 2.11. We won't go into details on the construction here, but further reading on both gamma matrices and other important SUSY algebra can be found here [55].

$$d_0^\mu = \frac{1}{\sqrt{2}}\Gamma^\mu \tag{3.71}$$

the gamma matrices satisfy a Dirac algebra

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}. \tag{3.72}$$

This means they are spinors which allow degeneracy. This is analogous to the four dimension Dirac equation 2.10 allowing spin up and down.

We will now use our energy-momentum tensor $T_{\alpha\beta}$ and supercurrent J_A^μ to define our super Virasoro generators and algebra. This derivation follows Becker [12]. Using our super-Virasoro constraints introduced in 3.60 to help define our Fourier components of T_{++} and J_+:

$$L_m = \frac{1}{\pi}\int_{-\pi}^{\pi} d\sigma e^{im\sigma} T_{++} = L_m^{(b)} + L_m^{(f)}. \tag{3.73}$$

The bosonic contribution is the same as defined previously

$$L_m^{(b)} = \frac{1}{2}\sum_{n\in\mathbb{Z}} : \alpha_{-n}\cdot\alpha_{m+n} : \quad m \in \mathbb{Z}. \tag{3.74}$$

the fermionic contribution is dependent on the sector, in the NS sector

$$L_m^{(f)} = \frac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}} \left(r + \frac{m}{2}\right) : b_{-r}\cdot b_{m+r} : \quad m \in \mathbb{Z} \tag{3.75}$$

and in the R sector

$$L_m^{(f)} = \frac{1}{2}\sum_{n\in\mathbb{Z}} \left(n + \frac{m}{2}\right) : d_{-n}\cdot d_{n+m} : \quad m \in \mathbb{Z}. \tag{3.76}$$

Once again, there are equivalent conditions for the left-movers. We can define the modes of the supercurrent by integrating over the string, giving us the supercurrent mode in our NS sector

$$G_r = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} d\sigma e^{ir\sigma} J_+ = \sum_{n\in\mathbb{Z}} \alpha_{-n}\cdot b_{r+n} \qquad r \in \mathbb{Z} + \frac{1}{2} \tag{3.77}$$

and in our R sector

$$F_m = \sum_{n\in\mathbb{Z}} \alpha_{-n}\cdot d_{m+n} \qquad m \in \mathbb{Z}. \tag{3.78}$$

This allows us to define the zero mode operator, which we will use to define our mass squared of our excited states

$$L_0 = \frac{1}{2}\alpha_0^2 + N_{NS/R}. \tag{3.79}$$

Here we have introduced the number operator $N_{NS/R}$, similar to the number operators introduced for the bosonic string 3.40. We have to remember that we also have the left-moving operators for our closed string. The left-moving operators all have exact definitions but are defined with a tilde. Our number operators, for our right-movers, can be defined as

$$N_{NS} = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n + \sum_{r=\frac{1}{2}}^{\infty} r b_{-r} \cdot b_r, \tag{3.80}$$

$$N_R = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n + \sum_{n=\frac{1}{2}}^{\infty} d_{-n} \cdot d_n. \tag{3.81}$$

The NS sector obeys the following super-Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{8} m(m^2-1)\delta_{m+n,0}, \tag{3.82}$$

$$[L_m, G_r] = \Big(\frac{m}{2} - r\Big) G_{m+r}, \tag{3.83}$$

$$\{G_r, G_s\} = 2L_{r+s} + \frac{D}{2}\Big(r^2 - \frac{1}{4}\Big)\delta_{r+s,0} \tag{3.84}$$

and the R sector follows a slightly different super-Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{8} m^3 \delta_{m+n,0}, \tag{3.85}$$

$$[L_m, F_n] = \Big(\frac{m}{2} - n\Big) F_{m+n}, \tag{3.86}$$

$$\{F_m, F_n\} = 2L_{m+n} + \frac{D}{2} m^2 \delta_{m+n,0}. \tag{3.87}$$

It is useful to note that there is no normal ordering constant when we define F_0, which we will later use this to find the normal ordering constant of the R sector a_R. When defining our Fock space, we used the definitions 3.68 and 3.69. We can expand this to the Virasoro generators by requiring the positive modes annihilate the physical state $|\Phi\rangle$ [12]. This gives us for our NS sector

$$G_r|\Phi\rangle = 0 \qquad r > 0, \tag{3.88}$$

$$L_m|\Phi\rangle = 0 \qquad m > 0, \tag{3.89}$$

$$(L_0 - a_{NS})|\Phi\rangle = 0 \tag{3.90}$$

and similarly the conditions on physical states in the R sector

$$F_n|\Phi\rangle = 0 \qquad n \geq 0, \tag{3.91}$$

$$L_m|\Phi\rangle = 0 \qquad m > 0, \tag{3.92}$$

$$(L_0 - a_R)|\Phi\rangle = 0. \tag{3.93}$$

The last equation in both sectors comes from the ambiguity in the ordering of the creation and annihilation operators. These are the superstrings normal ordering constants, similar to the bosonic case. Using our number operators defined in 3.80 and 3.81, we get a mass formula for our closed string by combining with the corresponding left-moving conditions

$$M^2 = \frac{2}{\alpha'}(N_A + \tilde{N}_B - a_A - a_B). \tag{3.94}$$

This is almost exactly as the bosonic string mass formula 3.43. Except, now our number operators for our right moving N_A and left moving $\tilde{N}_B$ can be either NS or R independently. We still need to determine our normal ordering constants, but from this equation we can see that the mass squared of our string is dependent on the pairing of boundary conditions. We require that both the left- and right-moving sectors of our closed string be level matched, giving us the condition:

$$N_A - a_A = \tilde{N}_B - a_B. \tag{3.95}$$

In order to calculate our normal ordering constants, we need to impose Lorentz invariance. We will do this in detail since we glossed over it for the bosonic string. This can be done in the NS sector and is seen in the references [12] [64]. We first construct an first excited state $|\phi\rangle$ of the form

$$|\phi\rangle = G_{-\frac{1}{2}}|\chi\rangle. \tag{3.96}$$

$|\chi\rangle$ must satisfy our physical state condition for our Virasoro generators 3.88

$$L_{m>0}|\chi\rangle = 0, \tag{3.97}$$

the physical state conditions for successive states of the supercurrent 3.89 and the zero mode oscillator 3.90 which we can equate

$$G_{\frac{1}{2}}|\chi\rangle = G_{\frac{3}{2}}|\chi\rangle = \Big(L_0 - a_{NS} + \frac{1}{2}\Big)|\chi\rangle = 0. \tag{3.98}$$

This constant of $\frac{1}{2}$ in the last equality comes from $G_{-\frac{1}{2}}$ lowering the eigenvalue of L_0 from a_{NS} to $a_{NS} - \frac{1}{2}$ [64]. All we have to do is prove that $G_{\frac{3}{2}}|\chi\rangle = G_{\frac{1}{2}}|\chi\rangle = 0$, but since $G_{\frac{3}{2}}$ is a consequence of $G_{\frac{1}{2}}$ then the only condition which needs to be checked is $G_{\frac{1}{2}}$ [12]. This is given by

$$G_{\frac{1}{2}}|\phi\rangle = G_{\frac{1}{2}}G_{-\frac{1}{2}}|\chi\rangle. \tag{3.99}$$

Rearranging of our anti-commutator relation 3.84 to give us

$$G_{\frac{1}{2}}G_{-\frac{1}{2}} = \{G_{-\frac{1}{2}}, G_{\frac{1}{2}}\} - G_{-\frac{1}{2}}G_{\frac{1}{2}} \tag{3.100}$$

and remembering that our anti-commutator is equal to

$$\{G_{-\frac{1}{2}}, G_{\frac{1}{2}}\} = 2L_0, \tag{3.101}$$

when we plug in $r = -\frac{1}{2}$ and $s = \frac{1}{2}$. Leaving the following relation:

$$G_{\frac{1}{2}}|\phi\rangle = \Big(\{G_{-\frac{1}{2}}, G_{\frac{1}{2}}\} - G_{-\frac{1}{2}}G_{\frac{1}{2}}\}\Big)|\chi\rangle = 2L_0|\chi\rangle = 2\Big(a_{NS} - \frac{1}{2}\Big)|\chi\rangle \overset{!}{=} 0 \tag{3.102}$$

which can only vanish if:

$$a_{NS} = \frac{1}{2}. \tag{3.103}$$

The Ramond normal ordering constant can be calculated using the same method, but it is easier to extract it as a consequence of $F_0^2 = L_0$ giving us:

$$a_R = 0. \tag{3.104}$$

To determine our critical dimension, the one which preserves Lorentz invariance, we can either go up another state in the NS sector or use our Ramond sector. We consider a state in the R sector of the form:

$$|\phi\rangle = F_0F_{-1}|\chi\rangle \tag{3.105}$$

with $|\chi\rangle$ once again satisfying our physical state conditions

$$F_1|\chi\rangle = (L_0 + 1)|\chi\rangle|\chi\rangle = 0. \tag{3.106}$$

Once again, the introduction of the constant in the last term comes from the F_1 operator lowering the eigenvalue of the state, now by an integer value. We need to show that $L_1|\phi\rangle$ is annihilated [12] [64]. Then using $a_R = 0$ we are left with

$$L_1|\phi\rangle = \Big(\frac{1}{2}F_1 + F_0F_1\Big)F_{-1}|\chi\rangle = \frac{1}{4}\Big(D - 10)|\chi\rangle \overset{!}{=} 0 \tag{3.107}$$

which is only satisfied when

$$D = 10. \tag{3.108}$$

Now we know that the superstring is a theory in $D = 10$, it is also useful to discuss the spectrum of theory. This will be done for the open string, which is actually just considering only one of the left- or right-moving sectors on it's own. We then can create the closed string spectrum by taking the tensor product of the left- and right-moving components. Due to the choice of boundary conditions for each sector, this can be rather complicated. Especially if we jumped straight into the closed string, so it's easier to consider the open string first. It is useful to move to the light-cone gauge, which allows us to create all physical excitation by acting up the ground state with the transverse raising modes, denoting this with a superscript i. To move to the light-cone gauge, we need to use conformal and super-conformal invariance, for both our bosonic and fermionic coordinates respectively. We can use our super-Virasoro constraints 3.60, for our bosonic coordinates we use the Energy-Momentum tensor to fix:

$$X^+(\sigma, \tau) = x^+ + p^+\tau. \tag{3.109}$$

Now fixing our fermionic coordinates, using the constraint on the super current, our spacetime fermionic coordinates (in the NS sector only) become:

$$\Psi^+(\sigma, \tau) = 0. \tag{3.110}$$

In the R sector, we have to keep the zero mode of the Ψ^+ expansion [12], which we will see why in the ground state. This means in our light-cone gauge that now our lightcone coordinates, X^+ and X^- as well as Ψ^+ and Ψ^-, are no longer independent degrees of freedom. This can be seen from the definitions

$$X^\pm = \frac{1}{\sqrt{2}}\big(X^0 \pm X^{D-1}\big) \tag{3.111}$$

$$\Psi^\pm = \frac{1}{\sqrt{2}}\big(\Psi^0 \pm \Psi^{D-1}\big). \tag{3.112}$$

This removes our reparametrisation invariance for both the bosonic and fermonic coordinates, as well as the manifest Lorentz invariance, but we fix the latter by working in the critical dimension $D = 10$.

Neveu-Schwarz Sector

Starting with the NS sector, we obtain the NS mass-shell condition by combining 3.94 with 3.80:

$$\alpha' M^2 = \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n + \sum_{r=\frac{1}{2}}^{\infty} r b^i_{-r} b^i_r - \frac{1}{2} \tag{3.113}$$

- **Ground state**:
  The ground state of the NS sector is defined as the state which is annihilated by α^i_n and b^i_r with $n, r > 0$:

  $$|0, p^\mu\rangle_{NS}, \tag{3.114}$$

  which must also satisfy,

  $$\alpha^\mu_0 |o, p^\mu\rangle_{NS} = \sqrt{2\alpha'} p^\mu |0, p^\mu\rangle_{NS}. \tag{3.115}$$

  The factor $\sqrt{2\alpha'}$ is a normalisation factor [64]. Our ground state has a mass squared of $M^2 = -\frac{1}{2\alpha'}$, meaning it is tachyonic. Using the same reasoning as for the bosonic string, this is an unstable maximum. We will discuss how this is removed from the spectrum shortly through the use of a GSO projection.

- **First excited state**:
  The first excited state is obtained by acting upon the ground state with the raising oscillator with the lowest frequency [12], this is $b^i_{-\frac{1}{2}}$. For clarity, our first excited state would not be the one which a^i_{-1} acts upon the ground state as instead this raises the eigenvalue by an integer unit rather than half integer.

  $$b^i_{-\frac{1}{2}} |0, p^\mu\rangle_{NS} \tag{3.116}$$

  Acting with this oscillator will give us a massless state $M^2 = 0$. The superscript i on our oscillator represents the transverse directions of our spacetime, in the light-cone gauge this actually means we are left with 8 transverse directions. Our first excited state is a massless vector (needed for Lorentz invariance), which actually confirms both our critical dimension $D = 10$ and our normal ordering constant $a_{NS} = \frac{1}{2}$.

Higher order excited states are left from discussion, as we are only interested in the massless states for phenomenology.

Ramond Sector

The Ramond sector has the following mass-shell condition, which we obtain in the same way as for the NS sector, but now instead using the Ramond number operator 3.81:

$$\alpha' M^2 = \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n + \sum_{n=1}^{\infty} n d^i_{-n} d^i_n \tag{3.117}$$

- **Ground state**:
  As discussed previously when we defined our Fock space, the Ramond ground state is degenerate due to d_0^μ not changing the mass of the state. It is the state which is annihilated by both α^i_n and d^i_n with $n > 0$:

  $$|0, p^\mu\rangle_R \tag{3.118}$$

  and must also satisfy

  $$F_0|0, p^\mu\rangle_R = 0. \tag{3.119}$$

  Using our explanation before and our redefinition of d_0 given in 3.71, we see that our ground state is a spinor (which allows degeneracy). This initially causes a problem when we realise that our zero mode d_0^μ actually contains the $D = 10$ gamma matrices, which is 32×32 matrix with 32 components [12]. A solution comes by first applying Majorana reality conditions, reducing our number of independent components by half to 16. A further condition comes from the ground state having to satisfy the Dirac equation. This reduces the number of independent components to 8. The ground state of the Ramond sector is an irreducible spinor of $Spin(8)$ or $SO(8)$.

- **First excited state**:
  We have two options for our first excited state in our Ramond sector

  $$\alpha^i_{-1}|0, p^\mu\rangle_R \quad \text{and} \quad d^i_{-1}|0, p^\mu\rangle_R \tag{3.120}$$

  However, it is obvious to see from 3.94 that the first and all higher excited states will be massive so for our discussion they aren't relevant.

The tachyonic ground state of the NS is a problem for us, as we want a theory which only consists of physical states. We will also want a theory which can sustain, if required, spacetime supersymmetry rather than just worldsheet supersymmetry. Upon inspection, we can see if we were actually able to shift the NS first excited state to the ground state and remove the tachyonic state then we'd have the same number of physical bosonic and fermionic degrees of freedom in the ground (and further excited states). The method of doing this is called a GSO projection, originally introduced Gliozzi, Scherk and Olive [37]. We will introduce this in the next section and this is something we will further generalise later for combination of states in the GGSO projection.

First, we will quickly discuss the closed string spectrum. This is formed by the tensor product of both left- and right-movers.Before we discussed the possible combinations before as being R-R, NS-NS, NS-R and R-NS coming from the pairings of boundary conditions in each sector. There are actually two different types of oriented closed superstring theories: type IIA and type IIB which actually depend on the relative chirality of the left- and right-moving R ground states [12]. This will be clearer after the next section, but for now it suffices to understand that type IIA have the left and right R sectors with opposite chirality and type IIB have the same chirality. Again, we are only interested in the massless states, with these the only states interesting for phenomenology. We will then explain why we actually use another type of theory, heterotic strings, for the construction of our models and the advantages of using heterotic strings over type IIA and type IIB strings.

The massless states of the type IIA theory:

$$|-\rangle_R \otimes |+\rangle_R, \tag{3.121}$$

$$\tilde{b}^i_{-\frac{1}{2}}|0\rangle_{NS} \otimes b^j_{-\frac{1}{2}}|0\rangle_{NS}, \tag{3.122}$$

$$\tilde{b}^i_{-\frac{1}{2}}|0\rangle_{NS} \otimes |+\rangle_R, \tag{3.123}$$

$$|-\rangle_R \otimes b^i_{-\frac{1}{2}}|0\rangle_{NS}. \tag{3.124}$$

The massless states of the type IIB theory:

$$|+\rangle_R \otimes |+\rangle_R, \tag{3.125}$$

$$\tilde{b}^i_{-\frac{1}{2}}|0\rangle_{NS} \otimes b^j_{-\frac{1}{2}}|0\rangle_{NS}, \tag{3.126}$$

$$\tilde{b}^i_{-\frac{1}{2}}|0\rangle_{NS} \otimes |+\rangle_R, \tag{3.127}$$

$$|-\rangle_R \otimes b^i_{-\frac{1}{2}}|0\rangle_{NS}. \tag{3.128}$$

From this, we can see these two theories differ only by the relative chirality of the R-R sector. It is possible to actually map these two theories together using a target space duality (T-duality) transformation, more reading focused on this can be found in reference [36]. Our type IIA/B strings have $\mathcal{N} = 2$ supersymmetry, and IIA is a non-chiral and IIB a chiral $D = 10$ supergravity theory [4]. We are going to move away from this to the heterotic string. The heterotic string only has $\mathcal{N} = 1$ supersymmetry which as mentioned in the supersymmetry section is the only possible way to construct realistic models [63]. After this, we will actually see that in a specific construction of these heterotic strings, namely the free fermionic formulation, that it's possible to construct many realistic models algorithmically (by computer generation) whilst showing the breaking patterns to the Standard Model gauge group.

3.3 GSO projection

The last section showed that not only was the superstring spectrum not spacetime supersymmetric, but more worryingly it fell to the same tachyonic fate as our bosonic string in the ground state. This can be fixed by introducing a GSO projection for the states, pioneered by Gliozzi, Scherk and Olive [37]. We apply a projection operator P_{GSO} to a physical state $|\psi\rangle$ [4]:

$$|\psi\rangle \rightarrow P_{GSO}|\psi\rangle. \tag{3.129}$$

This actually accomplishes two things, first it allows us to have a spacetime supersymmetry by matching the number of fermionic and bosonic states for a particular level. It additionally allows us to keep our theory modular invariant. Modular invariance will be properly introduced in the free fermionic models chapter but an overview comes from thinking of our theory as a conformal field theory, mapped onto a torus. A conformal field theory is really just a theory that looks the same at all different length scales and only depends on the angles [60]. Modular invariance allows us to re-scale and reorient the torus with a particular set of transformations, without changing the underlying theory.

In the NS sector our projection operator is

$$P_{GSO} = \frac{1}{2}\Big[1 - (-1)^F\Big] \tag{3.130}$$

where we have introduced the fermion number operator F, given by

$$F = \sum_{r=\frac{1}{2}}^{\infty} \eta_{\mu\nu} b^{\mu}_{-r} b^{\nu}_{r}. \tag{3.131}$$

When the GSO projection is applied to the NS sector, any states with an even number of b oscillators will be removed. Therefore, we see upon application of the GSO projection that we are able remove the tachyon from ground state of the NS sector. The R sector is different and must include a chirality operator Γ^{11}, which is the $D = 10$ analog of the $D = 4$ chirality operator γ^5 introduced in 2.13. This can be justified by our definition of our zero mode operator d_0 3.71.

In the R sector our projector operator is

$$P^{\pm}_{GSO} = \frac{1}{2}\Big[1 \mp \Gamma^{11}(-1)^F\Big]. \tag{3.132}$$

3.4 The heterotic string

As we have seen, a closed string allows the left- and right-moving fields to be decoupled. This allows us the flexibility on to treat each separately and is the basis of the heterotic string. In this construction, we impose supersymmetry on the left-moving fields only, whilst treating the right-moving fields as bosonic. This is different from our type IIA/B strings introduced above, as type II strings have supersymmetry on both the left- and right-movers. For a general superstring, the conformal anomaly is given by the sum of the individual anomaly contributions for each field and multiplied by their dimensional weighting [25]:

$$c_{total} = c_{bg} + c_{fg} + c_{X^\mu} \cdot D + c_{\psi^\mu} \cdot D \tag{3.133}$$

here c_{bg} and c_{fg} are the Faddeev-Popov ghost fields. These come from trying to maintain manifest Lorentz invariance during path integral quantisation. They are introduced to cancel the unphysical non-transverse degrees of freedom which arise [61]. It is easy to see that the left- and right-moving fields are going to have differing critical dimensions and conformal anomalies. Considering our fermions have conformal weight of $\frac{1}{2}$ and our bosons of weight 1:

$$c_L = -26 + 11 + D + \frac{D}{2} = 0 \qquad \Rightarrow D = 10 \tag{3.134}$$

$$c_R = -26 + D = 0 \qquad \Rightarrow D = 26 \tag{3.135}$$

The left-moving sector forms a superconformal algebra with $\mathcal{N} = 2$ supersymmetry and contains our superstring fields X_+^μ and ψ_+^μ with $\mu = 0, ..., 9$. The left-moving sector is hence of critical dimension $D = 10$. The right-moving sector forms a conformal algebra and has the usual bosonic fields X_-^μ n with index $\mu = 0, ..., 25$, critical dimension $D = 26$. We can then cancel our anomalies by compactifying our extra 16 space-time dimensions onto a flat torus. Modular invariance imposes some conditions on how we can do this, requiring even and self-dual lattices with only two possible options $E_8 \otimes E_8$ or $SO(32)$. These are the HO and HE heterotic strings respectively. We could then further compactify six dimensions onto a Calabi-Yau manifold, preserving $\mathcal{N} = 1$ space-time supersymmetry giving the E_6 group in four-dimensions [25].

Instead, we are going to use the free fermionic formulation which was introduced by two separate groups (Antoniadis, Bachas and Kounnas - ABK) [8][6] and Kawai, Lewellen and Tye - KLT) [43][42]. In this project, we will be using the ABK formalism. We consider our extra degrees of freedom as free fermions (we could also choose these to be bosons and this is called the free bosonic construction) on the string worldsheet. This modifies our anomaly equation: 3.135

$$c_L = -26 + 11 + D + \frac{D}{2} + \frac{N_{f_L}}{2} \tag{3.136}$$

$$c_R = -26 + D + \frac{N_{f_R}}{2}. \tag{3.137}$$

We want a four-dimensional spacetime $D = 4$, so we can plug this in and then setting both the left and right parts equal to zero [40], giving the result

$$N_{f_L} = 18 \quad \text{and} \quad N_{f_R} = 44. \tag{3.138}$$

Our four-dimensional heterotic string therefore has an extra 18 real Majorana-Weyl left-moving fermions and 44 real Majorana-Weyl right-moving fermions. Working in complex coordinates defined by

$$z = \tau + i\sigma \quad \text{and} \quad \bar{z} = \tau - i\sigma. \tag{3.139}$$

This means we now have the following fields, here z represents our left-movers and $\bar{z}$ represent our right movers

$$X^{\mu}(z, \bar{z}), \quad \mu = 1, 2 \tag{3.140}$$

$$\psi^{\mu}(z), \quad \mu = 1, 2 \tag{3.141}$$

$$\lambda^{i}, \quad i = 1, ..., 18 \tag{3.142}$$

$$\bar{\lambda}^{j}, \quad j = 1, ..., 44. \tag{3.143}$$

The spacetime bosons and fermions are restricted to two degrees of freedom, which are transverse, hence we are in the light-cone gauge. Our free fermionic heterotic string action is

$$S = \frac{1}{\pi} \int d^2 z \Big(\partial_z X_{\mu} \partial_{\bar{z}} X^{\mu} - 2i\psi^{\mu} \partial_z \psi_{\mu} - 2i \sum_{i=1}^{18} \lambda^i \partial_z \lambda^i - 2i \sum_{j=1}^{44} \bar{\lambda}^j \partial_{\bar{z}} \bar{\lambda}^j \Big) \tag{3.144}$$

The λ^i and $\bar{\lambda}^i$ do not have a spacetime index and are considered as internal degrees of freedom of the conformal field theory [46]. This finishes our review of string theory. We will now move onto the main topic of this project, which is construction of free fermionic models.

Chapter 4

Free Fermionic Models

The aim of this project is to be able to link the Standard Model to string theory. Analyse a realistic string model, which reproduces many of the features of the Standard Model. The best models are built in the free fermionic formulation [5, 6, 8]. We will now introduce the common notation of the fermionic fields and the one that we will be using for the rest of this project. It is useful to note that we will be complexifying some of our right-moving worldsheet fermions 3.143, this can be done by combining two real fermions [40]

$$\lambda_{ab} = \frac{1}{\sqrt{2}}(\lambda_a + i\lambda_b), \tag{4.1}$$

$$\lambda^*_{ab} = \frac{1}{\sqrt{2}}(\lambda_a - i\lambda_b). \tag{4.2}$$

We will only really be interested in the worldsheet fermions moving forward. We must remember we also have our space-time bosonic 3.140 fields on our left-moving SUSY sector and our right non-SUSY sector which we will package as

$$X^\mu, \quad \text{with} \quad \mu = 0, 1, 2, 3. \tag{4.3}$$

Now, our worldsheet 18 left-moving 3.142 and 44 right-moving 3.143 free fermions that we introduced in the previous section can be represented in the following way

$$\{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\psi}^{*1,\dots,5}, \bar{\eta}^{*1,2,3}, \bar{\phi}^{*1,\dots,8}\} \tag{4.4}$$

here, the complex fermions are represented with a $*$ superscript but we will drop this notation after this. The $\bar{\eta}^{1,2,3}$, $\bar{\psi}^{1,\dots,5}$ and $\bar{\phi}^{1,\dots,8}$ correspond to the gauge groups $U(1)^3$, $SO(10)$ and $SO(16)$ respectively [28].

4.1 Boundary Conditions and Modular Invariance

For perturbative theories, it's important to consider the construction of the one-loop partition function. This involves integrating over all the independent tori. We can determine which tori are independent from each other by considering the two non-contractible loops which form the tori and considering the set of transformations which preserve modular invariance [40]. Modular invariance is found to be a fundamental symmetry of all closed string models. These modular transformations are a discrete set of transformations that leave the world-sheet string action invariant [22]. Defining a complex parameter or 'modular parameter' τ which parametrises the inequivalent tori by

$$\tau = \tau_1 + i\tau_2, \tag{4.5}$$

here, τ_1 is considered a spatial coordinate and τ_2 a time coordinate [40]. We want to find transformations in which nothing changes if we re-scale and reorient the torus. we find that the torus invariant under transformations of the form

$$T: \quad \tau \to \tau + 1 \tag{4.6}$$

$$S: \quad \tau \to -\frac{1}{\tau} \tag{4.7}$$

Where T redefines the same torus and S swaps the coordinates and reorients the torus [40]. These transformations called the modular group, with the following group algebra [40]

$$\tau \to \frac{a\tau + b}{c\tau + d}. \tag{4.8}$$

We can interpret this as a matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{4.9}$$

this allows us to see our transformations more clearly as

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{4.10}$$

The fundamental domain of our modular parameter, which will be the domain we integrate over is:

$$\mathcal{F} = \{\tau \in \mathbb{C} : |\tau| \geq 1,\ -\frac{1}{2} \leq \mathrm{Re}(\tau) < \frac{1}{2},\ \mathrm{Im}(\tau) > 0\}. \tag{4.11}$$

Any region outside of this can be mapped, by our modular transformations, to inside our fundamental domain.

We must define our boundary conditions for each of our worldsheet fermions. As usual these will be NS or R but the way we define these will be slightly different. Each worldsheet fermion can be parallel transported around the non-contractible loops of the torus, picking up a phase

$$f \to -e^{i\pi\alpha(f)} f. \tag{4.12}$$

Real fermions pick up an integer phase $\alpha(f) = 0, 1$, which correspond to NS and R boundary conditions respectively [5]. Complex fermions instead pick up a phase of $\alpha(f) = (-1, 1]$ [40]. Our torus has two non-contractible loops, so we must define each fermion with the phase picked up on both of these loops. A single fermion f would be defined with the phases

$$\begin{bmatrix} \alpha(f) \\ \beta(f) \end{bmatrix} \tag{4.13}$$

but we must define something we call a spin structure for all 64 of our worldsheet fermions. Reintroducing our new notation from 4.4, we are left with the spin structure α of a single non-contractible loop [40]

$$\alpha = \{\alpha(\psi_1^\mu), ..., \alpha(\omega^6)|\alpha(\bar{y}^1), ..., \alpha(\bar{\phi}^8)\} \tag{4.14}$$

and similar for our second loop β. So our complete spin structure can be represented as

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \tag{4.15}$$

It is this large choice of combinations of different phases for our fermions which will allows us to construct our range of models, as shown in [23].

To compute the one-loop amplitude, which is our physical partition function we must integrate both the bosonic and fermionic contributions over the fundamental domain [25, 46]:

$$Z = \int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{(\operatorname{Im}\tau)^2} Z_B^2 \sum_{\text{Spin structure}} C\binom{\alpha}{\beta} \prod_{f=1}^{64} Z_F \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \tag{4.16}$$

Our bosonic contribution Z_B should look familiar

$$Z_B = \frac{1}{\sqrt{\operatorname{Im}\tau}|\eta(\tau)|^2} \tag{4.17}$$

here $\eta(\tau)$ is the Dedekind eta function defined by

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1-q^n), \quad \text{with} \quad q = e^{2\pi i\tau}. \tag{4.18}$$

The first two terms $\frac{d\tau d\bar{\tau}}{(\operatorname{Im} tau)^2}$ and Z_B are already modular invariant; the ABK rules [8] are defined by imposing modular invariance under our transformations S and T on the rest of of the contributions. The one-loop phase coefficients $C\binom{\alpha}{\beta}$ will be covered in the next section when we will state the ABK rules. A full derivation can be found in the original text [8] or appendix c in [4].

4.2 ABK Rules

In the last section, we stated the partition function for the heterotic string. Upon imposing modular invariance we gain the the ABK/KLT rules, which were originally derived by two separate research groups. The KLT formalism developed in [43, 42] is equivalent but we are going to be using the formalism developed by Antoniadis, Bachas and Kounnas in [8, 6]. Our fermions, as described in the previous section, pick up a phase when parallel transported around the two non-contractible loops of the torus. These correspond to our boundary conditions. Our models are going to be built by defining a set of basis vectors, which are themselves the set of 64 boundary conditions. A general basis vector can be defined as

$$b_i = \{\alpha(\psi_1^\mu), ..., \alpha(\omega^6)|\alpha(\bar{y}^1), ..., \alpha(\bar{\phi}^8)\} \tag{4.19}$$

and $\alpha(f)$ is our boundary condition for the fermion f taking values of

$$\alpha(f) = \begin{cases} 0 & \text{Anti-periodic (NS)} \\ 1 & \text{Periodic (R)} \\ \frac{1}{2} & \text{Complex} \end{cases} . \tag{4.20}$$

Our boundary conditions in our non-SUSY model will only ever be NS or R which simplifies a lot of the calculations. This will be especially apparent when we introduce the GGSO projection in the next section. The collection of all our basis vectors form an finite additive group Ξ which span a space of 2^{N+1} sectors defined by a linear combination of the basis vectors [30]:

$$\Xi = \sum_{i=1}^{N} m_j b_i, \quad \text{with} \quad m_j = 0, ..., N_j - 1 \tag{4.21}$$

where $N_j \cdot b_j = 0 \bmod 2$. This formula is just really a fancy way of saying all our basis vectors need to be linearly independent and a vector is not a basis vector if it can be made from a combination of the other basis vectors. This will be more apparent when we compare the basis vectors of our model to previous works, we will find that a vector common in other models z_2 can actually be created through a linear combination of our other basis vectors and hence should not be included. Our basis vectors have to be selected carefully. Thinking of our model as a conformal field theory means that our basis vectors need to be modular invariant and this is used to derive our ABK rules.

Rules on the Basis Vectors

Imposing modular invariance gives us the following 5 rules on our basis vectors:

1. The basis vector, where all our boundary conditions are periodic must be in our group

$$\mathbb{1} \in \Xi \tag{4.22}$$

2. There must be an even number of real fermions

3. Our basis, which can be chosen to be canonical

$$\sum_{i=1}^{N} m_i b_i = 0 \quad \Leftrightarrow \quad m_i = 0 \qquad \text{mod } N_i, \quad \forall i \tag{4.23}$$

4. The basis must obey the condition:

$$N_{ij} b_i \cdot b_j = 0 \qquad \text{mod } 4 \tag{4.24}$$

5. The basis must also obey the condition, if N_i is even:

$$N_i b_i = 0 \qquad \text{mod } 8 \tag{4.25}$$

The scalar product of two our our basis vectors, α and β, is given by:

$$\alpha \cdot \beta = \Big(\sum_{Left} - \sum_{Right} \Big) \alpha(f) \beta(f) \tag{4.26}$$

here complex fermions are weighted as 2 real fermions. This can be seen from our complexification formula 4.1, which will will repeat here just for clarity:

$$\lambda_{ab} = \frac{1}{\sqrt{2}}(\lambda_a + i\lambda_b),$$
$$\lambda^*_{ab} = \frac{1}{\sqrt{2}}(\lambda_a - i\lambda_b).$$

Rules on the Phase Coefficients

When imposing modular invariance we also get the following rule on the one-loop phase coefficients:

1.
$$C\begin{pmatrix} b_i \\ b_j \end{pmatrix} = \delta_{b_i} e^{\frac{2i\pi}{N_j} n} = \delta_{b_j} e^{\frac{i\pi}{2} b_i \cdot b_j} e^{\frac{2i\pi}{N_i} m} \tag{4.27}$$

2.
$$C\begin{pmatrix} b_i \\ b_i \end{pmatrix} = -e^{\frac{i\pi}{4} b_i \cdot b_i} C\begin{pmatrix} b_i \\ \mathbb{1} \end{pmatrix} \tag{4.28}$$

3.
$$C\begin{pmatrix} b_i \\ b_j \end{pmatrix} = e^{\frac{i\pi}{2} b_i \cdot b_j} C\begin{pmatrix} b_j \\ b_i \end{pmatrix}^* \tag{4.29}$$

4.
$$C\begin{pmatrix} b_i \\ b_j + b_k \end{pmatrix} = \delta_{b_i} C\begin{pmatrix} b_i \\ b_j \end{pmatrix} C\begin{pmatrix} b_i \\ b_k \end{pmatrix} \tag{4.30}$$

here, the $*$ means conjugate, but we will only have real boundary conditions in our models so this doesn't really affect us. δ_{b_i} is called the spacetime spin statistics index [40], which is either ± 1 depending whether the sector b_i contains our spacetime fermions ψ^μ:

$$\delta_{b_i} = \begin{cases} -1 & b_i \text{ contains } \psi^\mu \\ 1 & b_i \text{ does not contain } \psi^\mu \end{cases} \tag{4.31}$$

4.3 GGSO Projections and Mass Formulae

In section 3.3, we introduced the concept of being able to project out certain states of our fermionic string. Specifically, this was used to remove the tachyon from the spectrum and match the number of fermionic and bosonic states. We would now like to be able to perform GSO projections on each of our basis vectors, which contain combinations of states, this extension is the Generalised GSO (GGSO) projection [37]. Our GGSO projection is given by

$$e^{i\pi b_i \cdot F_\alpha}|s\rangle_\alpha = \delta_\alpha C\begin{pmatrix}\alpha\\ b_i\end{pmatrix}^*|s\rangle_\alpha, \tag{4.32}$$

here $|s\rangle_\alpha$ is a state in the sector $\alpha \in \Xi$. States which satisfy this expression are kept in the spectrum. Whereas states that do not are projected out, like our fermionic string tachyon.

The string we are dealing with is heterotic, which means the mass of the left- and right-moving sectors is decoupled and we can consider them separately. We still require our mass to by level matched, i.e. satisfying the Virasoro condition:

$$M_L^2 \overset{!}{=} M_R^2, \tag{4.33}$$

where the masses of the left- and right-moving sectors are given by

$$M_L^2 = -\frac{1}{2} + \frac{\alpha_L \cdot \alpha_L}{8} + N_L \tag{4.34}$$

$$M_R^2 = -1 + \frac{\alpha_R \cdot \alpha_R}{8} + N_R. \tag{4.35}$$

The N_L,N_R are the sum of the left- and right-moving oscillators

$$N_{L,R} = \sum_{f_{L,R}} \nu_f + \sum_{f^*_{L,R}} \nu_{f^*} \tag{4.36}$$

here ν_{f,f^*} are the frequencies of the fermion f or complex conjugate fermion f^* and are given by

$$\nu_f = \frac{1+\alpha(f)}{2}, \quad \nu_{f^*} = \frac{1-\alpha(f)}{2}. \tag{4.37}$$

We will be interested in the sectors which are tachyonic, so we can ensure they are removed, and for the sectors which are massless. We are now ready to construct our non-SUSY models.

Chapter 5

Non-supersymmetric models

A lot of work has been done in previous in the free fermionic formulation introduced in [8, 43, 5]. Early three generation standard-like models and their construction happened in the late eighties / early nineties and can be found in [7, 29, 27]. Later research focused on models containing a common basis of five basis vectors $\{\mathbb{1}, S, b_1, b_2, b_3\}$, called the NAHE set [26], which generates a gauge group of $SO(10) \otimes SO(6)^3 \otimes E_8$. In these models, the S-vector generates an $\mathcal{N} = 4$ spacetime supersymmetry which is broken to $\mathcal{N} = 2$ then $\mathcal{N} = 1$ by the b_1 and b_2 vectors respectively, whilst the final b_3 either preserves or removes the remaining spacetime supersymmetry [28]. Additional extensions of these models came from adding additional basis vectors which allowed the breaking of the $SO(10)$ gauge group into commonly recognised GUT gauge groups.

The latest papers use an extended basis and research has been heavily focused on the classification [9, 24, 23, 30]. The model presented is going to use a modified version of the unbroken SO(10) basis vectors, used in the classification paper [23] where the $e_i, i = 1, ..., 6$ basis vectors will instead be grouped in pairs as $T_i, i = 1, 2, 3$. This pairing of basis vectors has been used in the paper [32], focused on strings with positive cosmological constants. This differs from our selection of basis vectors in a few distinct ways.

5.1 Basis Vectors

We will use the basis presented in [32] with a few modifications:

$$
\begin{aligned}
\mathbb{1} &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \overline{y}^{1,\dots,6}, \overline{\omega}^{1,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,\dots,8}\},\\
\tilde{S} &= \{\psi^\mu, \chi^{1,\dots,6} \mid \overline{\phi}^{3,4,5,6}\},\\
T_1 &= \{y^{1,2}, \omega^{1,2} \mid \overline{y}^{1,2}, \overline{\omega}^{1,2}\},\\
T_2 &= \{y^{3,4}, \omega^{3,4} \mid \overline{y}^{3,4}, \overline{\omega}^{3,4}\},\\
T_3 &= \{y^{5,6}, \omega^{5,6} \mid \overline{y}^{5,6}, \overline{\omega}^{5,6}\},\\
b_1 &= \{\psi^\mu, \chi^{12}, y^{34}, y^{56} \mid \overline{y}^{34}, \overline{y}^{56}, \overline{\eta}^{1}, \overline{\psi}^{1,\dots,5}\}, && (5.1)\\
b_2 &= \{\psi^\mu, \chi^{34}, y^{12}, y^{56} \mid \overline{y}^{12}, \overline{y}^{56}, \overline{\eta}^{2}, \overline{\psi}^{1,\dots,5}\},\\
b_3 &= \{\psi^\mu, \chi^{56}, y^{12}, y^{34} \mid \overline{y}^{12}, \overline{y}^{34}, \overline{\eta}^{3}, \overline{\psi}^{1,\dots,5}\},\\
z_1 &= \{\overline{\phi}^{1,\dots,4}\},\\
\alpha &= \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}. && (5.2)
\end{aligned}
$$

The addition of the α vector breaks the $SO(10)$ gauge group to the Pati-Salam gauge group $SO(4) \otimes SO(6)$. Pati-Salam models were classified in [9]. Our model differs from those Pati-Salam classification paper in the following way:

1. The SUSY generating vector is replaced with a SUSY breaking vector $S \to \tilde{S}$
2. The e_i, $i = 1, \dots, 6$ are paired up into the $T_{1,2,3}$
3. b_3 is its own basis vector rather than a linear combination
4. z_2 appears as a linear combination so is not present in the basis
5. ψ^μ is Ramond in the $b_{1,2,3}$ rather than NS

These changes have a few consequences, which we will talk about briefly but later discuss in more detail throughout this section. The replacement of the SUSY-vector S with $\tilde{S}$ has a few key consequences, the obvious one is there is no generation of the $\mathcal{N} = 4$ spacetime supersymmetry. Lack of spacetime supersymmetry means further consideration of the tachyonic sectors is required. Tachyonic sectors and specifically the removal of these through choices of GGSO coefficients are now of primary importance and will be a major discussion of this work.

Another consequence of the change of basis vectors is now that the vector z_2 is not linearly independent and cannot be included as a basis vector. We can actually see that z_2 can be created from the combination:

$$z_2 = \mathbb{1} + \sum_i T_i + \sum_i b_i + z_1 = \{\bar{\phi}^{5,\dots,8}\} \tag{5.3}$$

and whenever z_2 is included following this it refers to this linear combination, rather than a independent basis vector. The inclusion of b_3 as a basis vector also comes from the change to $\tilde{S}$, as it can no longer be created from a linear combination. Later, we will discuss on the grouping of the e_i basis vectors into T_i's, how this constrains the moduli space and our models ability of the model to generate three generation models. Another useful combination is:

$$\tilde{x} = b_1 + b_2 = b_3 = \{\psi^\mu, \chi^{1,\dots,6} | \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}\} \tag{5.4}$$

What we are going to do next is present all the possible tachyonic and massless sectors $M_{L,R} \leq 0$ that can arise in our model. This is done quickly by running the basis vectors through some simple Python code and then ensuring our sectors are linearly independent. Running this code, we see that we are left with a total of 243 sectors to analyse. These are listed throughout the next sector, but we will focus our analysis on the observable sectors.

5.2 Tachyonic Sector Analysis

For our Tachyonic sectors, we are going to group them by mass level. This is just (M_L^2, M_R^2) and it allows us to more easily sort them. We require the tachyons to be level matched $M_L^2 = M_R^2$, so in some sectors we are going to need an addition of an oscillator and these are called vectorial sectors. The already level matched tachyonic states are the spinorial tachyons.

Mass Level	**Vectorials**	**Spinorials**	**Total**
$(-\frac{1}{2}, -\frac{1}{2})$	$\{\bar{\lambda}^i\}\|NS\rangle$	$z_1, \alpha, z_1+\alpha, z_2$	5
$(-\frac{1}{4}, -\frac{1}{4})$	$\{\bar{\lambda}^i\}T_i$	$T_i+z_1, T_i+\alpha, T_i+z_1+\alpha, T_i+z_2$	15

As we can see from the table, we have 5 tachyons in the $(-\frac{1}{2}, -\frac{1}{2})$ mass level and 15 in the $(-\frac{1}{4}, -\frac{1}{4})$ mass level. This gives us a total of 20 tachyonic states. We will now discuss the projector for each of these states.

5.2.1 Absence of Tachyons - Tachyonic Projectors

We can summarise the tachyons and the corresponding vectors will project them into a table:

Sector	**Projecting vectors**
$\{\bar{\lambda}^i\}\|NS\rangle$	all
z_1	$T_1, T_2, T_3, b_1, b_2, b_3, z_2$
α	$\tilde{S}, T_1, T_2, T_3, z_2$
$z_1+\alpha$	T_1, T_2, T_3, z_2
z_2	$T_1, T_2, T_3, b_1, b_2, b_3, z_1, \alpha$
$\{\bar{\lambda}^i\}T_i$	$\tilde{S}, T_j, T_k, b_i, z_1, \alpha, z_2, \tilde{x}$
T_i+z_1	$T_j, T_k, b_i, \tilde{x}, z_2$
$T_i+\alpha$	$\tilde{S}, T_j, T_k, z_2$
$T_i+z_1+\alpha$	T_j, T_k, z_2
T_i+z_2	$T_j, T_k, b_i, \tilde{x}, z_1, \alpha$

$\{\bar{\lambda}^i\}|NS\rangle$ Tachyonic Sectors Projection Conditions

The conditions on the phases to project the NS tachyonic sector are seen in the table below:

Oscillator	$C\binom{NS}{\mathbb{1}}$	$C\binom{NS}{\tilde{S}}$	$C\binom{NS}{T_1}$	$C\binom{NS}{T_2}$	$C\binom{NS}{T_3}$	$C\binom{NS}{b_1}$	$C\binom{NS}{b_2}$	$C\binom{NS}{b_3}$	$C\binom{NS}{z_1}$	$C\binom{NS}{\alpha}$	$C\binom{NS}{z_2}$	$C\binom{NS}{\tilde{x}}$
$\bar{y}^{1,2}$	+	-	+	-	-	-	+	+	-	-	-	-
$\bar{y}^{3,4}$	+	-	-	+	-	+	-	+	-	-	-	-
$\bar{y}^{5,6}$	+	-	-	-	+	+	+	-	-	-	-	-
$\bar{\omega}^{1,2}$	+	-	+	-	-	-	-	-	-	-	-	-
$\bar{\omega}^{3,4}$	+	-	-	+	-	-	-	-	-	-	-	-
$\bar{\omega}^{5,6}$	+	-	-	-	+	-	-	-	-	-	-	-
$\bar{\psi}^{1/2/3}$	+	-	-	-	-	+	+	+	-	-	-	+
$\bar{\psi}^{4/5}$	+	-	-	-	-	+	+	+	-	+	-	+
$\bar{\eta}^1$	+	-	-	-	-	+	-	-	-	-	-	+
$\bar{\eta}^2$	+	-	-	-	-	-	+	-	-	-	-	+
$\bar{\eta}^3$	+	-	-	-	-	-	-	+	-	-	-	+
$\bar{\phi}^{1/2}$	+	-	-	-	-	-	-	-	+	+	-	-
$\bar{\phi}^{3/4}$	+	+	-	-	-	-	-	-	+	-	-	-
$\bar{\phi}^{5/6}$	+	+	-	-	-	-	-	-	-	-	+	-
$\bar{\phi}^{7/8}$	+	-	-	-	-	-	-	-	-	-	+	-

$\{\bar{\lambda}^i\}T_1$ **Tachyonic Sector calculation**

First, we need to check the mass squared level of both the left- and right-movers

$$M_L^2 = -\frac{1}{2} + \frac{\alpha_L \cdot \alpha_L}{8} + N_L == -\frac{1}{2} + \frac{2}{8} + N_L = -\frac{1}{4} + N_L \tag{5.5}$$

$$M_L^2 = -1 + \frac{\alpha_R \cdot \alpha_R}{8} + N_R = -1 + \frac{2}{8} + N_R = -\frac{3}{4} + N_R. \tag{5.6}$$

From this, we can see we need the sum of the right-moving oscillators to be $N_R = \frac{1}{2}$. We can calculate the frequencies of these oscillators:

$$\nu_f = \frac{1 + \alpha(f)}{2} = \frac{1}{2} \tag{5.7}$$

and we see this is satisfied when $\alpha(f) = 0$. so single complex (or two real) NS right-mover is required for level matching. In this sector our NS right-movers are $\{\bar{y}^{3,4}, \bar{y}^{5,6}, \bar{\omega}^{3,4}, \bar{\omega}^{5,6}, \bar{\psi}^{1/2/3/4/5}, \bar{\eta}^{1/2/3}, \bar{\phi}^{1/2/3/4/5/6/7/8}\}$. Here, the superscripts separated with a comma indicate they are grouped together and the slash means each each is a separate oscillator. The list of our projectors will be calculated at the same way as the sectors which aren't level matched. We are effectively looking for the basis vectors/combinations which do not 'overlap' with T_1. Hence we can see that our projectors are $\tilde{S}, T_2, T_3, b_1, z_1, \alpha, z_2, \tilde{x}$. We must calculate the conditions of projection of the $\{\bar{\lambda}^i\}T_1$ tachyon for all the combinations of these possible oscillators and projectors.

- if oscillator is $\{\bar{y}^{3,4}\}$ then:
  $\tilde{S}$ projection:

  $$e^{i\pi \tilde{S}\cdot F_{\bar{y}^{3,4}}} e^{i\pi \tilde{S}\cdot F_{T_1}} |\xi\rangle = \delta_{T_1} C \binom{T_1}{\tilde{S}} |\xi\rangle \tag{5.8}$$

  We can see that $\tilde{S} \cdot F_{\bar{y}^{3,4}} = 0$ and $\tilde{S} \cdot F_{T_1} = 0$. Whilst clearly $\delta_{T_1} = 1$ as the sector T_1 does not contain spacetime fermions ψ^μ. Hence we are left with the condition on the phase:

  $$C\binom{T_1}{\tilde{S}} = \begin{cases} 1 & \implies \quad \text{in} \\ -1 & \implies \quad \text{projected} \end{cases} \tag{5.9}$$

  $T_3, z_1, \alpha, z_2, \tilde{x}$ projections: We can use the same reasoning to speed things up a bit here. We see that the projectors which also have no 'overlap' with the $\bar{y}^{3,4}$ oscillator and hence hold the same condition on the phase are $T_3, z_1, \alpha, z_2, \tilde{x}$. These calculations will be therefore left out as they are identical to the $\tilde{S}$ calculation.

$$C\binom{T_1}{T_3} = C\binom{T_1}{z_1} = C\binom{T_1}{\alpha} = C\binom{T_1}{z_2} = C\binom{T_1}{\tilde{x}} \tag{5.10}$$

$$= \begin{cases} 1 & \Longrightarrow \quad \text{in} \\ -1 & \Longrightarrow \quad \text{projected} \end{cases} \tag{5.11}$$

We will now show what happens when an oscillator overlaps with the projector by considering the T_2 and b_1 projections.

$\boldsymbol{T_2, b_1}$ **projections**:

we will firstly just consider the b_1 projector and show that T_2 is similar.

$$e^{i\pi b_1 \cdot F_{\bar{y}^{3,4}}} e^{i\pi b_1 \cdot F_{T_1}} |\xi\rangle = \delta_{T_1} C\binom{T_1}{b_1} \tag{5.12}$$

Now we can see that the difference is now $e^{i\pi b_1 \cdot F_{\bar{y}^{3,4}}} = -1$ and hence flips our phase coefficient. The same will be true of T_2. This gives us the conditions on the phase coefficients:

$$\binom{T_1}{T_2} = C\binom{T_1}{b_1} = \begin{cases} 1 & \Longrightarrow \quad \text{projected} \\ -1 & \Longrightarrow \quad \text{in} \end{cases} \tag{5.13}$$

- For the other oscillators each calculation is similar. The phases to project out the $\{\bar{\lambda}^i\}T_1$, $\{\bar{\lambda}^i\}T_2$ and $\{\bar{\lambda}^i\}T_3$ tachyons are summarised in the tables below.

$\{\bar\lambda^i\}T_i$ Tachyonic Sectors Projection Conditions

Oscillator	$C\binom{T_1}{\tilde S}$	$C\binom{T_1}{T_2}$	$C\binom{T_1}{T_3}$	$C\binom{T_1}{b_1}$	$C\binom{T_1}{z_1}$	$C\binom{T_1}{\alpha}$	$C\binom{T_1}{z_2}$	$C\binom{T_1}{\tilde x}$
$\bar y^{3,4}$	-	+	-	+	-	-	-	-
$\bar y^{5,6}$	-	-	+	+	-	-	-	-
$\bar\omega^{3,4}$	-	+	-	-	-	-	-	-
$\bar\omega^{5,6}$	-	-	+	-	-	-	-	-
$\bar\psi^{1/2/3}$	-	-	-	+	-	-	-	+
$\bar\psi^{4/5}$	-	-	-	+	-	+	-	+
$\bar\eta^{1}$	-	-	-	+	-	-	-	+
$\bar\eta^{2/3}$	-	-	-	-	-	-	-	+
$\bar\phi^{1/2}$	-	-	-	-	+	+	-	-
$\bar\phi^{3/4}$	+	-	-	-	+	-	-	-
$\bar\phi^{5/6}$	+	-	-	-	-	-	+	-
$\bar\phi^{7/8}$	-	-	-	-	-	-	+	-

Oscillator	$C\binom{T_2}{\tilde S}$	$C\binom{T_2}{T_1}$	$C\binom{T_2}{T_3}$	$C\binom{T_2}{b_2}$	$C\binom{T_2}{z_1}$	$C\binom{T_2}{\alpha}$	$C\binom{T_2}{z_2}$	$C\binom{T_2}{\tilde x}$
$\bar y^{1,2}$	-	+	-	+	-	-	-	-
$\bar y^{5,6}$	-	-	+	+	-	-	-	-
$\bar\omega^{1,2}$	-	+	-	-	-	-	-	-
$\bar\omega^{5,6}$	-	-	+	-	-	-	-	-
$\bar\psi^{1/2/3}$	-	-	-	+	-	-	-	+
$\bar\psi^{4/5}$	-	-	-	+	-	+	-	+
$\bar\eta^{2}$	-	-	-	+	-	-	-	+
$\bar\eta^{1/3}$	-	-	-	-	-	-	-	+
$\bar\phi^{1/2}$	-	-	-	-	+	+	-	-
$\bar\phi^{3/4}$	+	-	-	-	+	-	-	-
$\bar\phi^{5/6}$	+	-	-	-	-	-	+	-
$\bar\phi^{7/8}$	-	-	-	-	-	-	+	-

Oscillator	$C\binom{T_3}{\tilde{S}}$	$C\binom{T_3}{T_1}$	$C\binom{T_3}{T_2}$	$C\binom{T_3}{b_3}$	$C\binom{T_3}{z_1}$	$C\binom{T_3}{\alpha}$	$C\binom{T_3}{z_2}$	$C\binom{T_3}{\tilde{x}}$
$\bar{y}^{1,2}$	-	+	-	+	-	-	-	-
$\bar{y}^{3,4}$	-	-	+	+	-	-	-	-
$\bar{\omega}^{1,2}$	-	+	-	-	-	-	-	-
$\bar{\omega}^{3,4}$	-	-	+	-	-	-	-	-
$\bar{\psi}^{1/2/3}$	-	-	-	+	-	-	-	+
$\bar{\psi}^{4/5}$	-	-	-	+	-	+	-	+
$\bar{\eta}^{3}$	-	-	-	+	-	-	-	+
$\bar{\eta}^{1/2}$	-	-	-	-	-	-	-	+
$\bar{\phi}^{1/2}$	-	-	-	-	+	+	-	-
$\bar{\phi}^{3/4}$	+	-	-	-	+	-	-	-
$\bar{\phi}^{5/6}$	+	-	-	-	-	-	+	-
$\bar{\phi}^{7/8}$	-	-	-	-	-	-	+	-

5.3 Massless Sector Analysis

In total, we have 223 massless sectors. These can be split into three groups: 197 spinorials, 21 vectorials and 5 enhancements. We will present and group all the sectors, but for the analysis in this paper we will only be focusing on the observable sectors (for both the spinorial and vectorials).

5.3.1 Spinorials

There is a total of 197 spinorial massless sectors. These are sectors which are of mass level $(4, 8)$ and do not require an oscillator. We will group these by observable sectors, hidden and exotic sectors and then further split them into fermionic and bosonic sectors which depends on the presence of the spacetime fermions.

Observable

There are 12 (actually 13, which we explain below) twisted sectors corresponding to the fermionic observable $\mathbf{16}/\overline{\mathbf{16}}$ of $SO(10)$ which come from the combinations:

$$B^1_{pq} = b_1 + pT_2 + qT_3 \tag{5.14}$$

$$B^2_{pq} = b_2 + pT_1 + qT_3 \tag{5.15}$$

$$B^3_{pq} = b_3 + pT_1 + qT_2 \tag{5.16}$$

We obtain the projectors for our spinorial/anti-spinorial $\mathbf{16}/\overline{\mathbf{16}}$'s sectors of $SO(10)$:

$$P^1_{pq} = \frac{1}{2^3}\left[1 - C\binom{B^1_{pq}}{T_1}\right]\prod_{i=1}^{2}\left[1 - C\binom{B^3_{pq}}{z_i}\right] \tag{5.17}$$

$$P^2_{pq} = \frac{1}{2^3}\left[1 - C\binom{B^2_{pq}}{T_2}\right]\prod_{i=1}^{2}\left[1 - C\binom{B^3_{pq}}{z_i}\right] \tag{5.18}$$

$$P^3_{pq} = \frac{1}{2^3}\left[1 - C\binom{B^3_{pq}}{T_3}\right]\prod_{i=1}^{2}\left[1 - C\binom{B^3_{pq}}{z_i}\right] \tag{5.19}$$

We can distinguish between our spinorials and anti-spinorials with appropriate projections, for example for B^1_{pq}:

$$e^{i\pi(b_2+(1-q)T_3)\cdot F_{B^1_{pq}}}|\xi\rangle_{B^1_{pq}} = \delta_{B^1} C\binom{B^1_{pq}}{b_2+(1-q)T_3}|\xi\rangle_{B^1_{pq}} \tag{5.20}$$

$$\implies e^{i\pi(F(\psi^\mu)-F(\bar{\psi}^{1,\dots,5}))}|\xi\rangle_{B^1_{pq}} = -C\binom{B^1_{pq}}{b_2+(1-q)T_3}|\xi\rangle_{B^1_{pq}} \tag{5.21}$$

$$\implies e^{i\pi F(\bar{\psi}^{1,\dots,5})}|\xi\rangle_{B^1_{pq}} = -ch(\psi^\mu)C\binom{B^1_{pq}}{b_2+(1-q)T_3}|\xi\rangle_{B^1_{pq}} \tag{5.22}$$

we define our spacetime chirality $ch(\psi^\mu)=+1$. The chirality of our surviving $\mathbf{16}/\mathbf{\bar{16}}$'s is given by:

$$\chi^1_{pq} = -ch(\psi^\mu)C\binom{B^1_{pq}}{b_2+(1-q)T_3}P^1_{pq} \tag{5.23}$$

$$\chi^2_{pq} = -ch(\psi^\mu)C\binom{B^2_{pq}}{b_1+(1-q)T_3}P^2_{pq} \tag{5.24}$$

$$\chi^3_{pq} = -ch(\psi^\mu)C\binom{B^3_{pq}}{b_1+(1-q)T_2}P^3_{pq}. \tag{5.25}$$

What differs in our models in comparison to other models is the spacetime fermions are Ramond in b_1, b_2, b_3. This means our $\tilde{x}$ sector also contributes to our observable $\mathbf{16}/\mathbf{\bar{16}}$'s. Another feature of the $\tilde{x}$ sector is that it can act as a map from fermion $\leftrightarrow$ boson:

$$B^4 = \tilde{x} = b_1+b_2+b_3 = \{\psi^\mu, \chi^{1,\dots,6}|\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}\} \tag{5.26}$$

We can similarly construct a projector for the $\tilde{x}$ sector:

$$P^4 = \frac{1}{2^5}\prod_{i=1}^{3}\left[1-C\binom{\tilde{x}}{T_i}\right]\prod_{j=1}^{2}\left[1-C\binom{\tilde{x}}{z_i}\right]. \tag{5.27}$$

We actually get both a $\mathbf{16}$ and $\mathbf{\bar{16}}$ from the $\tilde{x}$ sector, so we will deal with this separately if the sector is present. We can then count the number of $16/\bar{16}$'s for $B^{1,2,3}$ to these equations we need to remember to add an addition $\mathbf{16}$ and $\mathbf{\bar{16}}$ to each:

$$N_{16} = \frac{1}{2}\sum_{A=1,2,3,4} P^A_{pq}(1+\chi^A_{pq}) \tag{5.28}$$

$$N_{\bar{16}} = \frac{1}{2}\sum_{A=1,2,3,4} P^A_{pq}(1-\chi^A_{pq}). \tag{5.29}$$

Here the factor $\frac{1}{2}$ is just a normalising factor from the construction of the equation. $\tilde{S}$ means that no superpartner sectors arise for the $SO(10)$ $\mathbf{16}/\overline{\mathbf{16}}$'s, i.e. we have no supermultiplets. The $\mathbf{16}/\overline{\mathbf{16}}$ representations of $SO(10)$ decomposed under the Pati-Salam group, i.e. $SO(6) \otimes SO(4) \equiv SU(4) \otimes SU(2)_L \otimes SU(2)_R$, given in 2.40:

$$\mathbf{16} \rightarrow (\mathbf{4},\mathbf{2},\mathbf{1}) + (\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) = N_L + \bar{N}_R \tag{5.30}$$

$$\overline{\mathbf{16}} \rightarrow (\mathbf{4},\mathbf{1},\mathbf{2}) + (\bar{\mathbf{4}},\mathbf{2},\mathbf{1}) = N_R + \bar{N}_L \tag{5.31}$$

We determine whether our spinorials/anti-spinorials have left or right chirality by a projection with the α vector. This will allow us to determine the chirality of the $\bar{\psi}^{4,5}$. We can show this more explicitly to explain:

$$e^{i\pi\alpha\cdot F_{B^A_{pq}}}|\xi\rangle = \delta_{B^A_{pq}} C\binom{B^A_{pq}}{\alpha}|\xi\rangle \tag{5.32}$$

$$\implies ch(\bar{\psi}^{4,5}) = -C\binom{B^A_{pq}}{\alpha}. \tag{5.33}$$

The convention, adopted in [31], for left/right chirality is:

$$C\binom{B^A_{pq}}{\alpha} = \begin{cases} +1 & \implies \quad \text{Left} \\ -1 & \implies \quad \text{Right.} \end{cases} \tag{5.34}$$

Combining this with our spinorial/anti-spinorial counting expressions, we are able to count the number of left and right moving spinorial/anti-spinorials:

$$N_L = \frac{1}{4} \sum_{A=1,2,3,4} P^A_{pq}(1+\chi^A_{pq}) \left[1 + C\binom{B^A_{pq}}{\alpha}\right] \tag{5.35}$$

$$\bar{N}_R = \frac{1}{4} \sum_{A=1,2,3,4} P^A_{pq}(1+\chi^A_{pq}) \left[1 - C\binom{B^A_{pq}}{\alpha}\right] \tag{5.36}$$

$$\bar{N}_L = \frac{1}{4} \sum_{A=1,2,3,4} P^A_{pq}(1-\chi^A_{pq}) \left[1 + C\binom{B^A_{pq}}{\alpha}\right] \tag{5.37}$$

$$N_R = \frac{1}{4} \sum_{A=1,2,3,4} P^A_{pq}(1-\chi^A_{pq}) \left[1 - C\binom{B^A_{pq}}{\alpha}\right]. \tag{5.38}$$

Finally, we can count the total number of fermion generations n_g. Here, we must be mindful of the degeneracy of the states. We actually get a factor of 4 for each sector present:

$$n_g = 4\left(N_L - \bar{N}_L\right) = 4\left(\bar{N}_R - N_R\right) \tag{5.39}$$

Hidden

These are the spinorial sectors which contain the hidden group $\bar{\phi}^{1,\dots,8}$, but do not contain the observable group $\bar{\psi}^{1,..,5}$. We get 24 hidden bosonic spinorial sectors of the form:

$$B^1_{pq} + \tilde{x} + z_1 \tag{5.40}$$
$$B^2_{pq} + \tilde{x} + z_1 \tag{5.41}$$
$$B^3_{pq} + \tilde{x} + z_1 \tag{5.42}$$
$$B^1_{pq} + \tilde{x} + z_2 \tag{5.43}$$
$$B^2_{pq} + \tilde{x} + z_2 \tag{5.44}$$
$$B^3_{pq} + \tilde{x} + z_2 \tag{5.45}$$
$$\tag{5.46}$$

We also get an additional 6 hidden bosonic sectors, which come from the addition of z_1 and z_2 to the scalars:

$$T_i + T_j + z_1 \tag{5.47}$$
$$T_i + T_j + z_2 \tag{5.48}$$

where $i \neq j$. There are a total of 48 hidden fermionic sectors:

$$B^1_{pq} + \tilde{S} + \tilde{x} \tag{5.49}$$
$$B^2_{pq} + \tilde{S} + \tilde{x} \tag{5.50}$$
$$B^3_{pq} + \tilde{S} + \tilde{x} \tag{5.51}$$
$$B^1_{pq} + \tilde{S} + \tilde{x} + z_1 \tag{5.52}$$
$$B^2_{pq} + \tilde{S} + \tilde{x} + z_1 \tag{5.53}$$
$$B^3_{pq} + \tilde{S} + \tilde{x} + z_1 \tag{5.54}$$
$$B^1_{pq} + \tilde{S} + \tilde{x} + z_2 \tag{5.55}$$
$$B^2_{pq} + \tilde{S} + \tilde{x} + z_2 \tag{5.56}$$
$$B^3_{pq} + \tilde{S} + \tilde{x} + z_2 \tag{5.57}$$
$$B^1_{pq} + \tilde{S} + \tilde{x} + z_1 + z_2 \tag{5.58}$$
$$B^2_{pq} + \tilde{S} + \tilde{x} + z_1 + z_2 \tag{5.59}$$
$$B^3_{pq} + \tilde{S} + \tilde{x} + z_1 + z_2 \tag{5.60}$$

Exotic

Exotic sectors are those which contain both the observable $\bar{\psi}^{1,..,5}$ and the hidden group $\bar{\phi}^{1,...,8}$. We get 48 exotic bosonic sectors of the form:

$$B^1_{pq} + \tilde{x} + \alpha \tag{5.61}$$
$$B^2_{pq} + \tilde{x} + \alpha \tag{5.62}$$
$$B^3_{pq} + \tilde{x} + \alpha \tag{5.63}$$
$$B^1_{pq} + \tilde{x} + z_1 + \alpha \tag{5.64}$$
$$B^2_{pq} + \tilde{x} + z_1 + \alpha \tag{5.65}$$
$$B^3_{pq} + \tilde{x} + z_1 + \alpha \tag{5.66}$$
$$B^1_{pq} + \tilde{S} + z_1 + \alpha \tag{5.67}$$
$$B^2_{pq} + \tilde{S} + z_1 + \alpha \tag{5.68}$$
$$B^3_{pq} + \tilde{S} + z_1 + \alpha \tag{5.69}$$
$$B^1_{pq} + \tilde{S} + z_1 + z_2 + \alpha \tag{5.70}$$
$$B^2_{pq} + \tilde{S} + z_1 + z_2 + \alpha \tag{5.71}$$
$$B^3_{pq} + \tilde{S} + z_1 + z_2 + \alpha \tag{5.72}$$

and an extra 6 sectors which come from combinations involving the scalars:

$$T_i + T_j + \alpha \tag{5.73}$$
$$T_i + T_j + \alpha + z_1 \tag{5.74}$$

where $i \neq j$. There are 48 fermionic hidden sectors of the form:

$$B^1_{pq} + \alpha \tag{5.75}$$
$$B^2_{pq} + \alpha \tag{5.76}$$
$$B^3_{pq} + \alpha \tag{5.77}$$
$$B^1_{pq} + z_1 + z_2 + \alpha \tag{5.78}$$
$$B^2_{pq} + z_1 + z_2 + \alpha \tag{5.79}$$
$$B^3_{pq} + z_1 + z_2 + \alpha \tag{5.80}$$
$$B^1_{pq} + \tilde{S} + \tilde{x} + z_1 + \alpha \tag{5.81}$$
$$B^2_{pq} + \tilde{S} + \tilde{x} + z_1 + \alpha \tag{5.82}$$
$$B^3_{pq} + \tilde{S} + \tilde{x} + z_1 + \alpha \tag{5.83}$$
$$B^1_{pq} + \tilde{S} + \tilde{x} + z_1 + z_2 + \alpha \tag{5.84}$$
$$B^2_{pq} + \tilde{S} + \tilde{x} + z_1 + z_2 + \alpha \tag{5.85}$$
$$B^3_{pq} + \tilde{S} + \tilde{x} + z_1 + z_2 + \alpha \tag{5.86}$$
$$\tag{5.87}$$

and then 4 additional hidden fermionic combinations:

$$\tilde{S} + \alpha \tag{5.88}$$
$$\tilde{S} + \alpha + z_2 \tag{5.89}$$
$$\tilde{x} + \alpha \tag{5.90}$$
$$\tilde{x} + z_1 + \alpha \tag{5.91}$$

5.3.2 Vectorials

There are a total of 21 vectorial massless sectors which we will now group..

Observable

There are 12 bosonic observable **10**'s of $SO(10)$ from the combinations:

$$V_{pq}^1 = B_{pq}^1 + \tilde{x} \tag{5.92}$$
$$V_{pq}^2 = B_{pq}^2 + \tilde{x} \tag{5.93}$$
$$V_{pq}^3 = B_{pq}^3 + \tilde{x}. \tag{5.94}$$
$$\tag{5.95}$$

We can write projectors for our observable **10**'s by considering two cases which we are interested in. These are bi-doublets (oscillator is $\bar{\psi}^{4,5}$) and triplets (oscillator is $\bar{\psi}^{1,2,3}$).The projectors for the bi-doublet case:

$$R_{pq}^1 = \left[1 + C\begin{pmatrix} V_{pq}^1 \\ T_1 \end{pmatrix}\right]\left[1 - C\begin{pmatrix} V_{pq}^1 \\ \alpha \end{pmatrix}\right]\prod_{i=1}^{2}\left[1 + C\begin{pmatrix} V_{pq}^1 \\ z_i \end{pmatrix}\right] \tag{5.96}$$
$$R_{pq}^2 = \left[1 + C\begin{pmatrix} V_{pq}^2 \\ T_2 \end{pmatrix}\right]\left[1 - C\begin{pmatrix} V_{pq}^2 \\ \alpha \end{pmatrix}\right]\prod_{i=1}^{2}\left[1 + C\begin{pmatrix} V_{pq}^2 \\ z_i \end{pmatrix}\right] \tag{5.97}$$
$$R_{pq}^3 = \left[1 + C\begin{pmatrix} V_{pq}^3 \\ T_3 \end{pmatrix}\right]\left[1 - C\begin{pmatrix} V_{pq}^3 \\ \alpha \end{pmatrix}\right]\prod_{i=1}^{2}\left[1 + C\begin{pmatrix} V_{pq}^3 \\ z_i \end{pmatrix}\right] \tag{5.98}$$

This allows us to count the number of bi-doublets. We need to once again remember the factor 4 which comes from degeneracy:

$$N_{10}^{\bar{\psi}^{4,5}} = 4\sum_{A=1,2,3} R_{pq}^A \tag{5.99}$$

here the sum over p, q implicit. We can also construct projectors for the triplets, which just differ in sign for the phase with α, just to differentiate we will denote these with a tilde:

$$\tilde{R}_{pq}^1 = \left[1 - C\begin{pmatrix} V_{pq}^1 \\ T_1 \end{pmatrix}\right]\left[1 + C\begin{pmatrix} V_{pq}^1 \\ \alpha \end{pmatrix}\right]\prod_{i=1}^{2}\left[1 - C\begin{pmatrix} V_{pq}^1 \\ z_i \end{pmatrix}\right] \tag{5.100}$$
$$\tilde{R}_{pq}^2 = \left[1 - C\begin{pmatrix} V_{pq}^2 \\ T_2 \end{pmatrix}\right]\left[1 + C\begin{pmatrix} V_{pq}^2 \\ \alpha \end{pmatrix}\right]\prod_{i=1}^{2}\left[1 - C\begin{pmatrix} V_{pq}^2 \\ z_i \end{pmatrix}\right] \tag{5.101}$$
$$\tilde{R}_{pq}^3 = \left[1 - C\begin{pmatrix} V_{pq}^3 \\ T_3 \end{pmatrix}\right]\left[1 + C\begin{pmatrix} V_{pq}^3 \\ \alpha \end{pmatrix}\right]\prod_{i=1}^{2}\left[1 - C\begin{pmatrix} V_{pq}^3 \\ z_i \end{pmatrix}\right] \tag{5.102}$$

which allows us to count the number of triplets:

$$N_{10}^{\bar{\psi}^{1,2,3}} = 4 \sum_{A=1,2,3} \tilde{R}^A \tag{5.103}$$

Scalar

There are $\binom{3}{2} = 3$ scalars. These are sectors with no observable or hidden groups. We would have to be careful if we were planning to proceed with classification here, as these combinations could contribute to the vectorial bi-doublets (if oscillator $\bar{\psi}^{4,5}$) or triplets (if oscillator $\bar{\psi}^{1,2,3}$). Hence, in theory should count their contributions. In this paper however, we are excluding the contributions from these. If we wanted we could set up projectors for these states and ensure they are projected. These scalars come from the combinations:

$$T_i + T_j \tag{5.104}$$

where $i \neq j$.

Exotic

The last 6 of our vectorial sectors are all exotic, at least without any oscillators. For classification, we would have to be careful again with what oscillators could be present, $\psi^{1,2,\bar{3},4,5}$ oscillators could affect the observable group and careful consideration would need to taken to ensure their projection. Likewise, these combinations could become purely hidden. We will not be considering these in our analysis. We have the following combinations:

$$\tilde{S} \tag{5.105}$$
$$\tilde{S} + z_1 \tag{5.106}$$
$$\tilde{S} + z_1 + \alpha \tag{5.107}$$
$$\tilde{S} + z_2 \tag{5.108}$$
$$\tilde{S} + z_1 + z_2 \tag{5.109}$$
$$\tilde{S} + z_1 + \alpha + z_2. \tag{5.110}$$

5.3.3 Enhancements

Extra gauge bosons may arise from the originally $(0,8)$ combinations. We will want these projected from our specific model when we come to our analysis. We can summarise these with their corresponding projectors in the table below:

Enhancement	**Projecting vectors**
$\psi^\mu\vert z_2+z_1\rangle$	$T_1, T_2, T_3, b_1, b_2, b_3, \tilde{x}$
$\psi^\mu\vert z_2+\alpha\rangle$	T_1, T_2, T_3
$\psi^\mu\vert z_2+z_1+\alpha\rangle$	T_1, T_2, T_3
$\psi^\mu\vert\tilde{S}+\tilde{x}+z_1+\alpha\rangle$	$T_1, T_2, T_3, z_1, \alpha$
$\psi^\mu\vert 1+\tilde{S}+T_1+T_2+T_3+\alpha\rangle$	$\tilde{S}, T_1, T_2, T_3, z_1, \alpha$

We need to be careful here and realise that if one of our projecting vectors contains ψ^μ, we will get a flip of sign in the GGSO projection, hence to project we will instead need a positive phase with the sector. We should also consider that we could also get extra enhancements from the originally tachyonic states (0,4) by both a left-moving ψ^μ and also a choice of right-moving oscillators:

$$\psi^\mu\{\bar{\lambda}^i\}|z_1\rangle \tag{5.111}$$
$$\psi^\mu\{\bar{\lambda}^i\}|\alpha\rangle \tag{5.112}$$
$$\psi^\mu\{\bar{\lambda}^i\}|z_1+\alpha\rangle \tag{5.113}$$
$$\psi^\mu\{\bar{\lambda}^i\}|z_2\rangle \tag{5.114}$$
$$\tag{5.115}$$

To ensure these enhancements are projected is little more complicated due the range of choice of oscillators. We will deal with them the same way we dealt with our tachyonic sectors with oscillators by tabulating them:

Oscillator	$C\binom{z_1}{T_1}$	$C\binom{z_1}{T_2}$	$C\binom{z_1}{T_3}$	$C\binom{z_1}{b_1}$	$C\binom{z_1}{b_2}$	$C\binom{z_1}{b_3}$	$C\binom{z_1}{\tilde{x}}$	$C\binom{z_1}{z_2}$
$\bar{y}^{1,2}$	+	-	-	+	-	-	+	-
$\bar{y}^{3,4}$	-	+	-	-	+	-	+	-
$\bar{y}^{5,6}$	-	-	+	-	-	+	+	-
$\bar{\omega}^{1,2}$	+	-	-	+	+	+	+	-
$\bar{\omega}^{3,4}$	-	+	-	+	+	+	+	-
$\bar{\omega}^{5,6}$	-	-	+	+	+	+	+	-
$\bar{\psi}^{1/2/3/4/5}$	-	-	-	-	-	-	-	-
$\bar{\eta}^{1}$	-	-	-	-	+	+	-	-
$\bar{\eta}^{2}$	-	-	-	+	-	+	-	-
$\bar{\eta}^{3}$	-	-	-	+	+	-	-	-
$\bar{\phi}^{5/6/7/8}$	-	-	-	+	+	+	+	+

Oscillator	$C\binom{\alpha}{\tilde{S}}$	$C\binom{\alpha}{T_1}$	$C\binom{\alpha}{T_2}$	$C\binom{\alpha}{T_3}$	$C\binom{\alpha}{z_2}$
$\bar{y}^{1,2}$	+	+	-	-	-
$\bar{y}^{3,4}$	+	-	+	-	-
$\bar{y}^{5,6}$	+	-	-	+	-
$\bar{\omega}^{1,2}$	+	+	-	-	-
$\bar{\omega}^{3,4}$	+	-	+	-	-
$\bar{\omega}^{5,6}$	+	-	-	+	-
$\bar{\psi}^{1/2/3}$	+	-	-	-	-
$\bar{\eta}^{1/2/3}$	+	-	-	-	-
$\bar{\phi}^{3/4}$	-	-	-	-	-
$\bar{\phi}^{5/6}$	-	-	-	-	+
$\bar{\phi}^{7/8}$	+	-	-	-	+

Oscillator	$C\binom{z_1+\alpha}{T_1}$	$C\binom{z_1+\alpha}{T_2}$	$C\binom{z_1+\alpha}{T_3}$	$C\binom{z_1+\alpha}{z_2}$
$\bar{y}^{1,2}/\bar{\omega}^{1,2}$	+	-	-	-
$\bar{y}^{3,4}/\bar{\omega}^{3,4}$	-	+	-	-
$\bar{y}^{5,6}/\bar{\omega}^{5,6}$	-	-	+	-
$\bar{\psi}^{1/2/3}$	-	-	-	-
$\bar{\eta}^{1/2/3}$	-	-	-	-
$\bar{\phi}^{1/2}$	-	-	-	-
$\bar{\phi}^{5/6/7/8}$	-	-	-	+

Oscillator	$C\binom{z_2}{T_1}$	$C\binom{z_2}{T_2}$	$C\binom{z_2}{T_3}$	$C\binom{z_2}{b_1}$	$C\binom{z_2}{b_2}$	$C\binom{z_2}{b_3}$	$C\binom{z_2}{\tilde{x}}$	$C\binom{z_2}{z_1}$
$\bar{y}^{1,2}$	+	-	-	+	-	-	+	-
$\bar{y}^{3,4}$	-	+	-	-	+	-	+	-
$\bar{y}^{5,6}$	-	-	+	-	-	+	+	-
$\bar{\omega}^{1,2}$	+	-	-	+	+	+	+	-
$\bar{\omega}^{3,4}$	-	+	-	+	+	+	+	-
$\bar{\omega}^{5,6}$	-	-	+	+	+	+	+	-
$\bar{\psi}^{1/2/3/4/5}$	-	-	-	-	-	-	-	-
$\bar{\eta}^{1}$	-	-	-	-	+	+	-	-
$\bar{\eta}^{2}$	-	-	-	+	-	+	-	-
$\bar{\eta}^{3}$	-	-	-	+	+	-	-	-
$\bar{\phi}^{1/2/3/4}$	-	-	-	+	+	+	+	+

Chapter 6

Analysis of a specific non-supersymmetric model

Here, we have ran a computer code to generate a possible model of the GGSO phase coefficients:

$$
C\binom{v_i}{v_j} = \begin{array}{c c}
 & \begin{array}{cccccccccc} \mathbf{1} & \tilde{S} & T_1 & T_2 & T_3 & b_1 & b_2 & b_3 & z_1 & \alpha \end{array} \\
\begin{array}{c} \mathbf{1} \\ \tilde{S} \\ T_1 \\ T_2 \\ T_3 \\ b_1 \\ b_2 \\ b_3 \\ z_1 \\ \alpha \end{array} &
\left(\begin{array}{cccccccccc}
-1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
-1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 \\
-1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 \\
1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1
\end{array}\right)
\end{array} \tag{6.1}
$$

We are going to check this model for the following:

1. Modular Invariance
2. Is the model tachyon free
3. What spinorial 16's or $\bar{16}$'s sectors are present and what are they under the Pati-Salam $(N_L, \bar{N}_L, N_R, \bar{N}_R)$?
4. How many generations are there?

5. What sector(s) give a vectorial bidoublet? i.e. $\{\bar{\psi}^{4,5}\}$ as the oscillator?
6. Are there any triplet vectorials? $\{\psi^{1,2,3}\}$ oscillators
7. Are the enhancements projected?

6.0.1 Modular Invariance

We can check modular invariance by checking our GGSO phases against our ABK rules for our phases , which are themselves derived by imposing modular invariance. The two rules we will be using correspond to the diagonal 4.28 and off-diagonal 4.29 elements of the GGSO matrix:

$$C\binom{b_i}{b_i} = -e^{\frac{i\pi}{4}b_i\cdot b_i}C\binom{b_i}{\mathbb{1}} \tag{6.2}$$

$$C\binom{b_i}{b_j} = e^{\frac{i\pi}{2}b_i\cdot b_j}C\binom{b_j}{b_i} \tag{6.3}$$

Here we will show the calculation for one diagonal element and one non-diagonal element. Starting with a diagonal element, we will test the following GGSO phase:

$$C\binom{T_1}{T_1} \tag{6.4}$$

It must satisfy the following:

$$C\binom{T_1}{T_1} = -e^{\frac{i\pi}{4}T_1\cdot T_1}C\binom{T_1}{\mathbb{1}} \tag{6.5}$$

from the choice of GGSO phase 6.1 we see that:

$$C\binom{T_1}{T_1} = -1 \quad \text{and} \quad C\binom{T_1}{\mathbb{1}} = +1. \tag{6.6}$$

Calculating the scalar product:

$$T_1 \cdot T_1 = 0 \implies e^{\frac{i\pi}{4}T_1\cdot T_1} = 0, \tag{6.7}$$

hence we can see this condition is satisfied. Repeating this for all our diagonal elements we see all of the diagonal satisfies modular invariance. The off-diagonal uses the second of the two equations presented, we will check the following phase:

$$C\binom{b_2}{\alpha}. \tag{6.8}$$

The phase must satisfy the following:

$$C\binom{b_2}{\alpha} = e^{\frac{i\pi}{2}b_2\cdot\alpha}C\binom{\alpha}{b_2}. \tag{6.9}$$

From our model 6.1 we see that:

$$C\binom{b_2}{\alpha} = -1 \quad \text{and} \quad C\binom{\alpha}{b_2} = +1. \tag{6.10}$$

The scalar product of the basis vectors:

$$b_2 \cdot \alpha = -2 \implies e^{\frac{ipi}{2}b_2\cdot\alpha} = -1 \tag{6.11}$$

and hence our condition is satisfied. Upon checking all the upper triangle, we see that our model satisfies modular invariance.

6.0.2 Tachyons

We can check if this model has any tachyons by inserting our GGSO phases of our model to our algebraic tachyon conditions contained in the appendix A. This is easy done by running a quick Python script to check. Here, we will be checking our model by hand rather than running a script by checking the conditions for each possible tachyon.

- $\{\bar{\lambda}\}|NS\rangle$ tachyon - For this, we could input our GGSO matrix into a code to check against the condition to check every sector, which can be seen in our appendix. However a single model like ours it's faster to check against the table 5.2.1 for the required conditions for projection for each oscillator. We can use a trick to massively simplify this. We can see in our table 5.2.1 that a negative phase with $\tilde{S}$ will project all NS tachyons except $\bar{\phi}^{3,4,5,6}$.

$$C\binom{NS}{\tilde{S}} = C\binom{\mathbb{1}+\mathbb{1}}{\tilde{S}} = C\binom{\tilde{S}}{\mathbb{1}+\mathbb{1}} \tag{6.12}$$

$$= \delta_{\tilde{S}}C\binom{\tilde{S}}{\mathbb{1}}C\binom{\tilde{S}}{\mathbb{1}} \tag{6.13}$$

  As $\delta_{\tilde{S}} = -1$ this means all these tachyonic states are projected. We are left to deal with the $\bar{\phi}^{3,4,5,6}$ oscillators and these are projected by any of the b_i phases.

- z_1 tachyon projected as $C\binom{z_1}{T_2} = -1$

- α tachyon is projected as $C\binom{\alpha}{\tilde{S}} = -1$
- $z_1 + \alpha$ tachyon projected because $C\binom{T_1}{z_1} = -C\binom{T_1}{\alpha}$
- z_2 tachyon: using the 'by hand' argument presented in appendix A. we can scan across the T_1 row and see there is a odd number of negatives, hence the z_2 tachyon is projected by T_1. This is effectively $C\binom{z_2}{T_1} = -1$.
- $\{\bar{\lambda}^i\}T_1$ tachyon: once again it's easier to scan across the the table for a single model like ours:
  - All oscillators except $\bar{\phi}^{3,4,5,6}$ are projected by $C\binom{T_1}{\tilde{S}} = -1$.
  - $\bar{\phi}^{3,4,5,6}$ projected by $C\binom{T_1}{T_3} = -1$
- $\{\bar{\lambda}^i\}T_2$ tachyon:
  - All oscillators except $\bar{\phi}^{3,4,5,6}$ are projected by $C\binom{T_2}{\tilde{S}} = -1$.
  - $\bar{\phi}^{3,4,5,6}$ projected by $C\binom{T_2}{T_3} = -1$
- $\{\bar{\lambda}^i\}T_3$ tachyon:
  - All oscillators except $\bar{\phi}^{3,4,5,6}$ are projected by $C\binom{T_3}{\tilde{S}} = -1$.
  - $\bar{\phi}^{3,4,5,6}$ projected by $C\binom{T_3}{T_1} = -1$
- $\{\bar{\lambda}^i\}T_2$ and $\{\bar{\lambda}^i\}T_3$ can use the two same GGSO phases to project all their possible tachyons. This is easily seen from their tables.
- $T_i + z_1$
  - $T_1 + z_1$ tachyon is projected by $C\binom{z_2}{T_1} = -C\binom{z_2}{z_1}$.
  - $T_2 + z_1$ tachyon is projected by $C\binom{\tilde{x}}{T_1} = -C\binom{\tilde{x}}{z_1}$.
  - $T_3 + z_1$ tachyon is projected by $C\binom{T_1}{T_3} = -C\binom{T_1}{z_1}$.
- $T_i + \alpha$
  - $T_1 + \alpha$ tachyon is projected by $C\binom{T_2}{T_1} = -C\binom{T_2}{\alpha}$.
  - $T_2 + \alpha$ tachyon is projected by $C\binom{T_1}{T_2} = -C\binom{T_1}{\alpha}$.
  - $T_3 + \alpha$ tachyon is projected by $C\binom{z_2}{T_3} = -C\binom{z_2}{\alpha}$.
- $T_i + z_1 + \alpha$ easy to look across our matrix and if a projector has odd number of negatives then will project the tachyon

- $T_1 + z_1 + \alpha$ tachyon is projected by $C\binom{T_2}{T_1}C\binom{T_2}{z_1}C\binom{T_2}{\alpha} = -1$
- $T_2 + z_1 + \alpha$ tachyon is projected by $C\binom{T_1}{T_1}C\binom{T_1}{z_1}C\binom{T_1}{\alpha} = -1$
- $T_1 + z_1 + \alpha$ tachyon is projected by $C\binom{z_2}{T_1}C\binom{z_2}{z_1}C\binom{z_2}{\alpha} = -1$

- $T_i + z_2$ we will use the by hand form which originally looks a lot less convenient, but really in this form we can just check across a row of the matrix and count the negatives of the sector phases.
  - $T_1 + z_2$ tachyon is projected by $C\binom{\alpha}{T_1} = -C\binom{\alpha}{z_2}$.
  - $T_2 + z_2$ tachyon is projected by $C\binom{T_1}{T_2} = -C\binom{T_1}{z_2}$.
  - $T_3 + z_2$ tachyon is projected by $C\binom{T_2}{T_3} = -C\binom{T_2}{z_2}$.

Hence, we can conclude that our model is tachyon free. We will now look at the observable sector of our model.

6.0.3 Observable Spinorials and Anti-Spinorials

For this, we can use our counting and chirality equations for each sector. We can see that the $\tilde{x}$ sector is projected out and does not contribute to the $\mathbf{16}/\overline{\mathbf{16}}$'s due to projection by the T_2 phase. Hence, we only need to count the contributions from the $B^{1,2,3}$ sectors. We will show this for the B^1 sectors and then just state the results for $B^{2,3}$ sectors. We first need to see which of the B^1 sectors survive the projection. We can neatly find conditions of survival on p and q by splitting our projectors:

$$T_1: \quad C\begin{pmatrix}T_1\\b_1\end{pmatrix}C\begin{pmatrix}T_1\\T_2\end{pmatrix}^p C\begin{pmatrix}T_1\\T_3\end{pmatrix}^q = -1(+1)^p(-1)^q \tag{6.14}$$

$$z_1: \quad C\begin{pmatrix}z_1\\b_1\end{pmatrix}C\begin{pmatrix}z_1\\T_2\end{pmatrix}^p C\begin{pmatrix}z_1\\T_3\end{pmatrix}^q = -1(+1)^p(-1)^q \tag{6.15}$$

$$z_2: \quad C\begin{pmatrix}z_2\\b_1\end{pmatrix}C\begin{pmatrix}z_2\\T_2\end{pmatrix}^p C\begin{pmatrix}z_2\\T_3\end{pmatrix}^q = -1(+1)^p(+1)^q. \tag{6.16}$$

Solving these, we actually get two solutions which will survive:

$$p = q = 0 \quad \text{and} \quad p = 1, q = 0 \tag{6.17}$$

which correspond to the sectors $B^1_{00} = b_1$ and $B^1_{10} = b_1 + T_2$ respectively. We now must determine the chirality of each sector independently, for b_1

$$\chi^1_{00} = -C\begin{pmatrix}b_1\\b_2+T_3\end{pmatrix} = -\delta_{b_1}C\begin{pmatrix}b_1\\b_2\end{pmatrix}C\begin{pmatrix}b_1\\T_3\end{pmatrix} = -(-1)(-1)(-1) = +1 \tag{6.18}$$

hence the sector b_1 corresponds to a **16**. On the other hand for our $b_1 + T_2$ sector we find:

$$\chi^1_{10} = -1 \leftrightarrow \mathbf{\bar{16}}. \tag{6.19}$$

Here, we have excluded the factor 4 for convenience in the calculation. We will reintroduce this back at the end but it is important to remember what looks like a single $\mathbf{16}/\mathbf{\bar{16}}$ is actually four due to the degeneracy mentioned previously. Now we have to determine the decomposition under the Pati-Salam group. Using our condition 5.34, we find:

$$b_1 \leftrightarrow N_L \tag{6.20}$$

$$b_1 + T_2 \leftrightarrow N_R. \tag{6.21}$$

Repeating this for the B^2 and B^3 sectors, we find two more surviving states and their corresponding decomposition's are:

$$B^2_{00} = b_2 \leftrightarrow \bar{N}_R \tag{6.22}$$

$$B^3_{11} = b_3 + T_1 + T_2 \leftrightarrow \bar{N}_R. \tag{6.23}$$

We can then summarise our spinorials as:

$$[N_L, \bar{N}_L, N_R, \bar{N}_R] = [1, 0, 1, 2] \tag{6.24}$$

and this allows us to calculate the number of generations. Remembering to account for the degeneracy factor of 4, as follows:

$$n_g = N_L - \bar{N}_L = \bar{N}_R - N_R = 4. \tag{6.25}$$

We see that this is actually a consequence of the grouping of our e_i's into the T_i's. This has restricted our ability to reproduce a three generation model. This is due to the y's and ω's being the six internal directions of the orbifold. Our twisted sectors, which give rise to our spinorials, correspond to fix points on the orbifold and we can only generate three generation models when we interpret the manifold as a lattice with:

$$\Gamma_{6,6} = \Gamma^3_{2,2} \rightarrow \Gamma^6_{1,1}. \tag{6.26}$$

Our T_i's are symmetric shifts and refer to complex Calabi-Yau manifolds $\Gamma_{6,6} = \Gamma^3_{2,2}$. This prevents us from creating three generation models, further information can be found in the paper [24]. Instead, we would have to use the non-grouped e_i's as basis vectors. These are not complex manifolds, but instead describe the internal real circles of the six internal directions of the orbifold.

6.0.4 Observable Vectorials

To analyse our vectorials, we take a similar approach to the spinorials and see for what p and q states will survive. Upon doing this, we find that the only surviving observable vectorial sector is V^3_{00}, i.e V^3 with $p = q = 0$:

$$V^3 = b_1 + b_2 \tag{6.27}$$

To see whether this sector gives us a bi-doublet or triplet, we need to check the GGSO phase of the sector with α:

$$C\begin{pmatrix} b_1 + b_2 \\ \alpha \end{pmatrix} = e^{\frac{i\pi}{2}(b_1+b_2)\cdot F_\alpha} C\begin{pmatrix} \alpha \\ b_1 + b_2 \end{pmatrix} \tag{6.28}$$

$$= \delta_\alpha C\begin{pmatrix} \alpha \\ b_1 \end{pmatrix} C\begin{pmatrix} \alpha \\ b_2 \end{pmatrix} = -1. \tag{6.29}$$

Therefore, we can see this sector will actually give us a bi-doublet rather than a triplet. In fact, like our spinorial sectors, we need to take into consideration the degeneracy of the states and we will actually get four bi-doublets in this model.

6.0.5 Enhancements

To test whether our model has any enhancements, we can check our GGSO phases against their projection conditions. We want a model which has no observable group enhancements, as this would affect our observable gauge group. Any hidden enhancements for the purpose of this analysis can be ignored.

- $\psi^\mu |z_1 + z_2\rangle$
  - Projected by the phase with T_1
- $\psi^\mu |z_2 + \alpha\rangle$
  - Projected by the phase with T_2
- $\psi^\mu |z_1 + z_2 + \alpha\rangle$
  - Projected by the phase with T_2
- $\psi^\mu |\tilde{S} + \tilde{x} + z_1 + \alpha\rangle$
  - Projected by the phase with T_1

- $\psi^\mu|1+\tilde{S}+T_1+T_2+T_3+\alpha\rangle$
  - Projected by the phase with T_3

The next set of enhancements can have oscillators and each oscillator case must be determined individually.

- $\psi^\mu\{\bar{\lambda}^i\}|z_1\rangle$
  - All enhancements excluding $\bar{y}^{5,6}/\bar{\omega}^{5,6}$ are projected by $C\binom{z_1}{T_3} = -1$
  - The case with oscillator $\bar{y}^{5,6}$ can be projected with $C\binom{z_1}{b_1} = -1$
  - The remaining enhancement with a $\bar{\omega}^{5,6}$ can't be removed giving us a hidden enhancement.
- $\psi^\mu\{\bar{\lambda}^i\}|\alpha\rangle$
  - All enhancements excluding $\bar{y}^{1,2}/\bar{\omega}^{1,1}$ are projected by $C\binom{\alpha}{T_1} = -1$
  - The remaining two enhancements are projected by $C\binom{\alpha}{T_2} = -1$
- $\psi^\mu\{\bar{\lambda}^i\}|z_1+\alpha\rangle$
  - All enhancements excluding $\bar{y}^{1,2}/\bar{\omega}^{1,1}$ are projected by $C\binom{z_1+\alpha}{T_1} = -1$
  - The remaining two enhancements are projected by $C\binom{z_1+\alpha}{T_2} = -1$
- $\psi^\mu\{\bar{\lambda}^i\}|z_2\rangle$
  - All enhancements excluding those with oscillators $\bar{y}^{1,2}/\bar{\omega}^{1,2}$ are projected by $C\binom{z_2}{T_1} = -1$
  - The remaining enhancements are projected by $C\binom{z_2}{T_3} = -1$

In our model, we are left with no observable enhancements, which is important not to change the overall observable gauge group. We only actually have a single enhancement with comes from the z_1 sector with the oscillator $\bar{\omega}^{5,6}$. This is a hidden group enhancement and the consequences of this are left out of this discussion.

Chapter 7

Conclusion

In this report, we have started from the beginning to try and motivate our work on a non-supersymmetric model in the free fermionic formulation. We started with recap of the highly successful and established Standard Model and highlighted some of it's key features which we aimed to be able to replicate. We then highlighted some of it's main flaws and current phenomena, which it currently cannot explain, the main issue being the non inclusion of gravity within it's QFT framework.

We then progressed to look at some theories which go beyond the Standard Model, namely grand unified theories, and how these embed the Standard Model gauge group in higher rank simple and semi-simple groups. These theories had a drawback of additional exotic gauge bosons in comparison to the Standard Model which would facilitate rapid proton decay. The lifetimes suggested by these groups in comparison to experimental limits ruled them out as viable candidates. Whilst these tried to unify the strong, weak and electromagnetic forces we were still missing gravity for total unification.
The bosonic and superstring were quickly derived using common quantisation techniques. We motivated the introduction of the GSO projection, which allowed to to solve the problems of the tachyonic states and level matching (required if we wanted spacetime supersymmetry) of both strings. We saw that the string introduced the graviton naturally into the spectrum but whilst the type IIA/B superstring was successful at allowing the introduction of fermions to our worldsheet, we weren't afforded much flexibility. Instead, we were able to take advantage of the decoupling of the left- and right-moving sectors of the closed string to form the heterotic string, focused on the free fermionic formulation.

The rest of the report focused on the construction of strings in the free fermionic formulation and we we're able to present a set of basis vectors which allowed the creation of non-supersymmetric Pati-Salam string models. We saw that the non-inclusion of usual SUSY basis vector meant that tachyonic states were not automatically projected, so only certain models with GGSO phases to ensure projection of these tachyons could be considered.

Finally, we were able to analyse a specific model, this model was generated by a computer code designed by Ben Percival. We saw that the choice of grouping the usual $e_i, i = 1, ..., 6$ into $T_{1,2,3}$ constricted the moduli space and actually prevented us to be able to present a three generation model, instead our model presented was four generational. Our choice of phases actually gave us a single bi-doublet vectorial sector, which corresponds to four bi-doublet states. All possible observable enhancements were projected out of the spectrum. However, we were left with a single hidden group enhancement, as this didn't affect our observable gauge group, we were happy to ignore this.

Further work is currently in progress by both my supervisors during this project, Alon Faraggi and Ben Percival, and also with another colleague Viktor Matyas. This looks into the partition function structure and classification of non-supersymmetric SO(10) models. In this work the e_i basis vectors are chosen, with interest in calculating the constant part of the partition function. The constant part of the partition function is linked to the cosmological constant and this task in general, for all moduli spaces is very difficult. This is due to non-level matched states also having contribution to the partition function, meaning it is beyond current technologies.

Appendices

Appendix A

Algebraic Tachyon Projectors

This are designed algebraically so that they can be inserted into a computer code which will check the existence of tachyons. We are only interested if the following formulas return 0. If they don't, then the model contains a tachyon in the stated sector. If they return something other than zero, then the number they return is the number of tachyons in that sector. For our analysis this won't be of interest, as we are only interested in finding tachyon free models.

A.1 $(-\frac{1}{2}, -\frac{1}{2})$ tachyon projections

NS tachyon projection equation

We start with the most complex which is the NS tachyon:

$$
\begin{aligned}
W^{NS} = \frac{1}{2^{12}} & \left(1 - C\binom{NS}{\mathbb{1}}\right)\left(1 + C\binom{NS}{\tilde{S}}\right)\left(1 - C\binom{NS}{T_1}\right) \\
& \left(1 + C\binom{NS}{T_2}\right)\left(1 + C\binom{NS}{T_3}\right)\left(1 + C\binom{NS}{b_1}\right) \\
& \left(1 - C\binom{NS}{b_2}\right)\left(1 - C\binom{NS}{b_3}\right)\left(1 + C\binom{NS}{z_1}\right) \\
& \left(1 + C\binom{NS}{\alpha}\right)\left(1 + C\binom{NS}{z_2}\right)\left(1 + C\binom{NS}{\tilde{x}}\right) \\
+ & \left(1 - C\binom{NS}{\mathbb{1}}\right)\left(1 + C\binom{NS}{\tilde{S}}\right)\left(1 + C\binom{NS}{T_1}\right) \\
& \left(1 - C\binom{NS}{T_2}\right)\left(1 + C\binom{NS}{T_3}\right)\left(1 - C\binom{NS}{b_1}\right) \\
& \left(1 + C\binom{NS}{b_2}\right)\left(1 - C\binom{NS}{b_3}\right)\left(1 + C\binom{NS}{z_1}\right) \\
& \left(1 + C\binom{NS}{\alpha}\right)\left(1 + C\binom{NS}{z_2}\right)\left(1 + C\binom{NS}{\tilde{x}}\right) \\
+ & \left(1 - C\binom{NS}{\mathbb{1}}\right)\left(1 + C\binom{NS}{\tilde{S}}\right)\left(1 + C\binom{NS}{T_1}\right) \\
& \left(1 + C\binom{NS}{T_2}\right)\left(1 - C\binom{NS}{T_3}\right)\left(1 - C\binom{NS}{b_1}\right) \\
& \left(1 - C\binom{NS}{b_2}\right)\left(1 + C\binom{NS}{b_3}\right)\left(1 + C\binom{NS}{z_1}\right) \\
& \left(1 + C\binom{NS}{\alpha}\right)\left(1 + C\binom{NS}{z_2}\right)\left(1 + C\binom{NS}{\tilde{x}}\right) \\
+ & \left(1 - C\binom{NS}{\mathbb{1}}\right)\left(1 + C\binom{NS}{\tilde{S}}\right)\left(1 - C\binom{NS}{T_1}\right) \\
& \left(1 + C\binom{NS}{T_2}\right)\left(1 + C\binom{NS}{T_3}\right)\left(1 + C\binom{NS}{b_1}\right) \\
& \left(1 + C\binom{NS}{b_2}\right)\left(1 + C\binom{NS}{b_3}\right)\left(1 + C\binom{NS}{z_1}\right) \\
& \left(1 + C\binom{NS}{\alpha}\right)\left(1 + C\binom{NS}{z_2}\right)\left(1 + C\binom{NS}{\tilde{x}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1+C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\quad\left(1-C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1+C\binom{NS}{b_1}\right)\\
&\quad\left(1+C\binom{NS}{b_2}\right)\left(1+C\binom{NS}{b_3}\right)\left(1+C\binom{NS}{z_1}\right)\\
&\quad\left(1+C\binom{NS}{\alpha}\right)\left(1+C\binom{NS}{z_2}\right)\left(1+C\binom{NS}{\tilde{x}}\right)\\
&+\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1+C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\quad\left(1+C\binom{NS}{T_2}\right)\left(1-C\binom{NS}{T_3}\right)\left(1+C\binom{NS}{b_1}\right)\\
&\quad\left(1+C\binom{NS}{b_2}\right)\left(1+C\binom{NS}{b_3}\right)\left(1+C\binom{NS}{z_1}\right)\\
&\quad\left(1+C\binom{NS}{\alpha}\right)\left(1+C\binom{NS}{z_2}\right)\left(1+C\binom{NS}{\tilde{x}}\right)\\
&+\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1+C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\quad\left(1+C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1-C\binom{NS}{b_1}\right)\\
&\quad\left(1-C\binom{NS}{b_2}\right)\left(1-C\binom{NS}{b_3}\right)\left(1+C\binom{NS}{z_1}\right)\\
&\quad\left(1+C\binom{NS}{\alpha}\right)\left(1+C\binom{NS}{z_2}\right)\left(1-C\binom{NS}{\tilde{x}}\right)\\
&+\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1+C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\quad\left(1+C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1-C\binom{NS}{b_1}\right)\\
&\quad\left(1-C\binom{NS}{b_2}\right)\left(1-C\binom{NS}{b_3}\right)\left(1+C\binom{NS}{z_1}\right)\\
&\quad\left(1-C\binom{NS}{\alpha}\right)\left(1+C\binom{NS}{z_2}\right)\left(1-C\binom{NS}{\tilde{x}}\right)\\
&+\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1+C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\quad\left(1+C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1-C\binom{NS}{b_1}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\left(1+C\binom{NS}{b_2}\right)\left(1+C\binom{NS}{b_3}\right)\left(1+C\binom{NS}{z_1}\right)\\
&\left(1+C\binom{NS}{\alpha}\right)\left(1+C\binom{NS}{z_2}\right)\left(1-C\binom{NS}{\tilde{x}}\right)\\
+&\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1+C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\left(1+C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1+C\binom{NS}{b_1}\right)\\
&\left(1-C\binom{NS}{b_2}\right)\left(1+C\binom{NS}{b_3}\right)\left(1+C\binom{NS}{z_1}\right)\\
&\left(1+C\binom{NS}{\alpha}\right)\left(1+C\binom{NS}{z_2}\right)\left(1-C\binom{NS}{\tilde{x}}\right)\\
+&\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1+C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\left(1+C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1+C\binom{NS}{b_1}\right)\\
&\left(1+C\binom{NS}{b_2}\right)\left(1-C\binom{NS}{b_3}\right)\left(1+C\binom{NS}{z_1}\right)\\
&\left(1+C\binom{NS}{\alpha}\right)\left(1+C\binom{NS}{z_2}\right)\left(1-C\binom{NS}{\tilde{x}}\right)\\
+&\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1+C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\left(1+C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1+C\binom{NS}{b_1}\right)\\
&\left(1+C\binom{NS}{b_2}\right)\left(1+C\binom{NS}{b_3}\right)\left(1-C\binom{NS}{z_1}\right)\\
&\left(1-C\binom{NS}{\alpha}\right)\left(1+C\binom{NS}{z_2}\right)\left(1+C\binom{NS}{\tilde{x}}\right)\\
+&\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1-C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\left(1+C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1+C\binom{NS}{b_1}\right)\\
&\left(1+C\binom{NS}{b_2}\right)\left(1+C\binom{NS}{b_3}\right)\left(1-C\binom{NS}{z_1}\right)\\
&\left(1+C\binom{NS}{\alpha}\right)\left(1+C\binom{NS}{z_2}\right)\left(1+C\binom{NS}{\tilde{x}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1-C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\left(1+C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1+C\binom{NS}{b_1}\right)\\
&\left(1+C\binom{NS}{b_2}\right)\left(1+C\binom{NS}{b_3}\right)\left(1+C\binom{NS}{z_1}\right)\\
&\left(1+C\binom{NS}{\alpha}\right)\left(1-C\binom{NS}{z_2}\right)\left(1+C\binom{NS}{\tilde{x}}\right)\\
&+\left(1-C\binom{NS}{\mathbb{1}}\right)\left(1+C\binom{NS}{\tilde{S}}\right)\left(1+C\binom{NS}{T_1}\right)\\
&\left(1+C\binom{NS}{T_2}\right)\left(1+C\binom{NS}{T_3}\right)\left(1+C\binom{NS}{b_1}\right)\\
&\left(1+C\binom{NS}{b_2}\right)\left(1+C\binom{NS}{b_3}\right)\left(1+C\binom{NS}{z_1}\right)\\
&\left(1+C\binom{NS}{\alpha}\right)\left(1-C\binom{NS}{z_2}\right)\left(1+C\binom{NS}{\tilde{x}}\right)
\end{aligned}
$$

z_1 tachyon projection equation

For the next four tachyonic sectors, the equations are much simpler as there is no oscillators to complicate things. We can see that all of $z_1, \alpha, z_1 + \alpha, z_2$ all do not contain spacetime fermions and hence we just require the phases with any of the listed projectors to equal -1 to project out the tachyonic state.

$$
W^{z_1} = \frac{1}{2^7}\left(1+C\binom{z_1}{z_2}\right)\prod_{i=1}^{3}\left[\left(1+C\binom{z_1}{T_i}\right)\left(1+C\binom{z_2}{b_i}\right)\right]
$$

α tachyon projection equation

$$
W^{\alpha} = \frac{1}{2^5}\left(1+C\binom{\alpha}{\tilde{S}}\right)\left(1+C\binom{\alpha}{z_2}\right)\prod_{i=1}^{3}\left[\left(1+C\binom{\alpha}{T_i}\right)\right]
$$

$z_1 + \alpha$ tachyon projection equation

For this one, we could proceed as normal or we could do a little rearrangement using our GGSO phase conditions to get our conditions in a more convenient form. Here, we are using Y as a generic projector for our state:

$$\begin{aligned} C\binom{z_1+\alpha}{Y} &= e^{\frac{i\pi}{2}(z_1+\alpha)\cdot Y} C\binom{Y}{z_1+\alpha} \\ &= e^{\frac{i\pi}{2}(z_1+\alpha)\cdot Y} \delta_{z_1+\alpha} C\binom{Y}{z_1} C\binom{Y}{\alpha}. \end{aligned}$$

Well we know our state has no spacetime fermions and we are taking dot product with a projector so:

$$C\binom{z_1+\alpha}{Y} = C\binom{Y}{z_1} C\binom{Y}{\alpha}.$$

This means for this state we just need to consider whether the GGSO phases of the projector with each of the basis vectors contained within our sector are of opposite sign or equivalently it can be written:

$$W^{z_1+\alpha} = \frac{1}{2^4}\left[C\binom{z_2}{z_1} + C\binom{z_2}{\alpha}\right] \prod_{i=1}^{3} \left[C\binom{T_i}{z_1} + C\binom{T_i}{\alpha}\right]$$

z_2 tachyon projection equation

In our GGSO phase matrix, we won't have the z_2 coefficient, we could produce a code to calculate this using the previous arguments giving us with a general projector Y:

$$C\binom{z_2}{Y} = C\binom{Y}{\mathbb{1}} C\binom{Y}{z_1} \prod_{i=1}^{3} C\binom{Y}{T_i} C\binom{Y}{b_i}. \tag{A.1}$$

To check this by hand, we could look across the row of a projector for our state and for these coefficients count the number of negatives. If this is odd then the tachyon will be projected. If we wanted to input this into our code, we can write this to check our coefficients for all our projectors as:

$$\begin{aligned} W^{z_2} = \frac{1}{2^8} &\left[1 + C\binom{z_1}{\mathbb{1}} C\binom{z_1}{z_1} \prod_{i=1}^{3} C\binom{z_1}{T_i} C\binom{z_1}{b_i}\right] \\ &\cdot \left[1 + C\binom{\alpha}{\mathbb{1}} C\binom{\alpha}{z_1} \prod_{i=1}^{3} C\binom{\alpha}{T_i} C\binom{\alpha}{b_i}\right] \\ \cdot \prod_{j=1}^{3} &\left[1 + C\binom{T_j}{\mathbb{1}} C\binom{T_j}{z_1} \prod_{i=1}^{3} C\binom{T_j}{T_i} C\binom{T_i}{b_i}\right] \\ \cdot \prod_{j=1}^{3} &\left[1 + C\binom{b_j}{\mathbb{1}} C\binom{b_j}{z_1} \prod_{i=1}^{3} C\binom{b_j}{T_i} C\binom{b_j}{b_i}\right] \end{aligned}$$

A.2 $(-\frac{1}{4}, -\frac{1}{4})$ Tachyon projections

$T_i + z_1$ tachyon projection equation

With similar arguments as the $z_1 + \alpha$ tachyon, we can write an equation for projection as:

$$W^{T_i+z_1} = \frac{1}{2^5}\left[C\binom{T_j}{T_i} + C\binom{T_j}{z_1}\right]\left[C\binom{T_k}{T_i} + C\binom{T_k}{z_1}\right]\left[C\binom{b_i}{T_i} + C\binom{b_i}{z_1}\right]$$
$$\left[C\binom{\tilde{x}}{T_i} + C\binom{\tilde{x}}{z_1}\right]\left[C\binom{z_2}{T_i} + C\binom{z_2}{z_1}\right]$$

where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$. This is easy to check by hand for a single model by checking that a projector has opposite phase with T_i and z_1.

$T_i + \alpha$ tachyon projection equation

Similar arguments for by hand and for construction of the equation:

$$W^{T_i+\alpha} = \frac{1}{2^4}\left[C\binom{\tilde{S}}{T_i} + C\binom{\tilde{S}}{\alpha}\right]\left[C\binom{T_j}{T_i} + C\binom{T_j}{\alpha}\right]\left[C\binom{T_k}{T_i} + C\binom{T_k}{\alpha}\right]$$
$$\left[C\binom{z_2}{T_i} + C\binom{z_2}{\alpha}\right]$$

where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$.

$T_i + z_1 + \alpha$ tachyon projection equation

Similar arguments for by hand and for construction of the equation just now need a factor $\frac{1}{3}$ for each projector and can't now use the opposite sign trick when checking by hand:

$$W^{T_i+z_1+\alpha} = \frac{1}{2^3}\left[1 + C\binom{T_j}{T_i}C\binom{T_j}{z_1}C\binom{T_j}{\alpha}\right]\left[1 + C\binom{T_k}{T_i}C\binom{T_k}{z_1}C\binom{T_k}{\alpha}\right]$$
$$\left[1 + C\binom{z_2}{T_i}C\binom{z_2}{z_1}C\binom{z_2}{\alpha}\right]$$

where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$. When checking this by hand, we can now use the the that there needs to be an odd number of minuses for the projector phases.

$T_i + z_2$ tachyon projection equation

$$W^{T_i+z_2} = \frac{1}{2^5}\left[C\binom{T_j}{T_i} + C\binom{T_j}{z_2}\right]\left[C\binom{T_k}{T_i} + C\binom{T_k}{z_2}\right]\left[C\binom{b_i}{T_i} + C\binom{b_i}{z_2}\right]$$
$$\left[C\binom{\tilde{x}}{T_i} + C\binom{\tilde{x}}{z_2}\right]\left[C\binom{z_1}{T_i} + C\binom{z_1}{z_2}\right]$$

where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$. Whilst this is a useful form for the computer code which will ensure to calculate the extra phase coefficients of the z_2 sectors which will speed up the processing, this is not a convenient form to check by hand. The way around this is to expand the z_2 sector into its components and express it as:

$$W^{T_i+z_2} = \frac{1}{2^5}\left[1 + C\binom{T_j}{\mathbb{1}}C\binom{T_j}{T_j}C\binom{T_j}{T_k}C\binom{T_j}{z_1}\prod_{a=1}^{3}C\binom{T_j}{b_a}\right] \tag{A.2}$$

$$\cdot\left[1 + C\binom{T_k}{\mathbb{1}}C\binom{T_k}{T_j}C\binom{T_k}{T_k}C\binom{T_k}{z_1}\prod_{a=1}^{3}C\binom{T_k}{b_a}\right] \tag{A.3}$$

$$\cdot\left[1 + C\binom{b_i}{\mathbb{1}}C\binom{b_i}{T_j}C\binom{b_i}{T_k}C\binom{b_i}{z_1}\prod_{a=1}^{3}C\binom{b_i}{b_a}\right] \tag{A.4}$$

$$\cdot\left[1 + C\binom{\tilde{x}}{\mathbb{1}}C\binom{\tilde{x}}{T_j}C\binom{\tilde{x}}{T_k}C\binom{\tilde{x}}{z_1}\prod_{a=1}^{3}C\binom{\tilde{x}}{b_a}\right] \tag{A.5}$$

$$\cdot\left[1 + C\binom{z_1}{\mathbb{1}}C\binom{`_1}{T_j}C\binom{z_1}{T_k}C\binom{z_1}{z_1}\prod_{a=1}^{3}C\binom{z_1}{b_a}\right] \tag{A.6}$$

$$\cdot\left[1 + C\binom{\alpha}{\mathbb{1}}C\binom{\alpha}{T_j}C\binom{\alpha}{T_k}C\binom{\alpha}{z_1}\prod_{a=1}^{3}C\binom{\alpha}{b_a}\right]. \tag{A.7}$$

This allow us to check across a row of a basis vector projector and once again count the number of negatives with each component of the sector.

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