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String Phenomenology: Deriving the Standard Model from String Theory

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Abstract

The methods and reasoning behind the composition of the Standard Model have been researched and are outlined. The Standard Model is presented as observed and to some extent these observations are explained using the well-fitting analogous mathematics of group theory. String Theory's foundations and its gauge invariance are briefly touched upon before a more in-depth later on. Finally, String Theory is shown to match the Standard Model's mathematics well, being able to break symmetries right down to the gauge group SU(3) × SU(2) × U(1), where the report begins.

Contents

1. Introduction

The majority of the observable universe can be described by what is known as the Standard Model of Physics. The Standard Model includes all known matter in the form of quarks and leptons. Further, it models interactions between this matter: the weak, strong and electromagnetic forces, as well as the Higgs interaction. An important feature of the Standard Model is that it works for all experimental observations. However, the Model is not perfect. For example, it is yet to accurately include gravity and, ergo, incorporate General Relativity.

The algebra behind the Standard Model, in particular that of Group Theory, lends itself analogously to that of String Theory. The aim is to resolve these two approaches of mathematics to unify String Theory with the Standard Model.

String Theory can go some way to resolve the problems thrown up by the Standard Model. Mainly in the hunt for a theorised “graviton” to quantise gravity in a unification with gauge theories. Since the mid-20th Century, String Theory has showed itself in numerous guises, until now when these are seemingly coming together under “M-theory”.

Before String Theory can be put to answering these big questions, it must however been shown to be compatible with what we already know. If it can explain what we already know and then go on to predict answers to the greatest questions of physics, then String Theory is the perfect candidate for our ultimate goal: a Grand Unified Theory (GUT). In the following pages, we will demonstrate how String Theory can be used to describe the elements of the universe we know to be true.

2. The Standard Model

[1] A key reference for this chapter is the lectures by Fuks & Rausch de Traubenberg, M. (2011).

2.1 Contents of the Standard Model

The Standard Model (currently) contains three key elements: fermions, spin 1 bosons and the Higgs boson. The key distinguishing feature between these elements is their signature spins of ½, 1 and 0, respectively.

✳ The fermionic section consists of quarks and leptons:

- There are varieties of quarks not only distinguishable by charge, but also by two other characteristics:
  - six "flavours": up (u), down (d), strange (s), charm (c), top (t) and bottom (b)
  - three "colours": red, green and blue.
- There are six leptons. These are the electron, muon and tau particles, each forming a doublet with a respective neutrino (i.e. the electron neutrino, the muon neutrino and the tau neutrino).

✳ The bosonic section has in it four particles, each corresponding to a known particle interaction:

- the photon, responsible for electromagnetic interactions,
- gluons, responsible for the strong nuclear interactions, which come in eight different "colours",
- the Z boson, responsible for weak nuclear interactions,
- the W boson, also responsible for weak nuclear interactions.

✳ The recently discovered Higgs boson is the quantisation of the Higgs field, which attributes mass to all of the other particles.

The Standard Model does not (currently) incorporate an element to describe gravity. The theorised particle for this, the graviton, is attributed spin 2. [2]

Gauge groups are those for which the Lagrangian remains invariant under local transformations. The interactions within the Standard Model are contained within the gauge group of three Lie groups,

$$SU(3)_C \times SU(2)_L \times U(1)_Y, \qquad [2.1]$$

the first corresponding to the strong, the second the weak and the third the weak hypercharge.

There is a division in the gauge groups used in the description of the Standard Model: Abelian and non-Abelian gauge groups. Abelian gauge groups are those for which the gauge transformations commute, i.e. the order of application does not affect the outcome. Non-Abelian groups are the opposite, they do not commute and the order in which transformations are applied may change the outcome. Here it will be shown how the differences between these types of groups lead to different outcomes found within the Standard Model. Eventually, it will become clear that there are more appropriate, higher rank groups that contain an outline of the Standard Model.

2.2 Abelian Gauge Group Transformations

The lectures by A. Singer (2002) are a key reference for this section [3].

Take the free Dirac Lagrangian density for an electron field, ψ^e. It is given by:

$$\mathcal{L} = \bar{\psi}^e(i\gamma^\mu\partial_\mu - m)\psi^e \qquad [2.2]$$

where γ^μ are Dirac matrices (see appendix) and ψ^e is the electron field, which is invariant under phase transformations of the field, e.g.

$$\psi^e \to e^{i\alpha}\psi^e \qquad \bar{\psi}^e \to e^{i\alpha}\bar{\psi}^e \qquad [2.3]$$

Hence the Lagrangian [2.2] is invariant under global transformations in the unitary group, rank 1, $U(1)$.

It is desirable to have these transformations invariant locally. Therefore, we introduce the dependence on the parameter x. The Lagrangian density [2.2] is not now invariant under the transformations of $U(1)$. Hence, we introduce a correction term in the form of a gauge, vector field, A_μ[1], via the term

$$-e\psi\gamma^\mu A_\mu\psi \qquad [2.4]$$

We introduce the covariant derivative:

$$D_\mu = \partial_\mu + ieA_\mu Q \qquad [2.5]$$

Adding [2.4] to the Lagrangian density and applying [2.5], it becomes

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \qquad [2.6]$$

To allow for this, assume

$$A_\mu \to A_\mu + i\partial_\mu\alpha \qquad [2.7]$$

The field strength tensor, $F_{\mu\nu}$, is defined as:

$$F_{\mu\nu} = [\partial_\mu A_\nu] - [\partial_\nu A_\mu] \qquad [2.8]$$

This term is invariant under [2.7] and hence can be added to [2.6], giving

$$\mathcal{L} = \frac{-1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \qquad [2.9]$$

---

1 The term A_μ can be interpreted as the photon field.

This is done in order to introduce a kinetic term for A_μ that remains invariant. Choosing the constant -1/4 matches the Lagrange equations to the Maxwell equations for electrodynamics, the first and simplest gauge theory. The above local gauge invariance for the electron field can be extended to other spin-½ particles and the photon field (or gauge boson field by extension) is required in order to maintain local gauge invariance.

2.3 Non-Abelian Gauge Group Transformations

Now to generalise to non-Abelian (i.e. non-commutative) internal symmetries. This follows the Yang-Mills theory [4]. Maintain the Lagrangian as being in the form of [2.2] but now ψ is summed over n fermion fields (i = 1,…,n),

$$\mathcal{L} = \bar{\psi}^i (i\gamma^\mu \partial_\mu - m)\psi_i \qquad [2.10]$$

However, now we necessitate the invariance under internal symmetry rotations:

$$\psi = U\psi \qquad\qquad \bar{\psi} = \bar{\psi}U^\dagger \qquad [2.11]$$

where U is an n × n matrix, with properties

$$\mathrm{UU}^\dagger = 1 \qquad\qquad \det[\mathrm{U}]=1 \qquad [2.12]$$

The matrices satisfying these conditions are Special Unitary matrices, SU(n), i.e.

$$U = e^{-i\sum_j \alpha_j(x)\tau_j} \qquad [2.13]$$

with n^2-1 generators represented by τ (that are independent, antihermitian, traceless matrices). The elements of SU(n) (in general) do not commute,

$$[\tau^a, \tau^b] \neq 0 \qquad [2.14]$$

In the two groups we will consider, SU(2) and SU(3), these generators, τ, represent the Pauli and the Gell-Man Matrices respectively.

2.4 *SU*(2)

Let us first discuss the SU(2) group formed with the Pauli matrices. These are three 2 × 2 matrices that are Hermitian and unitary:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad [2.15]$$

Spin-½ fields transform as doublets under transformations in the SU(2) group, with the Lagrangian as before. The requirement for invariance under the infinitesimal local gauge transformation

$$\psi(x) \rightarrow [1 - ig\alpha(x)\vec{T}]\psi(x) \qquad [2.16]$$

where T are the generators of of the gauge group. The generators of the group obey

$$[\tau^i, \tau^j] = if^{ijk}T^k \qquad [2.17]$$

where f_{ijk} are the group's structure constants. Hence, the gauge group is non-Abelian when operating on the doublets.

$$T_i = \tfrac{1}{2}\tau_i \qquad [2.18]$$

where τ_i are the aforementioned Pauli matrices, [2.14]. As before, in order to maintain local gauge invariance in the Lagrangian, a covariant derivative, D_μ, needs to be introduced

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \qquad [2.19]$$

where

$$D_\mu = \partial_\mu + igW_\mu T \qquad [2.20]$$

W_μ is a set of fields that transform in the adjoint representation and g is the gauge coupling.

The gauge fields W_μ transform as

$$W_\mu(x) \rightarrow W_\mu(x) + \partial_\mu\alpha(x) + g\alpha(x) \times W_\mu(x) \qquad [2.21]$$

The Lagrangian of the W-field part can be taken as

$$\mathcal{L}_W = \tfrac{-1}{4}W_{\mu\nu}W^{\mu\nu} \qquad [2.22]$$

with

$$W_{\mu\nu}(x) = \partial_\mu W_\nu - \partial_\nu W_\mu - gW_\mu \times W_\nu \qquad [2.23]$$

For each independent T_i, there is a gauge field, $W_{i\mu}$, and the Lagrangian is

$$\mathcal{L} = \tfrac{-1}{4}W_{i\mu\nu}W_i^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \qquad [2.24]$$

where

$$W_{i\mu\nu} = \partial_\mu W_{i\nu} - \partial_\nu W_{i\mu} - gf_{ijk}W_{j\mu}W_{k\nu} \qquad [2.25]$$

It can be shown that $W_{\mu\nu}$ transforms as

$$W_{i\mu\nu}(x) \rightarrow W_{i\mu\nu}(x) + gf_{ijk}\alpha_j(x)W_{k\mu\nu(x)} \qquad [2.26]$$

under the gauge transformations. Hence, given that the structure constants, f_{ijk} , are antisymmetric, $W_{i\mu\nu}W_i^{\mu\nu}$ is gauge invariant and therefore the Lagrangian term [2.23] is invariant, as required.

How can this algebra be used with practical observations? The weak interactions are experimentally observed to be non-Chiral (they are not right-left spin symmetric): the weak interaction only affects left-handed fermions (Wu et al 1956) [5]. Therefore, we have to consider the right and left handed parts of the fermion fields separately.

From experiment, the form of the weak interaction is determined to be the linear, Lorentz-invariant combination of vector minus axial-vector (V-A):

$$\bar{\psi}\gamma^{\mu}\frac{(1-\gamma^{5})}{2}A_{\mu}\psi \qquad [2.27]$$

This is a term in the Lagrangian for current of the weak interactions in the Fermi model.

Take the the chirality projectors

$$P_L = \frac{(1-\gamma^5)}{2} \qquad P_R = \frac{(1+\gamma^5)}{2} \qquad [2.28]$$

with

$$P_L^2 = P_L\,, \quad {P_R}^2 = P_R\,, \quad P_L P_R = P_R P_L = 0\,, \quad P_L + P_R = 1 \qquad [2.29]$$

Projecting these onto a spinor

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad [2.30]$$

gives

$$\psi_L = P_L\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} \qquad \psi_R = P_R\psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} \qquad [2.31]$$

From the currents represented in [2.28], only the left-handed chirality projector is acting upon the spinor. Hence, the weak interactions only act upon left-handed fermions. We hereby introduce the subscript *L* to the *SU*(2) group.

The mass of the fermions is given by

$$\mathrm{m}\psi = m(\bar{\psi_L}\psi_R + \bar{\psi_R}\psi_L) \qquad [2.32]$$

and ergo is not invariant under $SU(2)_L$. Since we do not want coupling between the right fields and gauge fields, we desire a Lagrangian that is invariant under left handed fields' rotations and leaves the right handed fields untransformed. Hence, we make up doublets of the left handed particles, such as the electron and electron neutrino or the up quark and the down quark, whilst putting the right handed fields into singlets. At this point, all of the fermion fields are massless.

One can group these doublets by particles that behave similarly under weak interactions, for example

$$\psi_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \psi_Q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad [2.33]$$

Putting these into the currents from the Lagrangian [2.28], we see

$$\bar{\psi}_e\gamma^\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \psi_e \qquad\qquad \bar{\psi}_e\gamma^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \psi_e \qquad [2.34]$$

and hence linear combinations of the aforementioned Pauli matrices appear. Therefore, $SU(2)_L$ is a sensible choice of a gauge group to represent the weak interactions.

As mentioned in §2.3, the number of generators is equal to n^2-1. Therefore there are three generators in $SU(2)_L$ and analogously there are three bosons: W^1, W^2 and W^3. These are observed to have positive, neutral and negative charges.

2.5 $SU(2)_L \times U(1)_Y$

According to the Glashow-Weinberg-Salam Theory [6], combining $SU(2)$ and $U(1)$ symmetries we find the $SU(2)_L \times U(1)_Y$ model of electroweak interactions. The subscript index, Y, here indicates that the unitary group $U(1)_Y$ is not the same unitary group as the electromagnetic, earlier denoted $U(1)$, but that of the weak hypercharge. The relationship between electric charge, Q, and the weak hypercharge, Y, is given by

$$\mathrm{Q} = \mathrm{T}_3 + \frac{Y}{2} \qquad [2.35]$$

from the Gell-Mann - Nishijima formula, where T_3 is as defined by [2.17] and represents isospin.

Left handed fermions transform under both $SU(2)_L$ and $U(1)_Y$. Right handed fermions only transform under $U(1)_Y$ and therefore no right-handed neutrino is included as it is without charge. Hence in $SU(2)_L \times U(1)_Y$ we have left handed doublets of:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad [2.36]$$

and right handed singlets of:

$$e_R \quad \mu_R \quad \tau_R \quad u_R \quad c_R \quad t_R \quad d_R \quad s_R \quad b_R \qquad [2.37]$$

In this model we have three bosons from the $SU(2)$ group and the $U(1)$ group provides the boson B^0. We will see that this combines with the neutral W boson from the $SU(2)$ group to give us the Z^0 and γ bosons. The model has a Lagrangian

$$\mathcal{L} = \frac{-1}{4} W^{i\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \bar{\psi} i \gamma^\mu D_\mu \psi \qquad [2.38]$$

where $W^{\mu\nu}$ is as [2.24], $B^{\mu\nu}$ is defined as

$$\mathrm{B}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \qquad [2.39]$$

and the covariant derivative is

$$\mathrm{D}_\mu = \partial_\mu + igW_\mu T + ig'\frac{1}{2} B_\mu Y \qquad [2.40]$$

[2.39] is invariant under local gauge transformations in both groups. Defining the isospin raising and lowering operators as

$$\mathrm{T} = \frac{T_1 \pm iT_2}{\sqrt{2}} \qquad [2.41]$$

and the weak bosons W^+ and W^- by

$$\mathrm{W}_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}} \qquad [2.42]$$

We cannot create mass terms for the fermions or gauge bosons because they would require the left and right handed fields and therefore would break the gauge symmetry. Thus, the Lagrangian in [2.39] only contains massless fields.

To unify the electromagnetic and weak interactions, the electromagnetic term $ieQA$ (seen in §2.2 in the Lagrangian [2.6]) must be contained in the $i(gT_3W_{3\mu}+g'\frac{1}{2}YB_\mu)$ of [2.41]. Given that all vector bosons normalise the same, W_3 and B must be linear combinations of A and Z, another neutral field. Hence, we can write

$$\begin{pmatrix} W_3 \\ B \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} Z^\mu \\ A_\mu \end{pmatrix} \qquad [2.43]$$

θ_W is the electroweak mixing angle. From $i(gT_3W_{3\mu}+g'\frac{1}{2}YB_\mu)$,

$$\mathrm{i}(\mathrm{gT}_3 W_{3\mu} + g'\tfrac{1}{2}YB_\mu) = iA_\mu[g\sin\theta_w + g'\cos\theta_w\tfrac{Y}{2}] + [g\cos\theta_w - g'\sin\theta_w\tfrac{Y}{2}] \qquad [2.44]$$

In order to coincide with Quantum Electro Dynamics, we want the first term to equal ieQ, i.e.

$$\mathrm{ieQ{=}ie}(\mathrm{T}_3 + \tfrac{1}{2}Y) \qquad [2.45]$$

Therefore we must have

$$\mathrm{e} = \mathrm{g}\sin\theta_w = g'\cos\theta_w \qquad [2.46]$$

and so

$$\sin^2\theta = \frac{g'^2}{g^2+g'^2} \qquad [2.47].$$

The Z term in the covariant derivative can now be written

$$\mathrm{D}_\mu^Z = ig_Z Z_\mu(T_3 - x_w Q) \qquad [2.48]$$

Where the Z^0 coupling constant is

$$\mathrm{g}_Z = \frac{e}{\sin\theta_w\cos\theta_w} \quad \text{and} \quad \mathrm{x}_w = \sin^2\theta_w \qquad [2.49]$$

Hence, if θ_W is fixed, then e determines all gauge couplings and so the weak and electromagnetic interactions are unified. The flaw in this model is that the gauge symmetry dictates massless gauge particles, which we know to be untrue from experiment. We see that in reality, this symmetry spontaneously breaks down resulting in the missing mass terms for some fermions and gauge bosons.

2.6 Spontaneous Symmetry Breaking

A report by A. Pich proved very useful in this section [7], as did an essay by X. Xin [8].

The symmetry of the electroweak unification is broken by the vacuum

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}.$$

We know this symmetry must break because, experimentally, not all of of the gauge bosons are massless. According to Goldstone, Salam & Weinberg (1962)[10], the symmetry breaking requires a zero mass, spin-0 boson and must be renormalisable, something which was unheard of previously

We introduce a scalar field, called the Higgs field, with the Higgs doublet, ϕ;

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad [2.50]$$

We choose the direction of the field fluctuation at the minimum of the potential so that

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad [2.51]$$

We then select the unitary gauge

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \quad [2.52]$$

h describes a Higgs boson.

This field exists in a vacuum and it's potential has a vacuum expectation value which is not stable around ϕ=0 (see appendix). Therefore the minimum, most stable, value of the potential, the vacuum expectation value, occurs at

$$\phi = \frac{v}{\sqrt{2}} \quad [2.53]$$

Expanding the Higgs field about its minimum potential energy by applying a phase transform to the vacuum presents

$$e^{i\Lambda\theta} \langle \varphi \rangle \neq \langle \varphi \rangle \quad [2.54]$$

and hence the symmetry is broken. The interaction between the scalar field and fermionic field is called the Yukawa Interaction.

Massive particles should have three degrees of freedom but the gauge fields have two, whilst the Higgs field has four. Expanding the Higgs field in the vacuum means each vector boson gains one extra degree of freedom and hence becomes massive. Goldstone's theory dictates that when a symmetry is spontaneously broken massless, spin-0 bosons appear, called “ghosts”. The ghost associated with each generator is “eaten” by the gauge bosons and provides them with an extra degree of freedom, allowing them to be massive.

This symmetry breaking is spontaneous and leaves us with the three massive bosons W^+, W^- and Z^0, as well as the massless photon, γ.

2.7 *SU*(3)

Quantum Chromo-Dynamics (QCD) belongs to the gauge group $SU(3)_C$. The subscript index *C* is added to signify the group is related to colour. Every quark is either “red”, “green” or “blue”.

Analogous to *SU*(2)’s three generators of Pauli matrices, there are eight generators in $SU(3)_C$, ($n^2-1 = 3^2-1 = 8$) and these are given by the Gell-Mann matrices mentioned in §2.3

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

[2.55]

These eight generators are representative of the eight colours of gluons; combinations of the red, green, blue, antired, antigreen and antiblue colours. The gluon states are:

$$g_1 = \tfrac{1}{\sqrt{2}}(r\bar{b} + b\bar{r}) \quad g_2 = \tfrac{1}{\sqrt{2}}(r\bar{b} - b\bar{r}) \quad g_3 = \tfrac{1}{\sqrt{2}}(r\bar{r} - b\bar{b})$$
$$g_4 = \tfrac{1}{\sqrt{2}}(r\bar{g} + g\bar{r}) \quad g_5 = \tfrac{1}{\sqrt{2}}(r\bar{g} - g\bar{r}) \quad g_3 = \tfrac{1}{\sqrt{2}}(b\bar{g} - g\bar{b})$$
$$g_7 = \tfrac{1}{\sqrt{2}}(b\bar{g} + g\bar{b}) \quad g_8 = \tfrac{1}{\sqrt{6}}(r\bar{r} + b\bar{b} - 2g\bar{g})$$

[2.56]

Unlike the earlier $SU(2)_L \times U(1)_Y$ undergoing spontaneous symmetry breaking (§2.6), this group does not. Hence, the gluons do not have mass. Overall, we now have a group

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

We look to expand on this by introducing higher rank groups.

2.8 *SU*(5)

Reference [11] was researched for information in this section.

The Standard Model works well for energies up to approximately 100GeV. If we find a Grand Unified Theory then it must correspond to the Standard Model at lower energies. One big motivation for a GUT is that the Standard Model as above does not assign masses to neutrinos, with them only being singlets. This contradicts experimental observation and so an extra particle, the right-handed neutrino must be introduced to give neutrinos their observed mass.

The $SU(3)_C \times SU(2)_L \times U(1)_Y$ group naturally fits into $SU(5)$. There are 15 fermions in five representation of the group:

$$Q \equiv \left(3, 2, +\frac{1}{6}\right)$$
$$L \equiv \left(1, 2, -\frac{1}{2}\right)$$
$$u^c \equiv \left(\bar{3}, 1, -\frac{2}{3}\right)$$
$$d^c \equiv \left(\bar{3}, 1, \frac{1}{3}\right)$$
$$e^c \equiv (1, 1, +1) \qquad [2.57]$$

$SU(3)$ is represented in the third column, where 3, $\bar{3}$ and 1 stand for fundamental, anti-fundamental and singlet respectively. The second column represents $SU(2)$: 2 stands for fundamental and 1 for singlet. The third column is the generator of $U(1)$, hypercharge, as mentioned in [2.25]. The diagonal of $SU(5)$ is traceless and generates $U(1)_Y$. It has 24 dimensions ($n^2-1 = 5^2 - 1 = 24$). These 24 dimensions are a sum of the eight generators of $SU(3)$, the three of $SU(2)$ and one from $U(1)$ along with 12 broken generators. $SU(5)$ requires a scalar field to break it into the aforementioned group, this leaves us with the 12 Goldstone bosons.

However, $SU(5)$ still fails to explain neutrino masses, predicts incorrect Yukawa couplings and non-unified gauge couplings. Hence, we look for a larger group and find our answer in $SO(10)$.

2.9 *SO*(10)

In this section, Howard Georgi, The state of the art--gauge theories, in *Particles and Fields* proved to be an important reference. [12]

The big advantage of $SO(10)$ is that it has 16 spinorial representation, which can unify all of the Standard Model's matter as well as the sought-after right-handed neutrino. This is found by considering all possible combinations of five spin-½ states with even numbers of "–"s, as represented by **Table 1**.

SU(5) Representation	Particle	Weight Vector (±½,±½,±½,±½,±½)	
		Colour Spins	**Weak Spins**
1	**νbar**	+ + +	+ +
10	**ebar**	+ + +	− −
	$\mathbf{u}_Y$	+ + −	+ −
	$\mathbf{u}_B$	+ − +	+ −
	$\mathbf{u}_R$	− + +	+ −
	$\mathbf{d}_Y$	+ + −	− +
	$\mathbf{d}_B$	+ − +	− +
	$\mathbf{d}_R$	− + +	− +
	u *bar* $_Y$	− − +	+ +
	u *bar* $_B$	− + −	+ +
	u *bar* $_R$	+ − −	+ +
5bar	**e**	− − −	− +
	ν	− − −	+ −
	d *bar* $_Y$	− − +	− −
	d *bar* $_B$	− + −	− −
	d *bar* $_R$	+ − −	− −

SU(3) acts on the colour spins by raising or lowering one spin and doing the opposite to another.

Table 1: Spinor representation of *SO*(10)

SU(2) acts on the weak spins; the terms where the signs are the same represent singlets of *SU*(2), the others represent doublets in *SU*(2). Importantly, the right-handed neutrino, $(\nu_R)^c$ can be represented by the singlet of *SU*(5), $(N^c)_L$, $\bar{\nu}$. Therefore, *SO*(10) contains all of the elements we desire from a group describing the Standard Model.

3. String Theory

3.1 Introduction To String Theory

Becoming increasingly well-known, String Theory is an option for a route towards a Grand Unified Theory. Somewhat a misnomer, "String Theory" isn't a theory, as such, but more a school of thought. That is to say, there is no "String Theory equation" that can explain everything in our universe (yet!). Rather, String Theory comprises of a number of approaches to explaining different elements of physics using the mathematics associated with a "string". This idea of a string is analogous to a string on a musical instrument: the different vibrating frequencies of an instrument's string produce different musical notes; similarly different modes of the fundamental strings in String Theory produce different particles/ fields in our universe. These strings therefore replace point particles.

We will see that String Theory could be the perfect explanation behind the Standard Model. Here on in we draw comparisons between the group theory representation and String Theory; meeting at the kissing point of the two descriptions - the $SO(10)$ representation. Further, String Theory may incorporate an explanation of quantum gravity; something the Standard Model fails to do.

3.2 Motivation for String Theory

Whilst there is a fairly solid understanding the three fundamental forces of electromagnetic, strong and weak interactions, physicists lack a complete theory of quantum gravity for the shortest scales. Trying to unify gravity and particle physics, or General Relativity and Quantum Mechanics, leads to infinities in the parameters required to renormalise at the Planck scale (1.22×10^{19} GeV). We also need an explanation for the behaviours of black holes. General Relativity and Quantum Field Theory are yet to be unified for small spacetime scales or large gravitational forces. Quantum Field Theory also fails to handle the strong nuclear force over large distances. Nor does it explain the need for the parameters required for the Standard Model.

3.3 What is String Theory?

Strings, as described above, are analogous to those on a musical instrument. The strings may be open or closed, like an elastic band, and they vibrate. These vibrations, or modes, superimpose as waves do. Each mode has an associated energy and so can describe a fundamental particle. Somewhat perfectly, the laws governing the forces we know, including those for the graviton, emerge from String Theory for particular modes. However, earlier branches of String Theory only described bosons. Further, they require 26 spacetime dimensions, compared to the four we observe.

3.4 Superstring Theory

An additional "super" symmetry (SUSY) to the universe we see was proposed, that suggests a symmetry between fermions and bosons. It states that every particle has a symmetric partner that differs by spin-½. These SUSY partners must be far heavier than the observable particles that we are familiar with and none have been observed as of yet.

SUSY has since been incorporated into String Theory, with positive results. String Theory can now be used to describe fermions as well as bosons and only requires ten dimensions to do so. These extra dimensions are folded down to the Planck scale and hence are undetectable with contemporary technology.

Five different varieties of Superstring Theory were derived and were rather separate until in 1995 they were unified by Edward Witten [13] under the banner of "M-theory". This theory requires an extra spacetime dimension. This links nicely to the theory of Supergravity, which also works with eleven dimensions. As with the above, M-theory is yet to have any grounding in experiment, however it is a leading field of thought, with even Stephen Hawking referring to it as "the *only* candidate for a complete theory of the universe."[9]

3.5 Using String Theory

The main goal for String Theory is to explain quantum gravity. This looks promising when one considers the singularities thrown up by Einstein's theory of General Relativity. It appears that String Theory is applicable at these points in spacetime, whereas previous theories break down. At the small distance scale, infinities in the parameters appear due to the assumption of a point particle. If these particles are treated as strings, then these infinities no longer present a problem. Furthering the study of singularities, string theorists look back to the Big Bang, the "initial singularity" of the universe.

There are further unknowns in the universe, such as dark matter, dark energy the asymmetry of matter and antimatter. These areas are being researched in String Theory today.

4. String Phenomenology

4.1 What is String Phenomenology?

As alluded to earlier, String Theory could be the perfect explanation behind the Standard Model. The Standard Model has a few key problems, some mentioned earlier. Most prominently, it neglects to describe quantised gravity, a key inspiration for String Theory research, as mentioned in §3.2. Furthermore, it doesn't explain why gravity is markedly weaker than the other three fundamental forces. There is also no explanation as to why the Standard Model only sees three generations of particles. String phenomenology is the area of study that aims to model particle physics and answer some of its problems using String Theory.

In this chapter, we aim to draw comparisons between the group theory representation and String Theory; meeting at the kissing point of the two descriptions - the *SO*(10) representation.

4.2 The basis for a model using String Theory

We begin by using light-cone quantisation, which removes negative states.

Physical states, $|\varphi\rangle$, obey the mass-shell condition

$$(\hat{\mathrm{L}}_0 - a)\,|\phi\rangle = 0 \tag{4.1}$$

where a is a constant, and

$$\hat{\mathrm{L}}_m\,|\phi\rangle = 0, m > 0 \tag{4.2}$$

When Lorentz invariance is imposed, we see that the normal ordering constant $a = 1$:

$$-a = \tfrac{D-2}{2}\textstyle\sum_{n=1}^{\infty} n \tag{4.3}$$

$$-a = \frac{D-2}{2}\frac{-1}{12}$$

$$\text{Since } a = 1 \Rightarrow D = 26$$

after using the Ramanujan summation of the sum.

Hence, we have $D-2$ dimensions since Ψ^0 and Ψ^D are non-physical in the light-cone system. If we look at two types of string, bosonic and fermionic, we see that for bosonic strings, we require D=26 and for fermionic strings we require D=10.

Classically, we fix the world-sheet dynamic to the flat sheet:

$$\mathrm{h}_{\alpha\beta} \rightarrow \eta_{\alpha\beta} \tag{4.4}$$

which is the conformal condition. Now the ghosts are a conformal anomaly. Introducing the world-sheet's degrees of freedom as η_f, η_f are simply fermionic fields living on the world sheet.

$-26^b + 11 + D + D/2 = 0 \Rightarrow D = 10$

$-26 + D = 0 \Rightarrow D = 26$

For D=4

$-26 + 11 + 4^b{}_{D=4} + 2 + \frac{1}{2}\eta_f = 0$

$\Rightarrow \eta_f = 18$, hence we require eighteen fermions to introduce into four dimensions.

$-26 + 11 + D/2 + D + \frac{1}{2}\eta_f^f = 0$

For the bosonic string, we only have bosonic ghosts;

$- 26 + 4 + (1/2)\, \eta_f^b = 0$

so $\eta_f^b = 44$

X^μ and Ψ^μ are bosonic spacetime coordinates. To produce the Standard Model, we will use left moving fermionic fields and right moving bosonic fields, called heterotic String Theory. Here we only consider closed strings, since the left and right are not independent on an open string.

Fermionic Left-Movers	Bosonic Right-Movers
$X^\mu{}_L(\tau,\sigma)$, $\mu = 0,1,2,3$	$X^\mu{}_R(\tau,\sigma)$, $\mu = 0,1,2,3$
$\Psi^\mu{}_L(\tau,\sigma)$, $\mu = 0,1,2,3$	
$(\chi^i(\tau,\sigma), y^i(\tau,\sigma), \omega^i(\tau,\sigma))$, $i = 1,\ldots, 6$	$\phi^a(\tau,\sigma)$, a = 1,…, 44

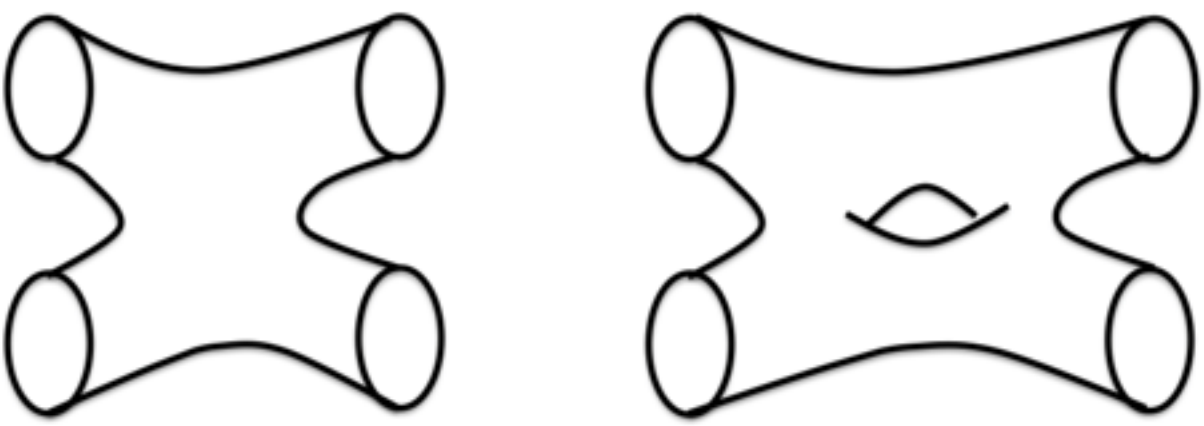

Figure 1: A string's worldsheet without and with a hole in

A string's worldsheet may form holes within it and can be mapped to a torus shape, as seen in **Figure 1**[]. The torus is parametrised by the complex parameter τ. If one flattens out the torus to a

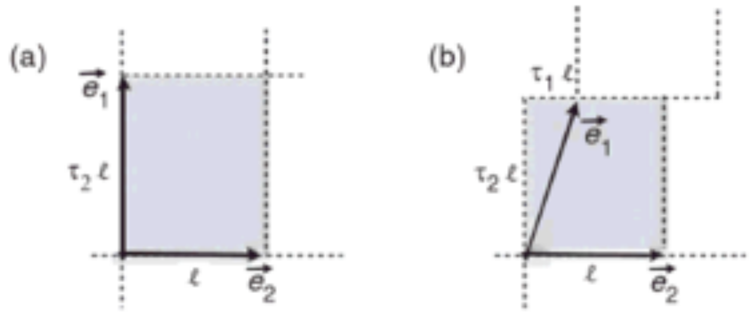

Figure 2: A

two-dimensional parallelogram, as shown in **Figure 2** [14], then τ_1 and τ_2 are related to τ and σ.

Picture the top of these quadrilateral meeting its bottom and side meets side and we get a cylinder then a torus. The amplitude of the torus is invariant under the modular transformations $\tau \rightarrow -1/\tau$ and $\tau \rightarrow \tau+1$.

Take Z to be the sum over all oscillations of the string. Picture a fermion, f, living on the worldsheet and propagate it around one of the closed loops on the string. After one complete loop it would either be $+f$ or $-f$. Hence there is the spin structure:

$$\mathrm{f} \rightarrow \begin{pmatrix} \pm \\ \pm \end{pmatrix} \qquad \begin{pmatrix} + \\ + \end{pmatrix}, \begin{pmatrix} + \\ - \end{pmatrix}, \begin{pmatrix} - \\ + \end{pmatrix}, \begin{pmatrix} - \\ - \end{pmatrix} \tag{4.5}$$

On a closed string we have 20 real left-moving fermions: 2 from Ψ^μ and six for each of χ^i, y^i and ω^i. We also have 44 real right-moving fermions We define T_f, the super current, as

$$\mathrm{T}_f = \psi^\mu \partial X_\mu + i \sum_{j=1}^{6} \chi_j y_j \omega_j \tag{4.6}$$

Taking a string around a closed loop is called the "vacuum to vacuum amplitude". There are four of these spin structure theta functions

$$\theta_1 = \begin{pmatrix} + \\ + \end{pmatrix}, \theta_2 = \begin{pmatrix} + \\ - \end{pmatrix}, \theta_3 = \begin{pmatrix} - \\ - \end{pmatrix}, \theta_4 = \begin{pmatrix} - \\ + \end{pmatrix} \tag{4.7}$$

Considering the possible combinations

$$\prod_{i=1}^{64} \theta_i \begin{bmatrix} a \\ b \end{bmatrix} \tag{4.8}$$

it is seen that $\prod \theta_i$ is not invariant under transforms, e.g. $\theta_1 \rightarrow \theta_2$. Therefore, we need to sum over all possible spin structures:

$$\mathrm{Z} = \sum_{SpinStructures} \prod_{i=1}^{64} \theta_i \begin{bmatrix} a \\ b \end{bmatrix} \tag{4.9}$$

which remains invariant under modular transformations.

Each spin structure may come with a phase, C. The phase of summing giving different ± comes from transformations under different theta functions

$$Z = \sum C \begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \end{pmatrix} \prod_{i=1}^{64} \theta_i \begin{bmatrix} a \\ b \end{bmatrix} \qquad [4.10]$$

This is a now a more accurate representation of Z, the partition function. To have a successful model, one needs a basis, phases and constraints.

Encode in the basis vectors and one-loop phases, C. From these, extract constraints on vectors and phases. The constraints were derived. To construct a model, we specify basis and phases to check if consistent with constraints.

Specify basis:

$$B = \vec{b_1}, \ldots, \vec{b_n} \qquad [4.11]$$

(vector has 64 entries). The phases between vectors in the basis:

$$\mathrm{C} \begin{pmatrix} b_i \\ b_j \end{pmatrix} \qquad [4.12].$$

This will generate the partition function. We impose the condition:

$$\mathbb{1} \in B \qquad [4.13],$$

therefore all the fermions are periodic:

$$f \rightarrow -e^{i(f)\pi} f \qquad [4.14].$$

We impose a second condition

$$N_{ij} b_i b_j = 0 mod 4 \qquad [4.15]$$

so that real fermions combine to one complex one,

$$b_i b_j = \sum b_i(f) b_j(f) + \frac{1}{2} \sum Real \qquad [4.16]$$

N_{ij} is the least common multiplier. In this case, 2, since $2\pi \equiv 0$ in phase shift.

Constraint number three:

$$N_i b_i^2 = 0 mod 8 \qquad [4.17]$$

Constraint number four:

$$\mathrm{C} \begin{pmatrix} b_i \\ b_i \end{pmatrix} = -\mathrm{e}^{i\pi \frac{b_i b_-}{4}} C \begin{pmatrix} b_i \\ \mathbb{1} \end{pmatrix} \qquad [4.18]$$

Constraint number five:

$$\mathrm{C} \begin{pmatrix} b_i \\ b_j \end{pmatrix} = \mathrm{e}^{i\pi \frac{b_i b_j}{2}} C \begin{pmatrix} b_j \\ b_i \end{pmatrix}$$

[4.19]

Constraint number six:

$$C\begin{pmatrix}\alpha\\ \beta+\gamma\end{pmatrix} = \delta_\alpha C\begin{pmatrix}\alpha\\ \beta\end{pmatrix} C\begin{pmatrix}\alpha\\ \gamma\end{pmatrix} \quad [4.20]$$

where

$$\delta_\alpha = e^{i\pi b_\alpha(\psi^{1,2})} \quad [4.21]$$

δ_α is known as the "spacetime spin statistic" and is dependent upon the boundary conditions of Ψ^μ:

$$\text{if } \psi^{1,2} = 1,1 \qquad \delta_\alpha = -1 \quad [4.22]$$

$$\text{if } \psi^{1,2} = 0,0 \qquad \delta_\alpha = 1$$

4.3 Applying GSO Projections

The condition states in the superstring spectrum must satisfy the operator acting on the states. We now apply the GSO projection, which acts by only keeping states that are even, i.e. the projection removes the presence of tachyons (non-physical particles with "negative" mass).

$$e^{i\pi \vec{b_j}\vec{F_\alpha}} |s\rangle_\alpha = \delta_\alpha C\begin{pmatrix}\alpha\\ b_j\end{pmatrix} |s\rangle_\alpha \quad [4.23]$$

F_α is the fermion number operator.

When we have a basis we can form all possible bases of vectors. The real, summed vector

$$\vec{\alpha} = \sum_{m_i=0}^{N-1} m_i \vec{b_i} \quad [4.24]$$

$$N_i \vec{b_i} = \vec{0} \quad [4.25]$$

2(1, 0) = (0,0) since 2 is the least common multiplier. So [4.25] is one phase complete.

The mass of the string states must be equal from the left and right.

$$M_L{}^2 = -\tfrac{1}{2} + \frac{\vec{\alpha_L}\vec{\alpha_L}}{8} + \hat{N}_L = -1 + \frac{\vec{\alpha_R}\vec{\alpha_R}}{8} + \hat{N}_R = M_R^2 \quad [4.26]$$

N_L is the sum of oscillators acting upon the vacuum, here they are fermionic oscillators. The number operator, N_i, depends on frequencies acting on the vacuum. The oscillator depends on the boundary conditions.

[4.27]

where v_f is the frequency of the fermion oscillators:

$$v_f = \frac{1-\alpha(f)}{2} \qquad v_f{*} = \frac{1+\alpha(f)}{2} \quad [4.28]$$

The fermion number operators:

$$F(f\,|0\rangle) = +f\,|0\rangle \qquad N_{\ldots} = \sum_{f,f^*}(v_f)_L$$
$$F(f^*\,|0\rangle) = -f^*\,|0\rangle \qquad F\,|\pm\rangle = \begin{cases} 0 & \text{if } |+\rangle \\ -1 & \text{if } |-\rangle \end{cases})_R \tag{4.29}$$

Define the charge for a complex fermion as:

$$Q(f) = \tfrac{1}{2}\alpha(f) + F(f) \tag{4.30}$$

The first basis vector sees all the elements as periodic as it is simply 𝟙.

$$B = (1,1,1,...(\times 20)...,1|1,1,1...(\times 44)...,1) \tag{4.31}$$

applying to [4.25]

$$\begin{aligned} N_i\vec{1}\cdot\vec{1} &= 2(\tfrac{1}{2}\times 20 - \tfrac{1}{2}\times 44) \\ &= -24 = 0mod8 \end{aligned} \tag{4.32}$$

We select 0mod8 in order to comply with the phase of 2≡0 so as to preserve invariance.

$$C\begin{pmatrix}1\\1\end{pmatrix} = \pm 1 \tag{4.33}$$

The form of phases explicitly is

$$C\begin{pmatrix}b_i\\b_j\end{pmatrix} = \delta_{b_i}e^{i\frac{2\pi m}{N_j}} = \delta_{b_i}e^{\frac{i_i b_j}{2}}e^{i\frac{2\pi m}{N_i}} \tag{4.34}$$

We can calculate the phase to be ±1. Now to look at the mass:

$$M_L^2 = \frac{-1}{2} + \frac{\vec{\alpha_L}\vec{\alpha_L}}{8} + N_L \tag{4.35}$$

$$\vec{\alpha_L}\vec{\alpha_L} = \text{half of the left mass} = 10 \tag{4.36}$$

$$M_L^2 = \frac{-1}{2} + \frac{10}{8} > 0 \tag{4.37}$$

We only seek states which obey [4.26] since others are not of low enough energy to feature in the Standard Model.

We can act with either one oscillator on the left, two on the right or a bosonic oscillator $\alpha_{-1}{}^{\mu}$. Using [4.26],

$$N_L = 0 \qquad N_R = \frac{-1}{2} \qquad \Rightarrow M_L^2 = \frac{-1}{2},\ M_R^2 = \frac{-1}{2} \tag{4.38}$$

Hence, we have a tachyonic state which is rejected as we find no tachyons in the Standard Model.

Next, we apply

$$e^{i\pi\vec{1}\cdot\vec{F_{NS}}}\,|s\rangle = \delta_{NS}C\begin{pmatrix}NS\\1\end{pmatrix}^*|s\rangle$$

$$e^{i\pi\vec{s}\cdot\vec{F}_{NS}}\psi^{\mu}_{\frac{1}{2}}\bar{\phi}^{a}_{\frac{1}{2}}\bar{\phi}^{b}_{\frac{1}{2}}\left|0\right\rangle_{L\times R}=\delta_{s}(\psi^{\mu}\bar{\phi}\bar{\phi})\left|0\right\rangle_{L\times R} \quad [4.39]$$

(NS signifies the Neveu-Schwarz 0-sector) and we see that

$$\delta_{NS}=+1 \quad [4.40]$$

as a result of $\Psi^{1,2}$ both being periodic and the application of Euler's identity. Since [4.40] and

$$\delta_{NS}C\binom{NS}{1}=-1 \quad [4.41]$$

both equal −1, the equation holds for $\Psi^{1,2}$ and these states are physical. The same can be repeated for $^{a}{}_{½}$ and for $\bar{\Psi}^{\mu}\bar{\phi}^{a}_{\frac{1}{2}}\bar{\phi}^{b}_{\frac{1}{2}}$

For the $SO(44)$ group there are 946 dimensions - too many!

We now add another basis vector, S, which has periodic conditions for Ψ and X.

$$\vec{s}=\{\psi^{1,2},\chi^{1,\dots,6}\}=1 \quad [4.42]$$

Only the terms above the diagonal on the matrix are independent, since it is symmetrical; those terms below the diagonal are fixed.

$$e^{i\pi\vec{s}\cdot\vec{F}_{NS}}\left|s\right\rangle_{NS}=\delta_{NS}C\binom{NS}{s}\left|s\right\rangle_{NS} \quad [4.43]$$

$$\delta_{s}=\delta_{NS}C\binom{NS}{s}=-1$$

$$e^{i\pi\vec{s}\cdot\vec{F}_{NS}}\bar{\phi}_{\frac{1}{2}}\left|0\right\rangle_{L\times R}>0>\delta_{s}\bar{\phi}^{a}\left|0\right\rangle_{L\times R} \quad [4.44]$$

Hence, this is another tachyonic state. We continue to apply these projections and find that some states are satisfactory. For example:

$$e^{i\pi\vec{s}\cdot\vec{F}_{NS}}\chi^{\mu}_{\frac{1}{2}}\bar{\phi}^{a}_{\frac{1}{2}}\bar{\phi}^{b}_{\frac{1}{2}}\left|0\right\rangle_{L\times R}=\delta_{s}(\chi^{\iota}\bar{\phi}\bar{\phi})\left|0\right\rangle_{L\times R} \quad [4.45]$$

in which both sides of the equation are negative, since s has periodic boundary conditions; and

[4.46]

Note that χ will get in but not y or ω, since they are zero in s.

Given that $\vec{S}_{L}\cdot\vec{S}_{L}=4$ and $S_{R}\cdot S_{R}=0$ [4.47]

going back to [4.26], we see that $N_L = 0$ and $N_R = 1$. Because $N_L = 0$, the vacuum on the left purely has Ramond fermions and is doubly degenerate. S gives rise to the $SO(44)$ gauge group. We shall work in the light cone gauge so that only transverse degrees of freedom are physical, i.e. there are no ghosts. In the right-moving sector, there is either one oscillator or two with magnitude one half each.

If
$$f \rightarrow +f = -e^{i\pi\alpha(f)} f$$
$$\Rightarrow \alpha(f) = 1 \qquad [4.48]$$

Counting the number of minus states in the vacuum gives us:

$$\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 16 \qquad [4.49]$$

which is the total number of possible states on the left.

Now let us consider the right:

$$|s\rangle_L \times \partial X^{\mu}_{-1} |0\rangle_R \qquad\qquad |s\rangle_L \times \bar{\phi}^a \bar{\phi}^b |0\rangle_R \qquad [4.50]$$

ϕ is an oscillator and $\bar{\phi}^a \bar{\phi}^b$ doesn't carry any spin under a Lorentz Group. The *s* term brings spin-½ and an extra +1 is picked up from the μ index. Thus, there we have spin three halves, which leads to the "gravitino", a particle of SUSY. In the second equation there is only the spin one half from the *s* term. This corresponds to the "gauginos" in SUSY.

Now we impose the GSO projection

$$e^{i\pi \vec{b}\cdot\vec{F}_s} |states\rangle_s = \delta_s C \begin{pmatrix} s \\ b \end{pmatrix} |states\rangle_s \qquad [4.51]$$

δ_s=−1 since in S(28) Ψ^1,Ψ^2 are periodic so δ_s depends on whether string theory fermions are periodic or not. If this projection condition is satisfied then states are "in". If not then they are incompatible.

Take the two basis vectors, $\vec{1}$ and $\vec{s}$, as shown in the matrix representation [4.52], and arbitrarily fix one of their values between ±1. Once one value is fixed the other four fix too.

$$\begin{matrix} & \vec{1} & \vec{s} \\ \vec{1} & \pm 1 & \pm 1 \\ \vec{s} & \pm 1 & \pm 1 \end{matrix} \qquad [4.52]$$

If we arbitrarily choose −1, then we again wish to look for balance

Let us see another projection:

$$e^{i\pi\vec{s}\vec{F}_s}\left|\vec{s}\right\rangle = \delta_s C\binom{s}{s}\left|\vec{s}\right\rangle = +\left|\vec{s}\right\rangle$$
$$\mathrm{C}\binom{s}{1} = \text{-}1 = \mathrm{C}\binom{s}{s} \qquad [4.53]$$

clearly the even numbered binomial coefficients will here be "in" as they produce a positive result. Those that are odd, [4.54] will produce negative coefficients and so will be excluded.

$$\binom{4}{1}\ \binom{4}{3} \qquad [4.54]$$

The projection of ***s*** is the same as that of 1 since it is periodic.

$\binom{1}{0}$ is the vacuum under $\psi^{1,2}$, the String Theory fermion.

$\binom{1}{1}$ is the vacuum under internal fermions (compacted fermions from SUSY within internal dimensions.

The state has N-degeneracy = 4 and analogously there are 4 gravitinos. Overall, we see {1,**s**} form an *SO*(44) gauge group with N=4 space time supersymmetry and no tachyonic particles.

4.4 Adding More Sets

Once all possible projections from ***s*** have been carried out, a new set can be introduced to continue the symmetry breaking. We now look at $\vec{b}$, which has six variations. For conciseness, in this paper we shall not mechanically turn through every projection of each of these sets but rather get an idea from the initial projections how they may pan out.

$$\vec{b} = \{\psi^{1,2}, \chi^{1,2}, y^{3\ldots6}|\bar{y}^{3\ldots6}, \bar{\psi}^{1\ldots5}\bar{\eta}^{1}\} \qquad [4.56]$$

When one considers the left and right movers, the group breaks down to 32 real, or 16 complex, fermions. It also churns out a fondly familiar *SO*(10) group from the five $\overline{\psi}$ terms. Let us now impose a projection from the set onto an untwisted (Neveu-Schwarz) sector.

$$e^{i\pi\vec{b_1}\cdot\vec{F_{NS}}}\psi^{1,2}\bar{\phi}^a\bar{\phi}^b\left|0\right\rangle_R = \delta_{NS}C\binom{NS}{b_1}\left|s\right\rangle_{NS} \qquad [4.57]$$

$\vec{b_1}$ and $\vec{F_s}$ act upon ψ to give a positive result. Hence, $e^{i\pi}$ provides a negative answer. The fermions in ϕ^a and ϕ^b are periodic and divide like so:

$$\{\bar{\psi}^{1...5}, \bar{\eta}^1, y^{3...6}\}\{\bar{\psi}^{1...5}, \bar{\eta}^1, y^{3...6}\}$$
$$\{^2, \bar{\eta}^3, \bar{\phi}^{1...8}, \bar{y}^{1,2}, \bar{\omega}^{1...6}\}\{\bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1...8}, \bar{y}^{1,2}, \bar{\omega}^{1...6}\} \quad [4.58]$$

The top two groups are both negative and the bottom two both positive. Hence the top and bottom both result in the same sign when their products are taken. The top group leads to an $SO(16)$ group and the bottom an $SO(28)$. Hence, we have symmetry breaking of

$$SO(44) \rightarrow SO(16) \times SO(28) \quad [4.59]$$

Continuing from this, applying a GSO projection to $\vec{b_1}$, given its twelve complex fermions, will lead to 2^{11} predicted states! We then begin to apply the next basis vectors, starting with, $\vec{b_2}$, and find that it breaks the symmetry from [4.59] even further. We see

$$SO(28) \rightarrow SO(6) \times SO(22) \quad [4.60]$$
$$SO(16) \rightarrow SO(10) \times SO(6) \quad [4.61]$$

We also have N breaking from four to one. Finally, we have the various decompositions of $\overline{\psi}^{1...5}$ to be in sets of sixteens. It is clear to see that repeated projections through string theory have lead us back to where we departed from the Standard Model: the spinorial-16 representation of $SO(10)$.

5. Conclusion

In the beginning, we looked at observable features of the Standard Model - our (almost) coherent description of everything that we see in our universe. We then progressed to abstract mathematical tools and found they made good analogies for the Standard Model. The group theory expanded to larger groups until it was accommodating all of the elements that we required.

Then we turned to String Theory; the notion of swapping point particles for vibrating strings.

It is a beauty of science that it usually lets you know when you have got it right. Clearly string theory isn't complete; we do not yet have one GUT to explain all. However, there are promising signs that indicate it could be the best hope we have for explaining the universe. One expects the beckon of a perfect formula waiting in the midst of the Theory. Perhaps String Theorists are dancing around the edge of the GUT, to the tunes of their beloved strings.

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7. Appendix

§2.5 Dirac matrices

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

§2.6 Plot of Higgs potential agains the Higgs field: