MATH423 - Introduction to String Theory
Set Work: Sheet 1

* **Reading**: Zwiebach chapters 1 and 2

1. Write down the weight lattice of the spinorial 16 representation of $SO(10)$ and how it decomposes under the $SU(5)\times U(1)$, $SO(6)\times SO(4)$ and] $SU(3)\times SU(2)\times U(1)^2$ subgroups.

Indentify how the Standard Model states fit into the spinorial 16 of $SO(10)$.

2. Consider the infinitesimal line element,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - dx^2.$$

(a) Write the metric $g_{\mu\nu}$ and its inverse in an explicit in matrix form.

(b) Find the set of independent transformations of the form

$$\begin{aligned} t &\to t + \epsilon A(t,x) \\ x &\to x + \epsilon B(t,x)\ , \end{aligned}$$

where ϵ is an infinitesimal constant and the functions A and B have to be determined by the requirement that ds^2 is invariant. State what each transformation represents in space time.

3. Consider two Lorentz vectors a^μ and b^μ. Write the Lorentz transformations $a^\mu \to a'^\mu$ and $b^\mu \to b'^\mu$. Verify that $a^\mu b_\mu$ is invariant under these transformations.

4. (a) Give the Lorentz transformations for the components a_μ of a vector under a boost along the x^1 axis.

(b) Show that the object $\frac{\partial}{\partial x^\mu}$ transforms under a boost along the x^1 axis as the a_μ vector considered in (a) do. This checks, in a particular case, that partial derivatives with respect to upper–index coordinates x^μ behave as a four–vector with lower indices, which is why they are written as ∂_μ.

(c) Show that, in quantum mechanics, the expression for the energy and momentum in terms of derivatives can be written compactly as $p_\mu = \frac{\hbar}{i}\frac{\partial}{\partial x^\mu}$.

5. Consider the infinitesimal line element on a two dimensional surface

$$ds^2 = g_{\mu\nu}d\theta^\mu d\theta^\nu = d\theta^2 + \sin^2\theta d\phi^2$$

(a convenient notation is $\theta^\mu \equiv (\theta, \phi)$ $(\mu = 1, 2)$)

(a) Write the metric $g_{\mu\nu}$ in an explicit matrix form.

Write $g^{\mu\nu}$ in matrix form.

(b) Consider the set of infinitesimal transformations of the form

$$\theta^\mu \to \theta^\mu + \epsilon \zeta^\mu(\theta, \phi) \ ,$$

for which the line element ds^2 is invariant and where ϵ is an infinitesimal constant. Derive the conditions that the functions ζ^1 and ζ^2 must satisfy for ds^2 to remain invariant.