MATH423 - Introduction to String Theory
Set Work: Sheet 10

1 Consider coordinates $X^\mu = (x^0, x^1, x^2, x^3)$ and the associated light–cone coordinates (x^+, x^-, x^2, x^3). Write the following Lorentz transformations in terms of the light–cone coordinates:

a. A boost with velocity parameter β in the x^1 direction.

b. A rotation with ang;e θ in the x^1, x^2 plane.

c. A boost with velocity parameter β in the x^3 direction.

2 Consider the "lightlike" compactification, in which we identify events with position and time coordinates related by

$$\begin{pmatrix} x \\ ct \end{pmatrix} \sim \begin{pmatrix} x \\ ct \end{pmatrix} + 2\pi \begin{pmatrix} R \\ -R \end{pmatrix} \tag{1}$$

a. Rewrite this identification using light–cone coordinates.

b. Consider coordinates (ct', x') related to (ct, x) by a boost with veclocity parameter β. Express the identifications in terms of the primed coordinates.

c. Consider the family of indentifications given by

$$\begin{pmatrix} x \\ ct \end{pmatrix} \sim \begin{pmatrix} x \\ ct \end{pmatrix} + 2\pi \begin{pmatrix} \sqrt{R^2 + R_s^2} \\ -R \end{pmatrix}. \tag{2}$$

Show that there is a boosted frame S' in which the identification (2) becomes a standard identification (*i.e.* the space coordinate is identified but the time coordinate is not).

3 The vacuum energy associated with the current acceleration of the universe is $\rho_{\text{vac}} = 7.7 \times 10^{-27}\text{kg/m}^3$. Derive a fundamental length scale, ℓ_{vac} associated with this vacuum energy in terms of ρ_{vac}, the Planck constant $\hbar$, and the speed of light c. Express the numerical value for ℓ_{vac} in μm where $1\mu\text{m} = 10^{-6}\text{m}$.