MATH423 - Introduction to String Theory
Set Work: Sheet 7

1.

The equations of motion for a relativistic string in the conformal gauge are

$$(\partial_0^2 - \partial_1^2) X^\mu = 0 \ .$$

The solutions must satisfy the constraints

$$\partial_0 X^\mu \partial_1 X_\mu = 0 \ , \quad \partial_0 X^\mu \partial_0 X_\mu + \partial_1 X^\mu \partial_1 X_\mu = 0$$

We consider open strings with boundary conditions

$$\partial_1 X_\mu = 0 \text{ for } \sigma^1 = 0, \pi$$

(a) Show that the equations of motions, the constraints and the boundary conditions are satisfied by

$$\begin{aligned} X^0 &= L\sigma^0 \\ X^1 &= L \cos\sigma^1 \cos\sigma^0 \\ X^2 &= L \cos\sigma^1 \sin\sigma^0 \\ X^i &= 0 \text{ for } i > 2 \end{aligned}$$

Explain in words how the string moves in this solution.

(b) Compute the mass, momentum and angular momentum of the string

(c) Compute the speed of the endpoints of string. Explain why the result you find holds for any solution of the open string theory.

2.

The equations of motion for a relativistic string in the conformal gauge are

$$(\partial_0^2 - \partial_1^2) X^\mu = 0$$

Solutions must satisfy the constraints

$$\partial_0 X^\mu \partial_1 X_\mu = 0 \ , \quad \partial_0 X^\mu \partial_0 X_\mu + \partial_1 X^\mu \partial_1 X_\mu = 0$$

We consider closed strings with boundary conditions

$$X^\mu(\sigma^0, \sigma^1) = X^\mu(\sigma^0, \sigma^1 + \pi)$$

1. Show that equations of motion and constraints are solved by

$$\begin{aligned} X^0 &= 2R\sigma^0 \\ X^1 &= R\cos(2\sigma^1)\cos(2\sigma^0) \\ X^2 &= R\sin(2\sigma^1)\cos(2\sigma^0) \\ X^i &= 0 \text{ for } i > 2 \end{aligned}$$

2. Describe in words how the closed string moves.
3. Compute the length, the mass and the momentum of the string.
4. Show that the string is at rest at time $X^0 = 0$. Express the total energy in terms of its length at time $X^0 = 0$ and interpret the result.