MATH423 - Introduction to String Theory

Set Work: Sheet 3

1.

(a)Show that for time independent fields, the Maxwell equation $T_{0ij} = 0$ implies that $\partial_i E_j - \partial_j E_i = 0$. Show that this condition is satisfied by the ansatz $\vec{E} = -\vec{\nabla}\Phi$.

(b) Show that in d spatial dimensions, with $d > 2$, the potential due to a point charge q is given by

$$\Phi(r) = \frac{\Gamma(\frac{d}{2} - 1)}{4\pi^{d/2}} \frac{q}{r^{d-2}}.$$

2.

(a) The Standard Bohr radius is $a_0 = \frac{\hbar^2}{me^2} \approx 5.29 \times 10^{-9}$cm, and arises from the electric potential $V = -\frac{e^2}{r}$. What would be the gravitational Bohr radius if the attraction force binding the electron to the proton was gravitational?

(b) In units where G, c and $\hbar$ are set equal to one, the temperature of a black hole is given by $kT = \frac{1}{8\pi M}$. Insert back the factors of G, c and $\hbar$ into this formula. Evaluate the temperature of a black hole of a million solar masses. What is the mass of a black hole whose temperature is room temperature.

3.

A string with tension T_0 is stretched from $x = 0$ to $x = 2a$. The part of the string $x \in (0, a)$ has constant mass density μ_1 and the part of the string $x \in (a, 2a)$ has constant mass density μ_2. Consider the differential equation

$$\frac{d^2y}{dx^2} + \frac{\mu(x)}{T_0}\omega^2 y(x) = 0.$$

that determines the normal oscillations

(a) What boundary conditions should be imposed on $y(x)$ and $\frac{dy}{dx}(x)$ at $x = a$?

(b) Write the conditions that determine the possible frequencies of oscillation.

(c) Calculate the lowest frequency of oscillation of this string when $\mu_1 = \mu_0$ and $\mu_2 = 2\mu_0$.