MATH423 - Introduction to String Theory

Set Work: Sheet 2

1. Consider the plane (x, y) with the identification

$$(x, y) \sim (x + 2\pi R, y + 2\pi R)$$

What is the resulting space.

2. (a) Consider the circle S^1. Presented as the real line with the identifications $x \sim x + 2$. Choose $-1 < x \leq 1$ as the fundamental domain. The circle is the space $-1 < x \leq 1$ with the points $x \pm 1$ identified. The orbifold S^1/Z_2 is defined by imposing the Z_2 identification $x \sim -x$. Describe the action of this identification on the circle. Show that there are two points on the circle that are left fixed by the Z_2 action find a fundamental domain for the two identifications. Describe the orbifold S^1/Z_2 in simple terms.

(b) Consider a torus T^2, presented as the (x, y) plane with the identifications $x \sim x + 2$ and $y \sim y + 2$. Choose $-1 < x, y \leq -1$ as the fundamental domain. The orbifold T^2/Z_2 is defined by imposing the Z_2 identification $(x, y) \sim (-x, -y)$. Prove that there are four points on the torus that are left fixed by the Z_2 transformation. Show that the orbifold $T^2/Z2$ is a two dimensional sphere, naturally presented as a square pillowcase with seamed edges.

3. The Lorentz force equation can be written relativistically as

$$\frac{dp^\mu}{ds} = \frac{q}{c} F^{\mu\nu} \frac{dx_\nu}{ds},$$

where p_μ is the four momentum. Check explicitly that that equation reproduce the Lorentz force law equation,

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right),$$

when μ is a spatial index. What does the relativistic equation give when $\mu = 0$

4. (a) Show explicitly that the source free Maxwell equations emerge from $T_{\mu\lambda\nu} = 0$.

(b) Show explicitly that the Maxwell equations with sources emerge from

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \frac{1}{c} j^\mu.$$