MATH423 - Introduction to String Theory Set Work: Sheet 10

1 Consider coordinates $X^{\mu} = (x^0, x^1, x^2, x^3)$ and the associated light-cone coordinates (x^+, x^-, x^2, x^3) . Write the following Lorentz transformations in terms of the light-cone coordinates:

- **a.** A boost with velocity parameter β in the x^1 direction.
- **b.** A rotation with ang; θ in the x^1 , x^2 plane.
- **c.** A boost with velocity parameter β in the x^3 direction.

2 Consider the "lightlike" compactification, in which we identify events with position and time coordinates related by

$$\begin{pmatrix} x \\ ct \end{pmatrix} \sim \begin{pmatrix} x \\ ct \end{pmatrix} + 2\pi \begin{pmatrix} R \\ -R \end{pmatrix}$$
(1)

a. Rewrite this identification using light–cone coordinates.

b. Consider coordinates (ct', x') related to (ct, x) by a boost with veclocity parameter β . Express the identifications in terms of the primed coordinates.

c. Consider the family of indentifications given by

$$\begin{pmatrix} x \\ ct \end{pmatrix} \sim \begin{pmatrix} x \\ ct \end{pmatrix} + 2\pi \begin{pmatrix} \sqrt{R^2 + R_s^2} \\ -R \end{pmatrix}.$$
 (2)

Show that there is a boosted frame S' in which the identification (2) becomes a standard identification (*i.e.* the space coordinate is identified but the time coordinate is not).