MATH423 - Introduction to String Theory
Set Work: Sheet 1

* **Reading**: Zwiebach chapters 1 and 2

1.

Write down the weight lattice of the spinorial 16 representation of $SO(10)$ and how it decomposes under the $SU(5) \times U(1)$, $SO(6) \times SO(4)$ and] $SU(3) \times SU(2) \times U(1)^2$ subgroups.

Indentify how the Standard Model states fit into the spinorial 16 of $SO(10)$.

2.

Consider the infinitesimal line element,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - dx^2.$$

(a)

Write the metric $g_{\mu\nu}$ and its inverse in an explicit in matrix form.

(b) Find the set of independent transformations of the form

$$\begin{aligned} t &\to t + \epsilon A(t, x) \\ x &\to x + \epsilon B(t, x) \ , \end{aligned}$$

where ϵ is an infinitesimal constant and the functions A and B have to be determined by the requirement that ds^2 is invariant. State what each transformation represents in space time.

3.

Consider two Lorentz vectors a^μ and b^μ. Write the Lorentz transformations $a^\mu \to a'^\mu$ and $b^\mu \to b'^\mu$. Verify that $a^\mu b_\mu$ is invariant under these transformations.

4.

(a) Give the Lorentz transformations for the components a_μ of a vector under a boost along the x^1 axis.

(b) Show that the object $\frac{\partial}{\partial x^\mu}$ transforms under a boost along the x^1 axis as the a_μ vector considered in (a) do. This checks, in a particular case, that partial derivatives with respect to upper–index coordinates x^μ behave as a four–vector with lower indices, which is why they are written as ∂_μ.

(c) Show that, in quantum mechanics, the expression for the energy and momentum in terms of derivatives can be written compactly as $p_\mu = \frac{\hbar}{i} \frac{\partial}{\partial x^\mu}$.