ST.1 Liverpool lectures on string theory 26/6/09.1 Abingdon

Lecture 1. Why strings?

Motivation: The road to unification? Heaven & earth?

Modern physics: predict out come of experiment

Initial conditions : mathematical model :

↕

Predicted out come.

↕

Experiment / observations

Practical: An acceptable mathematical model is the one most successful in accounting for wide range of experimental observations.

Themes: Reductionism: Large to small.

Celestial, Atomic, Nuclear, sub-nuclear, string.

Newton	19^{th} century	Rutherford	Gellman	Schwarz
	Faraday	Fermi	Feynman	Scherk
	Maxwell		Bjorken.	
	Einstein			

Unification: encompassing mathematical model.

Newton	Maxwell	Einstein	gauged S.M.	gauged GUTs
Earth & heaven.	Electric & magnetic.	Mechanical & E.M.	Weak & E.M.	S.M. + strong.

String : Gravity + gauged Q.F.T. (S.M./GUTs)

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Inventory |

Forces | E.M. Weak Strong spin 1 particles
U(1) SU(2) SU(3).

GRAVITY spin 2 particle.

Matter | Quarks & leptons.

Quarks	Leptons
strong	
weak	weak
E.M.	E.M.

quarks: $\begin{pmatrix} \text{up} \\ \text{down} \end{pmatrix}$ $\begin{pmatrix} \text{charm} \\ \text{strange} \end{pmatrix}$ $\begin{pmatrix} \text{top} \\ \text{bottom} \end{pmatrix}$.

leptons $\begin{pmatrix} \nu_e \\ \text{electron} \end{pmatrix}$ $\begin{pmatrix} \nu_\mu \\ \text{muon} \end{pmatrix}$ $\begin{pmatrix} \nu_\tau \\ \text{tau} \end{pmatrix}$.

Problem? Mass? → spontaneous symmetry breaking.
→ Higgs particle; LHC?

Unification | GUTs e.g. SO(10) → Rank = 5 $D = 45 = \frac{10 \cdot 9}{2}$

SO → Simple orthogonal 10 × 10 matrices. | generators $O^T = -O$ (Asymmetric)

$\left(\diagdown\right)$ $\frac{n^2-n}{2} + n = \frac{n(n+1)}{2}$ → symmetric.

$\left(\diagdown\right)$ $\frac{n^2-n}{2} = \frac{n(n-1)}{2}$ → asymmetric $\cdot (i)$
→ (hermitian.)
$O^+ = O$.

LST.3 | 26/6/09. 3 | Abingdon |

Physical theories $\rightarrow$ unitary $U^{\dagger}U = 1$.

time evolution $|a(t)\rangle = U\,|a(0)\rangle \Rightarrow \langle a(t)|a(t)\rangle = \langle a(0)|U^{\dagger}U|a(0)\rangle$
$= \langle a(0)|a(0)\rangle$

Representation of unitary operator: $U = e^{iH}$; $H \leftarrow$ hermitian.

$H \rightarrow$ generators of Lie algebra ; $SU(n)$, $SO(2n)$.

$$\begin{array}{ccccccccc} n=5 & = & 2 & & 1 & & 1 & & 1 \\ SO(10) & \rightarrow & SU(3) & \times & SU(2) & \times & U(1)_x & \times & U(1)_y \end{array}$$

$$45 = (8,1,0,0) + (1,3,0,0) + (3,2,x,y) + (\bar{3},2,-x,y) +$$

$\psi^x\psi$
ψ^x
$$+ (6,1,x',y') + (\bar{6},1,-x',-y')$$

$$+ (1,3,x'',y'') + (1,3,-x'',-y'')$$

$$+ (1,1,+x''',y''') + (1,1,-x''',-y''').$$

For our purpose: chiral ; spin 16 Repeesentation of $SO(10)$.

$\rightarrow$ even - in 5 slots

3	2	
+ + +	+ +	e_L^c
+ + +	− −	N_L^c
+ + −	+ −	$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$
+ − +	+ −	
− + +	+ −	
+ + −	− +	
+ − +	− +	
− + +	− +	
− − +	+ +	D_L^c
− + −	+ +	
+ − −	+ +	

3	2	
− − +	− −	U_L^c
− + −	− −	
+ − −	− −	
− − −	− +	$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$
− − −	+ −	

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OR- using $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$$16 = \binom{3}{0}\binom{2}{0} + \binom{3}{0}\binom{2}{2} + \binom{3}{1}\binom{2}{1} + \binom{3}{2}\binom{2}{0} + \binom{3}{2}\binom{2}{2} + \binom{3}{3}\binom{2}{1}.$$

$(1,1)_{..}$ $(1,1)_{..}$ $(3,2)_{..}$ $(\bar{3},1)_{..}$ $(\bar{3},1)_{..}$ $(1,2)_{..}$

E_L^c N_L^c Q_L D_L^c U_L^c L_L.

These are the sixteen states in a one S. M. generation.

+ R. H. N.

There is something right about GUTs (gauged).

Problems | Hierachy ? $\left.\begin{array}{l} M_W \sim 100\ \text{GeV} \\ M_{GUT} \sim 10^{16}\ \text{GeV} \end{array}\right\}$?

Flavour ? Not one but three ?

GUTs: gauge theories → renormalizable quantum field theories

Renormalizable : infinities absorbed in a finite number of measurable parameters. →

Gravity → nonrenormalizable.

→ Quantum & Gravity ?

→ String → finite → Gravity & Gauge unification.

LLST.5 | 3/7/09.1 / Abingdon / Friday /

special relativity and extra dimensions

Einstein's special relativity: 1) C = speed of light = constant in all inertial frames

$C = 3 \times 10^{8} \frac{m}{s}$

2) Laws of physics are the same in all inertial frames

events are labeled by their time & position in

Inertial frames. $\rightarrow$ 4-vector: $X^{\mu} = (ct, \vec{X})$

$X^0 = ct \qquad \vec{X} = (X^1, X^2, X^3) = (X, y, z) \qquad \mu = 0, 1, 2, 3$

The length of four vector is given by:

$$X \cdot X = -c^2t^2 + X^2 + y^2 + z^2$$

$$= \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu} X^{\mu} X^{\nu} = X_{\mu} X^{\mu}$$

Einstein summation convention

where
$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix} \begin{matrix} \leftarrow 1 \text{ time} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} 3 \text{ space} \end{matrix}$$

is the Minkowski metric.

In general the scalar product of two four vectors

$X^{\mu}; y^{\nu}$ is given by $X \cdot y = \sum_{\mu\nu=0}^{3} \eta_{\mu\nu} X^{\mu} y^{\nu} = X_{\nu} y^{\nu}$

$\boxed{X_{\nu} = \eta_{\mu\nu} X^{\mu}}$

Lorentz transformations:

From one inertial frame X^{μ}

To another " " X'^{μ}.

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The Lorentz transformations preserve the scalar product.

$$-c^2t^2 + \vec{x}^2 = X \cdot X = \eta_{\mu\nu} X^\mu X^\nu = X' \cdot X' = \eta_{\mu\nu} X'^\mu X'^\nu = -c^2t'^2 + \vec{x}'^2$$

where X and X' are related by a Lorentz transformation.

$$X^\mu \rightarrow X'^\mu = \Lambda^\mu{}_\nu X^\nu$$

Think for simplicity with the more familiar case of ordinary Rotations in 3 dimensions.

$$\vec{x} \rightarrow \vec{y} = R\,\vec{x}.$$

Rotations preserve the length of a three vector

similarly; Lorentz transformations preserve the length of a four vector in 4 dimensional Minkowski space-time.

*** Properties of Lorentz transformations

$\eta_{\mu\nu} X^\mu X^\nu \rightarrow$ invariant under L.T.
No change in size or shape

$$\eta_{\mu\nu} X^\mu X^\nu \longrightarrow \eta_{\mu\nu} X'^\mu X'^\nu = \eta_{\mu\nu} \Lambda^\mu{}_\alpha X^\alpha \Lambda^\nu{}_\beta X^\beta$$

$$= \eta_{\alpha\beta} X^\alpha X^\beta$$

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$\Rightarrow$ (*) $\eta_{\mu\nu} \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$

In matrix notation $\Lambda^{T} \eta \Lambda = \eta$.

$\rightarrow$ Defines the Lorentz transformations.

$\Rightarrow |\det \Lambda|^2 = 1 \Rightarrow \det \Lambda = \pm 1$.

The physical case $\det \Lambda = +1$.

$\det \Lambda = 1 \longrightarrow$ Proper Lorentz transformations.

$\det \Lambda = -1 \longrightarrow$ improper Lorentz "

Look at 00 component of $\Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} \eta_{\mu\nu} = \eta_{\alpha\beta}$.

$$\eta_{\mu\nu} \Lambda^{\mu}{}_{0} \Lambda^{\nu}{}_{0} = \eta_{00} = -1$$

$$-(\Lambda^{0}{}_{0})^2 + \sum_{i=1}^{3} (\Lambda^{i}{}_{0})^2 = -1$$

$$(\Lambda^{0}{}_{0})^2 = 1 + \sum_{i} (\Lambda^{i}{}_{0})^2 \geq +1.$$

$$(\Lambda^{0}{}_{0})^2 \geq +1 \begin{cases} \Lambda^{0}{}_{0} \geq +1 & \text{orthochronos} \\ \Lambda^{0}{}_{0} \leq -1 & \text{non-orthochronos} \end{cases}$$

Examples of Lorentz transformations

1) $\Lambda^{\mu}{}_{\nu} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \rightarrow$ Proper - non - orthochronos

$\rightarrow$ reflection.

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2) $t \to -t \qquad \vec{x} \to \vec{x}$.

$\Lambda^\mu{}_\nu = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \to$ improper $\to$ non-orthochronos

3) Physical Lorentz transformations $\to$

$\to$ continousely connected to the identity $\to$

$\to$ ⓐ $\det \Lambda = 1 \leftarrow$ Proper.

ⓑ $\Lambda^0{}_0 \geq 1 \leftarrow$ orthochronos

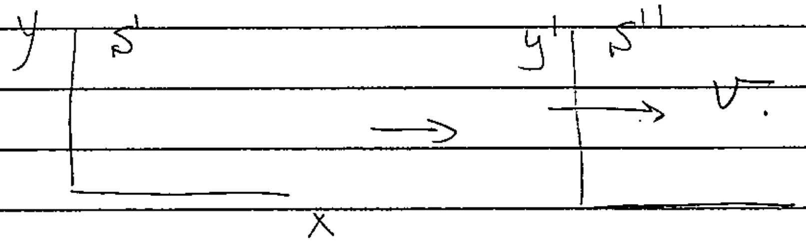
Examples! $\Lambda^\mu{}_\nu = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \qquad \Lambda^\mu{}_\nu = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$.

① Rotations $\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & \vec{R} \end{pmatrix}$ $\qquad \det \Lambda = \det R$

$\det R = \pm 1$

$\det R = 1 \to$ proper.

② boosts



$ct' = \gamma(ct - \beta x) \qquad \gamma = \dfrac{1}{\sqrt{1-\beta^2}} \qquad \beta = \dfrac{v}{c}$

$x' = \gamma(x - \beta ct)$

$y' = y$

$z' = z$

OR $\quad x'^0 = \gamma(x^0 - \beta x^1)$

$x'^1 = \gamma(-\beta x^0 + x^1)$

$x'^2 = x^2$

$x'^3 = x^3$

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or $$\Lambda^{\mu}{}_{\nu} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$\det \Lambda = \gamma^2 - \gamma^2\beta^2 = \frac{1}{(1-\beta^2)}(1-\beta^2) = 1.$$

$$\Lambda^0{}_0 = \gamma = \frac{1}{\sqrt{1-\beta^2}} \geq 1 \quad \text{as } \beta = \frac{v}{c} < 1.$$

The Poincare group.

consider the infinitesimal relativistic line element.

(*) $$-ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2.$$

So far we discussed rotation & boosts which are the Lorentz transformations.

The most general transformations also include translations. $X^\mu \to X'^\mu = X^\mu + a^\mu$

where a^μ is a constant 4-vector

obviously $dt' = dt \quad dX'^i = dX^i$

we have: 4 – translations dt, dx, dy, dz,
3 – boost $dt\,dx, dt\,dy, dt\,dz$
3 – Rotations $dx\,dy, dx\,dz, dy\,dz$,

These are the poincare transformations.

→ These form the Poincare group

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Reprensentation of the Poincare group are labeled by:

1) spin → label of the Lorentz group.

2) mass

spin Examples

	spin	component	
	0	1	singlet
	1/2	2	Weyl spinor.
	1/2	4	Dirac spinor = W-spinor + W spinor.
	+1	4	vector.

Relativistic energy and momentum

Non Relativistic energy-momentum relation: $E = \frac{P^2}{2m}$

Relativistic " " " $\frac{E^2}{c^2} = \vec{P}\cdot\vec{P} + m^2c^2$

elativistic $E = \gamma mc^2$ $\vec{P} = \gamma m\vec{v}$

Energy-momentum 4-vector $P^\mu = (P^0, P^1, P^2, P^3) = \left(\frac{E}{c}, P^1, P^2, P^3\right)$.

$= \left(\frac{E}{c}, \vec{P}\right) = m\gamma(c, \vec{v})$

$\Rightarrow P_\mu P^\mu = \eta_{\mu\nu} P^\mu P^\nu = -\frac{E^2}{c^2} + \vec{P}\cdot\vec{P} = -m^2c^2 = P^2.$

$P_\mu P^\mu = -m^2c^2 \rightarrow$ Lorentz scalar (invariant)

time intervals are not Lorentz invariant.

Proper time - time measured in the moving-frame relative to itself.

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consider a particle moving along the x - axis.

consider: $-ds^2 = -c^2 dt^2 + dx^2 = -c^2 dt^2 (1-\beta^2)$.

Evaluate the interval in a frame moving with the Particle

$dx' = 0 \quad dt' = dt_p \rightarrow$ proper time.

$ds^2 = c^2 dt_p^2 \Rightarrow ds = c\, dt_p \Rightarrow \frac{ds}{c} = dt_p$ ← proper time

also $ds = c\,dt\sqrt{1-\beta^2} \Rightarrow \frac{dt}{ds} = \frac{\gamma}{c}$.

$ds \rightarrow$ Lorentz scalar $\rightarrow$ construct new Lorentz vectors.

velocity four vector: $u^\mu = c\frac{dx^\mu}{ds} = c\left(\frac{d(ct)}{ds}, \frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}\right)$.

e.g. $\frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = v_x \frac{\gamma}{c}$

$\Rightarrow \quad u^\mu = \gamma(c, v_x, v_y, v_z)$.

$\Rightarrow \quad p^\mu = m u^\mu$

any Lorentz 4-vector transforms under Lorentz transformations as x^μ,

$$p'^\mu = \Lambda^\mu{}_\nu p^\nu$$

For x-boost

$$\begin{pmatrix} E'/c \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \gamma\left(\frac{E}{c} - \beta p_x\right) \\ \gamma\left(-\frac{\beta E}{c} + p_x\right) \\ p_y \\ p_z \end{pmatrix}$$

LST.12 | 5/7/09.2 | Abingdon | sanday |

Lorentz invariance in extra dimensions

$$-ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 + dw^2 \dots$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & & \\ & +1 & & & \\ & & +1 & & \\ & & & +1 & \\ & & & & +1 \end{pmatrix}$$

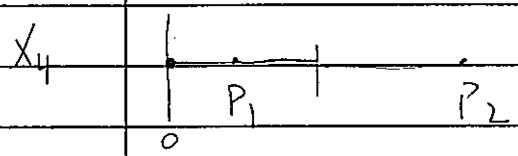
etc...

more components but otherwise the same.

compact extra dimensions.

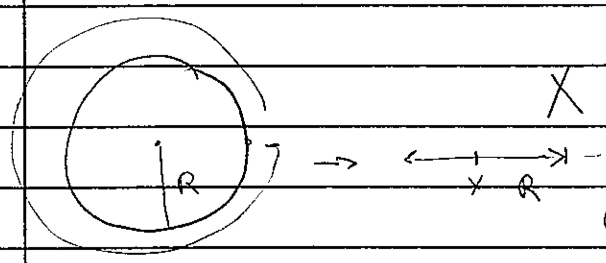
only 4 - are observed.

$\Rightarrow$ Extra dimensions are compact - curled.



$$x(P_2) = x(P_1) + 2\pi R n.$$

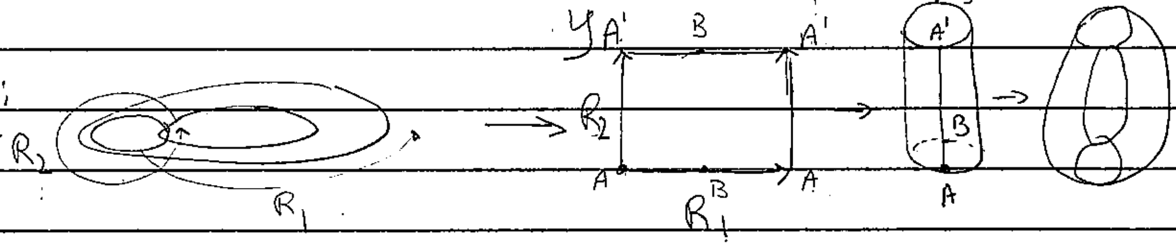
n - integer $\quad$ R - constant.



$x \sim x + 2\pi R n.$ $\leftarrow$ circle.

$0 \le x \le 2\pi R$ - fundamental domain.

Torus:



$$x \sim x + 2\pi R_1 n \qquad y \sim y + 2\pi R_2 n$$

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consider the real line $-\infty \leq X \leq \infty$

impose the condition $X \sim -X$.

points are identified under the Z_2 reflection.

The fundamental domain is now $X \geq 0$.

$X = 0$ is a fixed point.

This is the orbifold R_1 / Z_2

The orbifold is singular at the fixed point.

Quantum mechanics and the square well

Planck constant expresses the relation between the energy of a photon and its angular frequency.

$$E = \hbar \, \omega.$$

it is the fundamental constant of quantum mechanics

it appears in the basic commutation relations of quantum mechanics.

$$[x, p] = i\hbar.$$

and the limit $\lim_{\hbar \to 0}$ gives the classical limit.

From $E = \hbar \omega$ its units are

$$[E] = [M] \cdot \frac{[L]^2}{[T]^2} = [\hbar] \cdot \frac{1}{[T]} \Rightarrow [\hbar] = [M] \frac{[L^2]}{[T]} = [E][T] = [S]$$

[action] in Lagrangian mechanics