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Lecture one - The Standard Model.

1. String theory has a peculiar history. The original activity in the field sprang from the Veneziano amplitude and its relation to the observed dualities in the hadronic resonanses. The original interest in **string theory** therefore arose from its possible **application to strong interactions**. However, in the early 70's quantum field theories and in particular non-Abelian gauge theories came into prominence. **QCD**, with its basic property of "asymptotic freedom" became the prevalent theory of strong interactions. String theory therefore, to a large extent, was abandoned, and only a hand ful of hard core advocates continue to activily work on string theory. A lingering interest in string theory arose from its possible application to **gravity**. During the 1970's and early 1980's the structure of the standard Model was established experimentally. The structure of the standard Model motivated the studies of Grand Unified theories and Supersymmetry. In 1984 and 1985 two important papers appeared. The first by Green & Schwarz showed that **string theory produce an anomoly free theory of quantum gravity in 10 dimensions**. The second and which generated the wide-spread interest in string theory was the paper by Candelas, Horowitz, Strominger & Witten which showed that string theory **can reproduce the particle physics structures** that are observed in the standard model and grand unified theori. What we see is therefore that it is the standard Model it self that led to the wide spread interest in string theory.

OXFORD Lectures on String phenomenology

This will be a set of 16 lectures on particle physic phenomenology in the framework of string theory.

The goals of this course are:

motivation 1) to motivate the relevance of string theory phenomeno

tools 2) to introduce you the practical tools of string theor in phenomenological application

state of the art 3) to introduce you to the state of the art in many of the research topics that are of interest in particle phenomenology.

with this goals in mind therefore much of the discussion on the string theory background will be qualitative and the application will be discussed in more de

The plan of the course is as follows

1. The Standard Model - the most important features relevant for s
2. Grand unification and supersymmetry.
3. phenomenological aspects of grand unification.
4. General elements of string theory: bosonic string
5. fermionic, & heterotic str
6. four dimensional constructions: orbifold compactificati
7. 4D free fermionic formulation: Generalities,
8. partition functions and modular invariance.
9. Model building in the fermionic formulation
10. NAHE set & $Z_2 \times Z_2$ orbifold
11. Realistic three generation models.
12. calculation of superpotential terms.
13. flat directions & phenomenological studies
14. Beyond the Standard Model
15. string dualities
16. Non-perturbative considerations and outlook.

The other turn is that string theory, as a theory that consistently unify gravity with the gauge interactions can, in principle, shed light on how the standard model particle structures arise from a fundamental theory of quantum gravity.

historical milestones

1) 1968 – "Veneziano amplitude" Nuovo Cim. A57 (1968) 190
2) 1973 – "Asymptotic freedom" Phys. Rev. D8 (1973) 363
3) 1974 – "Grand unification" PRD 10 (1974) 275 (
   PRL 32 (1974) 438 (G.
   PRL 33 (1974) 451 (GQ
4) 1984 – "anomaly cancelation" PLB 149 (1984) 117 (GS
5) 1985 – "Vacuum configurations" NPB 258 (1985) 46 (CHSW

we see that it is the standard model itself that gave rise to the original interest in string theory. In this first lecture I will therefore review the general structure of the standard model. The emphasis in particular is on the structures that we will try to extract from string theory.

The Standard Model is based on the *general framework* of **second quantized renormalizable perturbative quantum field theories.**

The central principles that underly the Standard Model are:

gauge invariance: Local phase invariance + internal symme.

Renormalizable: divergences absorbed in to a finite number of parameters that are measured Experimentally

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$U(1)_{e.m.}$ produces the theory of QED, which was the first gauge theory and is the simplest.

Lagrangian $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} (i\gamma^\mu D_\mu - m)\psi$.

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \rightarrow$ E.M. field tensor.

$D_\mu = \partial_\mu + ie A_\mu Q \rightarrow$ covariant derivative

A^μ — spin 1 field

ψ — spin ½ field with ~~mass m and charge Q~~

This Lagrangian is invariant under.

$$\begin{cases} \psi(x) \rightarrow U(x) \psi(x) \\ A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \alpha(x), \end{cases}$$

with $U(x) = \exp(-ieQ\alpha(x))$.

and arbitrary $\alpha(x)$

For small $\alpha(x)$.

$$\psi(x) \rightarrow (1 - ieQ\alpha(x))\psi(x).$$

Local gauge invariance demands the existence of the gauge field $A^\mu(x)$ plus the interac terms.

$\mathcal{L}_{int} = -e J^\mu_{em} A_\mu$ where $J^\mu_{em} = \bar{\psi}\gamma^\mu Q$

This simple physical principle underlies the interaction of the standard model

The **Standard Model** is composed of **three sectors**:

* **Interactions** – spin-1 point gauge particles.
* **Matter** – spin-½ point particles
* **Higgs** – spin-0 point particle.

There is one additional sector that is not part of the standard model. !!!

* **Gravity** – spin 2 particle.

Interactions

The interaction sector of the standard Model is made of three gauge groups.

$$\underbrace{SU(3)_c}_{\text{Strong}} \times \underbrace{SU(2)_L}_{\text{weak}} \times \underbrace{U(1)_Y}_{\text{Weak-hypercharge}},$$

The Electromagnetic interaction arises from the spontaneous symmetry breaking.

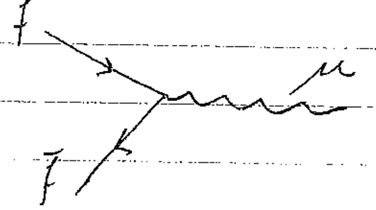
$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{e.m.}$$

After which we are left with a theory which is invariant under

$$SU(3) \times U(1)_{e.m.} \quad \leftarrow$$

gauge transformations.

in terms of Feynman Diagrams. we have a single interaction vertex.



$-i Q_f e \gamma^\mu$.

Non-Abelian gauge theories

The structure of QED (local phase invariance under U(1) phase transformations). generalizes to the case when the invariance is under non-Abelian internal symmetries.

we still write the Lagrangian as.

Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$.

we now require invariance under

$$\psi(x) \to [1 - ig\,\vec{\alpha}(x)\cdot\vec{T}]\,\psi(x)$$

where $\vec{\alpha}(x)$ is a vector of infinitesimal transformation and $\vec{T}$ are generators of the gauge group in the adjoint Reps.

FOR SU(2) $T_i = \frac{1}{2}\tau_i$ $i=1,2,3$. τ_i = Pauli matric
SU(3) $T_i = \frac{1}{2}\lambda_a \to$ Adjoint Rep of SU(3).

The generators of the group obey:

$$[T^i, T^j] = i f^{ijk} T^k$$

where f^{ijk} are the structure constants of the group.

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The Lagrangian is invariant under gauge transformations with the covariant derivative.

$$D_\mu = \partial_\mu + i g \vec{W}_\mu \cdot \vec{T}$$

where W^i_μ is a set of fields that transform in the adjoint representation.

→ g is the gauge coupling g_2, g_3

The gauge fields $\vec{W}_\mu$ transform as

$$\rightarrow \vec{W}_\mu(x) \longrightarrow \vec{W}_\mu(x) + \partial_\mu \vec{\alpha}(x) + g\, \vec{\alpha}(x) \times \vec{W}_\mu(x).$$

The Lagrangian of the gauge sector is:

$$\mathcal{L}_W = -\tfrac{1}{4} \vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu}$$

where

$$W_{i\mu\nu} = \partial_\mu W_{i\nu} - \partial_\nu W_{i\mu} - g f_{ijk} W_{j\mu} W_{k\nu}$$

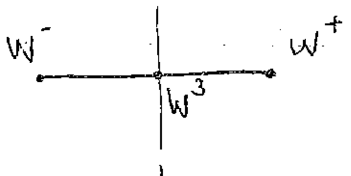
There is a new feature.

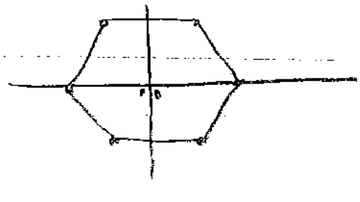
1) The gauge bosons are charged under the U(1) sub generators.

$SU(2)$: $R=1$ $W^{\pm}$, W^3

$$W^{\pm} = \frac{1}{\sqrt{2}}(W^1 + i W^2)$$

$SU(3)$: $R=2$





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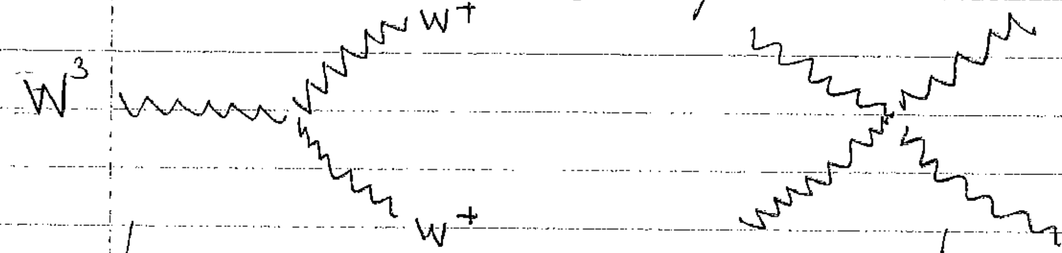
⇒ The vector bosons are charged. !!!

⇒ 2. The vector bosons self interact.



cubic quartic

Important: the mass term $M^2 A_\mu A^\mu$ is not gauge invariant.

⇒ gauge invariance demands that the vector bosons are massless.

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OXSP.8 **unbroken $SU(2)_L \times U(1)_Y$.**

The gauge sector of the standard model consist of. $SU(3)_c \times SU(2)_L \times U(1)_Y$.

The matter **states** transform as **representations** of this gauge group.

The Dirac spinor.

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \Rightarrow \quad \psi_L = \left(\frac{1-\gamma_5}{2}\right)\psi$$

$$\psi_R = \left(\frac{1+\gamma_5}{2}\right)\psi,$$

only the left-handed fields transform under $SU(2)_L$.

The mass term

$$m\bar{\psi}\,\psi = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

is not Invariant under $SU(2)_L$.

Therefore at this stage all the Standard Model fermion fields are masst

In addition to $SU(2)_L$ we introduce The weak hypercharge $U(1)_Y$.

The electric charge is given by.

$$Q_{em.} = T_3 + Y.$$

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The quantum numbers of the SM states

			T	T_3	Y	Q_{em}	$SU(3)$
ν_{e_L}	ν_{μ_L}	ν_{τ_L}	$1/2$	$+1/2$	$-1/2$	0	1
e_L	μ_L	τ_L	$1/2$	$-1/2$	$-1/2$	-1	1
u_L	c_L	t_L	$1/2$	$+1/2$	$1/6$	$2/3$	3
d_L	s_L	b_L	$1/2$	$-1/2$	$1/6$	$-1/3$	3
$u_R = \bar{u}_L$	$\bar{c}_L$	$\bar{t}_L$	0	0	$-2/3$	$-2/3$	$\bar{3}$
$d_R = \bar{d}_L$	$\bar{s}_L$	$\bar{b}_L$	0	0	$+1/3$	$+1/3$	$\bar{3}$
$e_R = \bar{e}_L$	$\bar{\mu}_L$	$\bar{\tau}_L$	0	0	$+1$	$+1$	1
$N_{R_\nu} = \bar{N}_e$	$\bar{N}_\mu$	$\bar{N}_\tau$	0	0	0	0	1

In addition I introduced the <u>right-handed neutrino</u>

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The Weinberg-angle.

Covariant derivative of $SU(2)_L \times U(1)_Y$

$$D_\mu = \partial_\mu + i g_2 \vec{W}_\mu \cdot \vec{T} + i g_Y B_\mu Y$$

$$W \cdot T = W^+ T^+ + W^- T^- + W^3 T^3$$

with $T^\pm = \frac{1}{\sqrt{2}}(T_1 \pm i T_2)$ $\quad W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp i W_2)$.

The electromagnetic current $ieQA_\mu$ is conta[ined in] the neutral term.

$$i\left(g W_{3\mu} T_3 + g' B_\mu Y\right).$$

Therefore W_3 and B must be linear combinations of A and a new vector Z

$$\begin{pmatrix} W_3 \\ B \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}.$$

where θ_w is the weak mixing angle.

Inserting, we have,

$$i g W_3 T_3 + i g' B Y = i A\left[g \sin\theta_w T_3 + g' \cos\theta_w Y\right] + i Z\left[g \cos\theta_w T_3 - g' \sin\theta\, Y\right].$$

The coefficient of A must equal $ieQ(T_3 + Y)$,

we get $\quad g = \frac{e}{\sin\theta_w} \quad g' = \frac{e}{\cos\theta_w}.$

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Hence $\frac{1}{g^2} + \frac{1}{g'^2} = \frac{1}{e^2}$ / $\tan\theta_\omega = g'/g$

$\sin^2\theta_\omega = \frac{g'^2}{g'^2+g^2}$

The Z-term in the covariant derivative can be written as

$$D_\mu^Z = i g_Z Z_\mu \left(T_3 - X_\omega Q\right)$$

with $g_Z = \frac{e}{\sin\theta_\omega \cos\theta_\omega}$ $X_\omega = \sin^2\theta_\omega$

The Higgs mechanism

up until now: $M_{W^\pm} = M_Z = 0$

In nature $M_W \simeq 80$ GeV $M_Z \sim 91$ G

The weak interactions are weak! (slow

we add a scalar field $\Phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$.

with

	T	T_3	Y	Q_{em}	$SU(3)$
ϕ^+	$+1/2$	$+1/2$	$1/2$	1	1
ϕ^0	$1/2$	$-1/2$	$+1/2$	0	1

The Lagrangian is given by

$$\mathcal{L}(\phi) = \underbrace{|D_\mu \phi|^2}_{\text{kinetic}} - \underbrace{V(|\phi|^2)}_{\text{Potential}} + \underbrace{\mathcal{L}_\phi^F}_{\text{Yukawa}}$$

The most general renormalizable form of the scalar potential.

$$V = \mu^2 |\phi|^2 + \lambda |\phi|^4$$

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minimize the potential

$$\frac{dV}{d\varphi^*} = 0 \Rightarrow \quad \varphi(\mu^2 + 2\lambda\varphi^*\varphi) = 0.$$

$$|\varphi|^2 = \frac{-\mu^2}{2\lambda}$$

if $\mu^2 > 0 \Rightarrow \varphi_{min} = 0$

if $\mu^2 < 0 \Rightarrow \varphi_{min} = \frac{|\mu|^2}{2\lambda} = v$

The field Φ has a non-vanishing vacuum expecta value in the minimum (vacuum)

We can expand the field Φ around this vacuum.

$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i\frac{\xi(x)\cdot\tau}{2v}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $\vec{\xi}$ and H have zero VeVs.

by performing a finite gauge transformation with $\alpha(x) = \xi(x)/v$ we can eliminate the dependence on $\vec{\xi}(x)$.

Therefore $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$

one real degree of freedom is left

$\rightarrow$ Higgs particle.

Lets analyze the kinetic term in the Lagrangian.

$$\mathcal{L} = |D^\mu \Phi|^2 \quad \text{with} \quad \langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

The vacuum Expectation value v produces the gauge boson masses, with.

$$\begin{cases} M_{W^\pm}^2 = \frac{1}{2} g^2 v^2 \\ M_Z^2 = \frac{1}{2} g_Z^2 v^2 \\ \frac{M_W^2}{M_Z^2} = \cos^2\theta_W \\ M_\gamma = 0 \end{cases}$$

$\Rightarrow$ the photon remains mass-

End lecture 1

Fermion masses

The fermion masses are obtained from the coupling of the fermions to the Higgs field.

$$\mathcal{L}_\phi^F = \lambda_e \left[(\bar{\nu}_e, \bar{e})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \bar{e}_R (\phi^-, \bar{\phi}^0) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right]$$

setting $\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ we get

$$\mathcal{L} = \frac{\lambda_e}{\sqrt{2}} v (\bar{e}_L e_R + \bar{e}_R e_L) \Rightarrow m_e = \frac{\lambda_e v}{\sqrt{2}}.$$

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we can rotate to the mass eigen basis by performing a bi-unitary transformation.

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

we can transform $\mathcal{M}^u$ and $\mathcal{M}^d$ to a diagonal basis

$$U_R^{-1} \mathcal{M}^u U_L = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}.$$

$$D_R^{-1} \mathcal{M}^d D_L = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}.$$

where U_R, U_L, D_R, D_L

are unitary matrices. and $m_{u,c,t}$ $m_{d,s,b}$ are the **physical quark masses**.

<u>the weak eigenstates u_1, u_2, u_3 / $d_1\ d_2\ d_3$</u> are linear combinations of <u>the mass eigenstates u, c, t / d, s, b.</u>

In the charged current sector, of the weak Interaction we have.

$$(\overline{u_1, u_2, u_3})_L \gamma^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L = (\overline{u, c, t})_L U_L^\dagger D_L \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

we will therefore in general have generation mixing of the mass eigenstates.

In this way we generate masses for the charged-leptons, up-quarks, down quark

$\lambda_e \begin{pmatrix} \nu \\ e \end{pmatrix} E h$ $\quad \lambda_u \begin{pmatrix} u \\ d \end{pmatrix} U \tilde{h}$, $\lambda_d \begin{pmatrix} u \\ d \end{pmatrix} D h$

note that it is $\tilde{h}$ which gives up quark mass terms. $\tilde{h} = (i\tau^2 h^*)$.

The fermion masses are then

$$m_{e,u,d} \sim \lambda_{e,u,d}\, v$$

quark mixing. $Q_j = \begin{pmatrix} u_j \\ d_j \end{pmatrix}_L$; u_{jR} d_{jR} ;

In the charged-lepton sector we can always rotate to a basis in which the mass eigenstates coincide with gauge eigenstates.

The most general quark Yukawa interactions.

$$\mathcal{L} = \lambda_u^{ij}\, \bar{U}_i\, (\tilde{h}^\dagger Q_j) + \lambda_d^{ij}\, \bar{d}_i\, (h^\dagger Q_j) + \ldots$$

where λ_u and λ_d are non-diagonal matrices

from the vevs of h and $\tilde{h}$ we get the mass terms for the $+2/3$ and $-1/3$ charge quarks

$(\bar{U}_1, \bar{U}_2, \bar{U}_3)_R\, \mathcal{M}_u \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}_L + \text{h.c.}$ $\quad (\bar{d}_1, \bar{d}_2, \bar{d}_3)_R\, \mathcal{M}_d \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L + \ldots$

$\mathcal{M}_u^{ij} = \frac{v}{\sqrt{2}} \lambda_u^{ij}$ $\qquad \mathcal{M}_d^{ij} = \frac{v}{\sqrt{2}} \lambda_d^{ij}$

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described by the matrix.

$$V = U_L^{\dagger} D_L \rightarrow \text{ gives physical states}$$

In the **neutral sector** however we have,

$$\left(\overline{\upsilon_1 \upsilon_2 \upsilon_L}\right)_L \gamma^\mu \begin{pmatrix} \upsilon_1 \\ \upsilon_2 \\ \upsilon_3 \end{pmatrix}_L = \left(\overline{u, c, t}\right)_L U_L^{\dagger} U_L \gamma^\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$$

unitarity $\Rightarrow$ $U_L^{\dagger} U_L = \mathbb{1} \Rightarrow$

GIM mechanism $\Rightarrow$ no Flavor changing neutral current.

Mixing matrix parametrization

For a 3×3 unitary matrix there are 9 independent parameters (9 are removed by unitarity). We can remove additional five by redifinition of the quark fields.

We are left with **4 physical parameters.**

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 c_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

where $0 \leq \theta_i \leq \pi/2$ $\quad 0 \leq \delta \leq 2\pi$

three mixing angle and one phase.

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This is the KM mixing matrix.

The measurement of its entries constitute much of the theoretical and experimental activ in particle physics

The Standard Model $\to$ data.

1st	m_e	m_u	m_d	9 masses.
2nd	m_μ	m_c	m_s	
3rd	m_τ	m_t	m_b	

CKM: θ_1, θ_2, θ_3, $\delta \Rightarrow 4$

gauge couplings: $\alpha_3(M_Z)$, $\alpha_2(M_Z)$, $\alpha_Y(M_Z)$.

$\alpha_3(M_Z)$ $\sin^2\theta_w(M_Z)$ $\alpha_{e.m.}(M_Z)$.

Higgs: (λ, v) ; (G_F, m_h)

Strong CP $\quad \bar{\theta} \leq 10^{-10} \quad \left(\frac{\theta}{16\pi^2} \operatorname{tr} F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$

This is the structure + data that we would like to derive from String theory.

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Grand unified theories

The standard model is made of three sectors

gauge $SU(3)_c \times SU(2)_L \times U(1)_Y$.

matter $3 * \left[\underset{6}{(3,2)_{1/6}} + \underset{3}{(\bar{3},1)_{-2/3}} + \underset{3}{(\bar{3},1)_{1/3}} + \underset{2}{(1,2)_{-1/2}} + \underset{1}{(1,1)_{+1}} \right] =$

generations Q_L $\quad U^c_L$ $\quad d^c_L$ $\quad L_L$ $\quad e^c_L$

Higgs. $h = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$ $\quad (1,2)_{+1/2}$. ?

Parameters g_1, g_2, g_3 $\quad m_{1..9}$ $\quad \theta_{1,2,3}, \delta$, λ, v $\quad \bar{\theta}_{QCD}$: 1

accounts for all experimental data → beautiful.

virtues: renormalizable gauge theory → no lepton, baryon # violations/n

but ad hoc → ugly

Problems: in SM

1) why 3, 2, 1
2) why Q_Y charges.
3) why 3 gen'ts
4) no unified description. g_1 g_2 g_3 different.
5) neutrino masses. ?
6) fermion masses and mixing parameters.
7) the weak scale is fixed ! by hand.
8) Higgs sector is arbitrary → # of Higgs c

we want to find a more economical description

assume $\exists\, G \supset (3,2,1)$.

Rank: $R_3 = 2$ $\quad R_2 = 1$ $\quad R_1 = 1 \Rightarrow R_T = 4$.

Dimensions $D_3 = 8$ $\quad D_2 = 3$ $\quad D_1 = 1 \Rightarrow D_T = 12$.

(The first step toward unification was done by Pati and Salam, which embedded quarks and lepton into one representation).

P.S.	$SU(4)$	$\times\ SU(2)_L$	$\times\ SU(2)_R$
Q_L	4	2	1
$(U_R, D_R)\ Q_R$	$\bar{4}$	1	2
L_L	1	2	1
$(e_R N_R)\ L_R$	1	1	2

Georgi - Glashow model:

$SU(5)$ $\quad R = 4 \quad D = 24$

$$SU(5) \Rightarrow SU(3)_c \times SU(2)_L \times U(1)_y$$

$R:\ 4 = 2 + 1 + 1$

$D:\ 24 > 8 + 3 + 1$

$$24: \ = \underset{SU(3)_c}{(8,1)_0} + \underset{SU(2)_L}{(1,3)_0} + \underset{U(1)_y}{(1,1)_0} + \underset{(X,Y)}{(3,2)_{-5/6}} + (\bar{3},$$

$X + 1/2 = -1/3$
$X - 1/2 = -4/3$
$2X = -5/3$

we obtain the 24 decomposition by multiply

$5 \cdot \bar{5}$

$$5 \times \bar{5} = 24 + 1$$

with $5 = (3,1)_{-1/3} + (1,2)_{1/2}$

$\bar{5} = (\bar{3},1)_{1/3} + (1,2)_{-1/2}$

$$5 \times \bar{5} = (8,0)_0 + (1,1)_0 + (1,3)_0 + (1,1)_0 + \underset{(X,Y)}{(3,2)_{-5/6}} + \ldots$$

$$Q_X = -\frac{1}{3} \qquad Q_Y = -\frac{4}{3}$$

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The matter representations are embedded in

$$\bar{5} = (\bar{3}, 1)_{1/3} + (1, 2)_{-1/2}$$

$$\quad d_L^c \qquad\qquad L_L$$

$$(5 \cdot 5)_A = 10 = (3, 2)_{1/6} + (\bar{3}, 1)_{-2/3} + (1, 1)_{+1}$$

$$\quad Q_L \qquad\qquad u_L^c \qquad\qquad e_L^c$$

SO(10): Adjoint Rep: $45 = D$
Rank $5 = R$

$$R(SO(10)) = R(SU(5)) + 1$$

$$4 + 1$$

There are five diagonal generators in the cartan subalgebra of SO(10).

Represent those as five boxes.

Later on we will call these boxes: compactified bosons or complex fermions.
OR we can think of them as fermions

$$\psi^1 \; \psi^2 \; \psi^3 \; \psi^4 \; \psi^5$$

The adjoint rep $\quad \psi^{i*} \psi^i |0\rangle \Rightarrow$ cartan su

$\psi^{i*} \psi^{j*} \qquad \psi^i \psi^{j*}$

$\psi^i \, \psi^j \qquad \psi^{i*} \psi^j$

$\frac{4\cdot 5}{2}$ $\pm 1 \ \pm 1$

how many do we have; $i,j = 1,5$

$i,j =$
$$5 + 4\cdot 10 = 45$$

FOR $SO(2n)$ $D = \frac{2n(2n-1)}{2}$ $R = n$

$SU(n)$ $D = n^2 - 1$ $R = n$

$SO(10) \to SU(5) \times U(1)$
$45 = 24_0 + 1_0 + 10_x + \overline{10}_{-x}$

<u>what about the matter</u>

spinorial rep. of $SO(10)$.

$$\begin{array}{ccccc} \psi^1 & \psi^2 & \psi^3 & \psi^4 & \psi^5 \\ \pm & \pm & \pm & \pm & \pm \end{array} \qquad \prod_{i=1}^{5} S_\psi = +1$$

how many possibilities.

$$\left[\binom{5}{0} + \binom{5}{2} + \binom{5}{4}\right]$$
$$16 = 1 + 10 + \bar{5}$$

So if we add a singlet all the reps
(R HN)

one generation of the Standard Model falls into a single 16 representation of $SO(10)$!!!

very, very nice!!!

End 2

OXSP 20.1

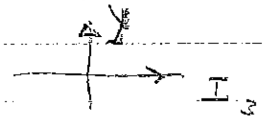
A bit of useful group theory (R. Slansky, Phys Rep. PRP 79 (1981) 1.

In Euclidean Root space

maximal set of ℓ simultaneously diagonalized generators

$\{H_i\} \rightarrow$ Cartan subalgebra $[H_i, H_j] = 0$

SU(3) : I_3 Y



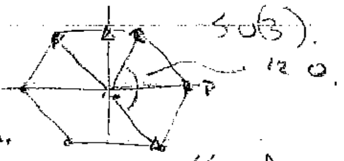
Rank = 2

These generators produce a 2-dimensional Euclidean

The remaining generators have commutation relati

$[H_i\ E_\alpha] = \alpha_i\ E_\alpha$ with Cartan weig

The E_α are represented by root vectors that are linear combinations of unit vectors pointing along the cartesian axis in an ℓ-dimensio space → The root lattice.



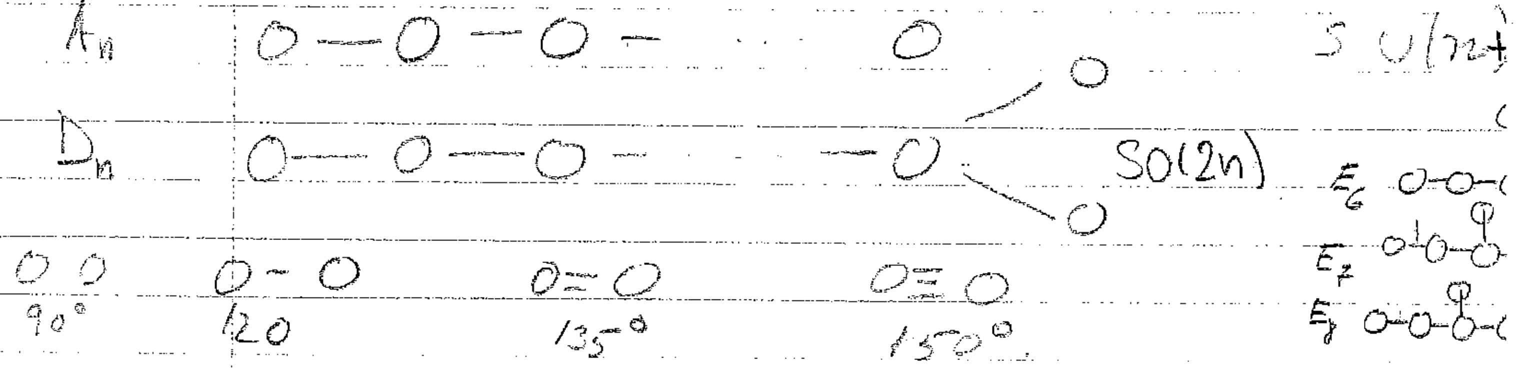
simple roots: Roots with 1) first component positive - P.R.
2) cannot be written as a sum of other P.R. with
⇒ exactly 2 - linearly independent simple roots.

The simple roots form a basis that generate the Group.

The length and angle between simple roots completly characterize any simple Lie algebra.

OXSP, 20, 2
20.2

Dynkin diagrams: classification of Lie algebra



Simply laced: all root vectors of equal length

non " " , Roots with 2-different length su
G 2

long ; short.

All representations of a group can be represented by their charges in root space → weight vecto

Ex SO(2n) $(\pm 1 \ \pm 1, \ldots ;)$ Root lattice

$(\pm\frac{1}{2} \pm\frac{1}{2} \pm\frac{1}{2} \ldots \pm\frac{1}{2})$ $\prod_i = +$ spinorial rep.

$\prod_i = -1$ antispinorial rep.

OXSP.22 | Higgs $10_H = 5 + \bar{5}$ $\rightarrow$ vectorial rep of so(10)

E_6:

Adjoint $= 78. = D$
Rank $= 6 = R.$

$E_6 \rightarrow SO(10) \times U(1)$

$78 \rightarrow 45 + 1 + 16 + \overline{16}$

$27 \rightarrow 16 + 10 + 1$

Do E_8 ~~from next page~~ first

Back to SU(5)

Fermion masses: $\bar{5}_f \, 10_f \, \bar{5}_h$
(gauge invariant combinations) $10_f \, 10_f \, 5_h$
(singlet of su(5)): G

$\langle 5 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}$

$\underset{(d^c, L)}{\bar{5}_f} \; \underset{\begin{pmatrix} Q_L \\ U^c_L \\ e^c_L \end{pmatrix}}{10_f} \; \underset{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}}{\bar{5}_h} \Rightarrow m_b = m_\tau$ at $M_{GUT}.$

In terms of SO(10)

$16_f \, 16_f \, 10_h.$

$m_t = m_b = m_\tau$ at m_{GUT}

E_6 $\quad 27 \; 27 \; 27.$

$27 \rightarrow 16 + 10 + 1 \rightarrow$ matter + Higgs. + singlet.

E_8 $\quad D = 248$
$\quad\quad R = 8$

only one Rep. Adjoint = 248

$E_8 \to SO(16)$. $\quad (\mathbb{I}\,|\,\mathbb{I}, \dots)$

$248 \to 128_{Sp.} + 120_{Ad.}$ $\quad (\tfrac{\mathbb{I}}{2} \cdots \tfrac{\mathbb{I}}{2})$

Phenomenological Aspects of Grand unified theor

gauge coupling unification

$$SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(5)$$
$$g_3 \quad g_2 \quad g_Y \to g_5.$$

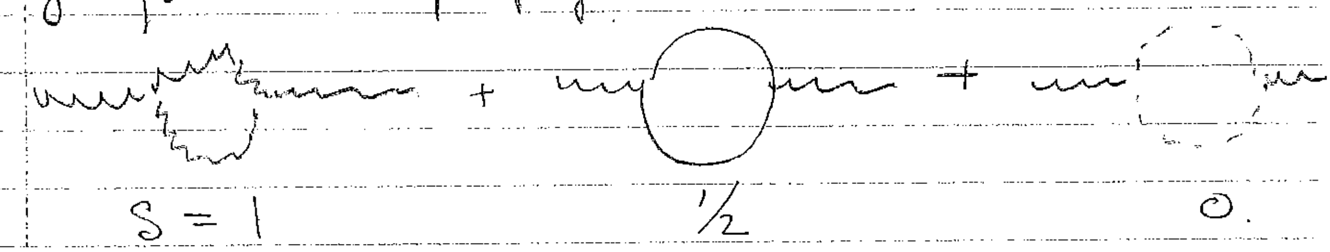
coupling constants Run !!!

Because of Renormalization we must define the coupling constants at a particular scale. However, at a different μ scale the coupling will, in general, change. This dependence of the couplings on the scale μ is governed by the Renormalization Group equations.

$$\frac{dg_n}{d(\ln\mu)} = -b_n g_n^3$$

OXSP.29

where b_n are the beta function coefficients and are obtained by calculating the radiative corrections to the gauge boson propogator



FOR SU(n)

$$b_n = \frac{1}{16\pi^2}\left[-\frac{11}{3}N + \frac{1}{3}n_f + \frac{1}{6}n_s\right]$$

n_f = number of fermion flavors.

n_s = number of scalar flavors.

→ Since we unified all the group generators into a single group. they should all have the same normalization this is fine for the NA groups SU(2), SU since the normalization of these generators is fixed by the structure functions However, the normalization of $U(1)$ y is arbitrary.

To fix its normalization we must insure the the $U(1)_Y$ generator has the same normalization as the non-Abelian generators. We take the trace over a single rep. and demand equality.

$so(5)$ 10 : $\dfrac{\mathrm{Tr}\, T_3^2}{\mathrm{Tr}(cY)^2} = \dfrac{\left(\frac{1}{4}\right)\cdot 3}{c^2\left(6\cdot\frac{1}{36} + 3\cdot\frac{4}{9} + 1\right)} = \dfrac{3/2}{c^2\left(\frac{1}{6} + \frac{8}{6} + \frac{6}{6}\right)} = \dfrac{3/2}{c^2(}$

$= \dfrac{3/2}{c^2\, 15/6} = \dfrac{18}{c^2\, 30} = \dfrac{3}{c^2 5}$

$\mathrm{tr}\, Y^2 = 5/3\ \mathrm{tr}\, T_3^2$

we must therefore define

$U(1)_{Y'} = \sqrt{\dfrac{3}{5}}\ U(1)_Y$

The Lagrangian must remain unchanged.

$i g_Y Y \rightarrow i g_{Y'} Y'$

So

$g_{Y'} = g_1 = \sqrt{\dfrac{5}{3}}\, g_Y$.

OXSP,26

recall $\sin^2\theta_w = \frac{g_Y^2}{g_2^2+g_Y^2} = \frac{\frac{3}{5}g_1^2}{g_2^2+\frac{3}{5}g_1^2}$

At the unification scale we have,

$g_1 = g_2 = g_3$.

Then, $\sin^2\theta_w = \frac{\frac{3}{5}}{8/5} = \frac{3}{8}$

hence unification predicts $\sin^2\theta_w = 3/8$ at M_{GUT}.

From the RGE's we get

$$\frac{1}{g_1^2(\mu)} = \frac{1}{g_{GUT}^2} + \frac{b_1}{2\pi}\ln\frac{\mu}{M_{GUT}}$$

$$\frac{1}{g_2^2(\mu)} = \frac{1}{g_{GUT}^2} + \frac{b_2}{2\pi}\ln\frac{\mu}{M_{GUT}}$$

$$\frac{1}{g_3^2(\mu)} = \frac{1}{g_{GUT}^2} + \frac{b_3}{2\pi}\ln\frac{\mu}{M_{GUT}}.$$

we can manipulate these eq. and get

$$\ln\left(\frac{M_X}{\mu}\right) = \frac{\pi}{11}\left[\frac{1}{\alpha(\mu)} - \frac{8}{3\alpha_s(\mu)}\right]$$

$$\sin^2\theta_w = \frac{3}{8} - \frac{55}{24\pi}\alpha(\mu)\ln\left(\frac{M_X}{\mu}\right)$$

where α, α_s are the electromagnetic and strong coupling at scale μ.

use experimental values for α, α_s.

get $\sin^2\theta_w \sim 0.21$

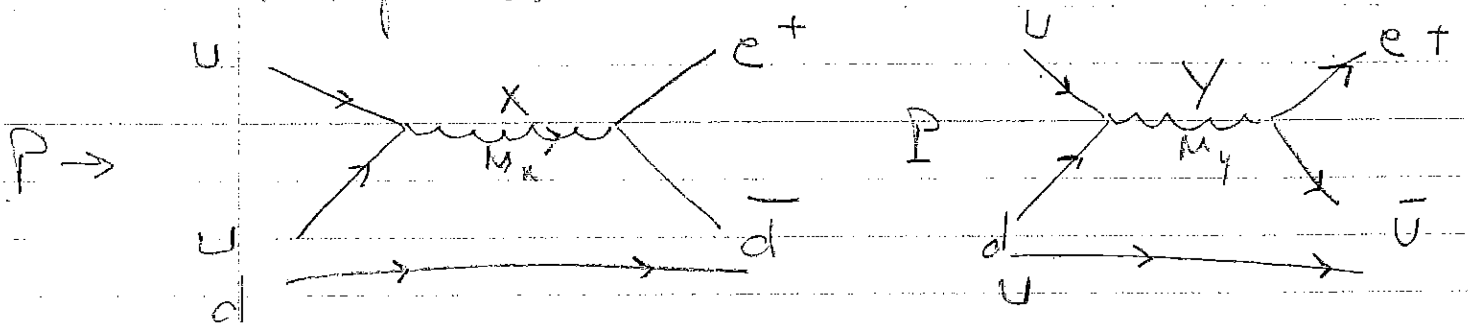
$M_X \sim 4 \cdot 10^{14}$ GeV

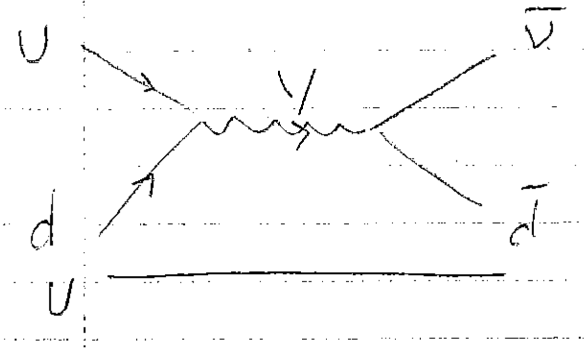
Experimentally $\sin^2\theta_w(M_Z) \sim 0.231 \longrightarrow$ amazing !!!

Proton decay

The most important consequence of GUTs is proton decay.

Proton decay is a result of embedding quarks and leptons in the same multiplets.





dominant mode

$P \longrightarrow e^+ \pi^0$

OXSP.28

what is the proton life time from the diagram.

$$\Gamma \sim \tau^{-1} \sim \frac{1}{M_X^4}$$

on dimensional grounds $\tau \propto \frac{1}{\alpha_G^2} \frac{M_X^4}{M_p^5}$

inserting $M_X \sim 10^{14}$ GeV $M_p \sim 1$ GeV.

we obtain Roughly $\tau_p \sim 10^{30}$ years

Experimentaly. $\tau_p \geq 10^{32-33}$ years.

Most extensions of SM $\to$ Proton decay.

In SM 1) Renormalizibility
2) B & L conserved global symmetries $\Big\} \Rightarrow$ no P-de

Extensions $\to$ cutoff $\to$ non-renormalizable interacti

$\Rightarrow$ most extensions $\to$ Proton decay.

$\Rightarrow M_X \to \uparrow \geq 10^{15-16}$ GeV

solutions are often ad-hoc !!! (e.g. symmet.

$M_P \sim 10^{19}$ GeV $\to$ natural gravity scale

End lecture 3

OXS P. 29/

(18)

Neutrino masses - see saw mechanism.

In SM $m_\nu \equiv 0$.

$m_\nu LL \rightarrow$ not gauge invaria

add a singlet. $N_R \rightarrow \lambda L N_R h^* \rightarrow$ Dirac mass

$M_X N_R N_R \rightarrow$ N_R majorana mass term.

$$M_\nu = \begin{array}{cc} \nu_L & N_R \end{array}$$
$$\begin{pmatrix} 0 & \lambda v \\ \lambda v & M_X \end{pmatrix}.$$

Eigenvalues $\frac{(\lambda v)^2}{M_X}$; M_X.

$\Rightarrow m_{\nu_L} \sim \frac{m_t^2}{m_X}$ OR $\frac{m_e^2}{M_X}$

$m_{\nu_L} \sim \frac{(100 \text{ GeV})^2}{10^{15} \text{ GeV}} \sim \frac{10^4}{10^{15}} \sim 10^{-11}$ GeV

$\sim 10^{-2}$ eV

m_{ν_L} in good agreement with recent Super K data.

Neutrino sector $\rightarrow$ intensive activity.

another indication for large M_X

OXSP, 301

1/5/01

Running masses

we have $\lambda_f \bar{5}_f \, 10_f \, \bar{5}_h \Rightarrow$ fermion mass

$$\left[(\bar{3},1)+(1,2)\right]\left[(3,2)+(\bar{3},1)+(1,1)\right]\left[(\bar{3},1)+(1,2)\right] \Rightarrow$$

$$\lambda_f \Big[\underbrace{\underset{d^c}{(\bar{3},1)}\ \underset{Q}{(3,2)}\ \underset{h}{(1,2)}}_{m_b} + \underset{L}{(1,2)}\ \underset{e^c}{(1,1)}\ \underset{h}{(1,2)} \Big]$$

$$m_b \quad = \quad m_\tau \quad \text{at} \quad M_{GUT}$$

$\lambda_b = \lambda_\tau$ at M_{GUT} is a consequence of the $SU(5)$ group structure.

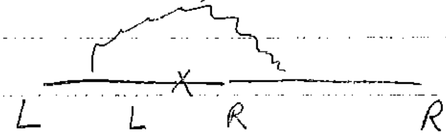
(similar relations for lighter generations)

$$m_b = (\lambda_b \cdot v) \qquad m_\tau = \lambda_\tau \cdot v$$

$$\Rightarrow \frac{m_b}{m_\tau} = \frac{\lambda_b}{\lambda_\tau}$$

$g, B \rightarrow$ no W

couplings evolve with energy



$$\frac{d}{d \ln \mu^2}\left[\ln \frac{m_b(\mu)}{m_\tau(\mu)}\right] = -\frac{\alpha_3(\mu)}{\pi} + \frac{\alpha_1(\mu)}{4\pi}$$

solution

$$\frac{m_b(\mu_1)}{m_\tau(\mu_1)} = \frac{m_b(\mu_2)}{m_\tau(\mu_2)} \left[\frac{\alpha_3(\mu_1)}{\alpha_3(\mu_2)}\right]^{\frac{1}{4\pi^2 b_3}} \left[\frac{\alpha_1(\mu_1)}{\alpha_1(\mu_2)}\right]$$

$$\mu_1 = M_X \longrightarrow \mu_2 \simeq 2\, m_b$$

$$\left(\frac{m_b}{m_\tau}\right)_{\mu_1} = 1 \longrightarrow \left(\frac{m_b}{m_\tau}\right)_{\mu_2} \simeq 2.5$$

Ellis, Chanowitz

Buras, Ellis, G.,

Experimentally: $\left(\frac{m_b}{m_\tau}\right) \sim 2.65$

OXSP.31 | <u>symmetry breaking</u>.

$$SU(5) \xrightarrow{\langle H \rangle} SU(3) \times SU(2) \times U(1)_Y \xrightarrow{\langle \phi \rangle} SUB$$

$H = 24$ $\qquad\qquad \phi = \bar{5}$

$$V(H, \phi) = V(H) + V(\phi) + \lambda_4 (\mathrm{tr}\, H^2)\, \phi^+ \phi + \lambda_5\, \phi^+ H^2$$

with

$$V(H) = -m_1^2\, \mathrm{tr}\, H^2 + \lambda_1 (\mathrm{tr}\, H^2)^2 + \lambda_2 (\mathrm{tr}\, H^4)$$

$$V(\phi) = -m_2^2 (\phi^+ \phi) + \lambda_3 (\phi^+ \phi)^2$$

minimize the potential. $\lambda_2 > 0$ $\quad \lambda_1 > -\frac{7}{30}$

$V(H)$, has an extremum at.

$$\langle H \rangle = v_1 \begin{bmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{bmatrix}.$$

$$v_1^2 = \frac{m_1^2}{60 \lambda_1 + 14 \lambda_2}.$$

$\langle H \rangle$ commutes with G of $SU(3) \times SU(2) \times U(1)$

and $M_X = M_Y = \sqrt{\frac{25}{2}}\, g\, v_1$

symmetry breaking → cumbersome → string theory
→ [illegible].

OXSP32
1/5/01 <u>The hierarchy problem.</u>

1) First incarnation → GUTs.

two scales M_W ; M_X

10^2 GeV 10^{15} GeV

Higgs: ⇒ $\bar{5} = [(\bar{3},1) + (1,2).]$

The triplet mediate proton decay from dimension 6 operator $QQQL \sim \frac{1}{M_H^2} QQQL$.

⇒ $M_3 \sim M_X$ but $M_2 \sim M_W$.

⇒ <u>doublet-triplet splitting problem.</u> → <u>solution cumbers</u>
$V(\phi, H)$. fine tuning of param
large Reps.

(strings can help!!!)

2) <u>without GUTS</u>
Higgs sector: M_W $SU(3)\times SU(2)\times U(1)$; M_{planck} G.

ⓐ "natural aspect": how to generate M_W from M
without fine tuning

OX SP. 33
1/5/01

(b) "technical aspect"

The Higgs sector is an essential component of the Standard Model.

t'hooft-Veltman - renormalizability of NA Yang-Mills

The Higgs mass term.

$$\Delta \mathcal{L} = -\mu^2 \phi^\dagger \phi + \lambda^2 (\phi^\dagger \phi)^2$$

respects all SM symmetries.

(a) why $\mu^2 \sim M_W$ and not M_P.

(b) if we set $\mu^2 = 0$ at tree level.

The Higgs mass squared is corrected by radiative corrections

$$-i m^2 = -i\lambda^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} = -i \frac{\lambda^2}{16\pi^2} \Lambda^2$$

where Λ is the cut off scale where the SM breaks down.

$\Rightarrow$ contribution of radiative correction to the Higgs boson

OX SP.34
1/5/01

mass is non-zero, divergent and positive.

$\Rightarrow$ fine tuning between 1^{st}-order - 2^{nd}-order

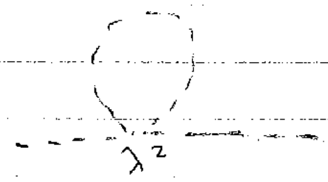
to obtain a negative mass term !!!

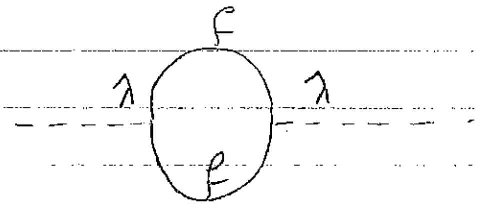
$\Rightarrow$ unnatural fine tuning.

Technicolor: Higgs $\to$ strong sector at $O(1\,\text{TeV})$ / Large extra dimension

<u>Supersymmetry</u> : for every spin-0 state -

a a spin 1/2 - superpartner





$$\sim + \lambda^2 \int \frac{d^4k}{k^2} \qquad - \lambda^2 \int \frac{d^4k}{k^2}$$

The difference between the fermion and boson loop.

$$\delta m^2 \sim (n_B - n_f)\lambda^2\Lambda^2 + (m_B^2 - m_f^2)$$

supersymmetry $n_B = n_f$

$$\Rightarrow |\delta m^2| \propto |(m_B^2 - m_f^2)| \leq 1\ \text{TeV}.$$

corrections are under control

$\Rightarrow$ set it and forget it.

Additional + of supersymmetry

1) local => spin 3/2 → spin 2 => Gra

2) gauge coupling unification in MSSM =>
in good agreement with data.

=> − of supersymmetry.

spectrum is doubled →
3) electro weak symmetry breaking with large λ_t. $\frac{dm^2}{dt^2} \propto$ (
Where in the world is supersymmetry.

Essential Supersymmetry.

we won't go into the details of supersymmetry.
which will require a dedicated course in
it self

Rather we will discuss the aspects that
are important from the perspective of
Superstring phenomenology.

PRIMARY. 1) mass terms → superpotential.

2) Supersymmetry breaking

end of lectu

OXSP. 36

global supersymmetry

so far: two kind of symmetries in particle physi

1) Poincare group - translation, rotation boost.

$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu}$

10 generators $3+3+4$.

2 casimirs : 1) m^2 2) spin ⇝ statistics.

all reps are labeled by their masses & spin.

2) internal symmetries G.

Coleman - Mandula theorem. [poincare, G] = 0.

cannot find bigger group that contains both Poincare & G as subgroups.

⇒ internal symmetries cannot relate different spin

Exceptions: Graded Lie algebras. -

generators obey anticommutation relations

OX SP. 37
1/5/01 $\Rightarrow$ Internal $[T_a, T_b] = i f_{abc} T_c$

Poincare $[P_\mu, P_\nu] = 0$

$[M_{\mu\nu}, M_{\rho\sigma}] = -i(\eta_{\mu\sigma} M_{\nu\rho} + \ldots)$

$[M_{\mu\nu}, P_\rho] = -i g_{\mu\rho} P_\nu + i g_{\nu\rho} P_\mu$

graded Lie algebra: N=1

$Q_\alpha |\text{boson}\rangle = |\text{fermion}\rangle$

$Q_\alpha |\text{fermion}\rangle = |\text{boson}\rangle$

GL algebra $[P_\mu, Q_\alpha] = 0$ $\rightarrow$ translation invaria

$[M_{\mu\nu}, Q_\alpha] = -(\sigma_{\mu\nu} Q)_\alpha$ $\rightarrow$ Q spinor of P. G.

$\{Q_\alpha, \bar{Q}_\beta\} = 2\gamma^\mu_{\alpha\beta} P_\mu$

$\sigma_{\mu\nu} = \frac{1}{4} i [\gamma_\mu, \gamma_\nu]$

Extended supersymmetry: $i = 2, 3, 4 \ldots N$

$Q^i_\alpha \rightarrow N$ susy generat

$\{Q^i_\alpha, \bar{Q}^j_\beta\} = 2\delta^{ij} \sigma^\mu_{\alpha\beta} P_\mu + \varepsilon_{\alpha\beta} Z$

(4 component notation). central charges.

OXSP. 38

1/5/01 → 7/5/01 (corrected)

In the string model we will identify a supersymmetry generator with a sector that takes bosons ⟷ fermion.

we will count the number of supersymmetries by the # of spin 3/2 states in the spectrum.

$N=4 \rightarrow N=1 \rightarrow$ only $N=1$ allows chiral Rep

Representations in superspace: $\underset{\text{vector}}{X^\mu}$, $\underset{\text{spinor}_L}{\theta_\alpha}$, $\underset{\text{spinor}}{\bar{\theta}_{\dot\alpha}}$

under Lorentz group: $\left(\frac{1}{2},\frac{1}{2}\right)$ $\left(\frac{1}{2},0\right)$ $\left(0,\frac{1}{2}\right)$

$[X^\mu, X^\nu]=0 \quad \{\theta,\bar\theta\}$

N=1 chiral rep. : matter / Higgs.

$$\begin{pmatrix} f \\ \tilde f \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} \rightarrow \text{depend on } \theta \text{ not } \bar\theta$$

Example: $\begin{pmatrix} \nu \\ e \end{pmatrix} \rightarrow SU(2) \begin{pmatrix} \nu_1 & \nu_2 & \tilde\nu^R & \tilde\nu^I \\ e_1 & e_2 & \tilde e^R & \tilde e^I \end{pmatrix}$ (spin, SUSY)

Expand in superspace. $\{\theta_1, \theta_1\} = 0 \Rightarrow \theta_1^2 =$

Define : $P^\mu = -i\partial_\mu \rightarrow$ Q.M. momentum opera

Similarly $D_\alpha = i\left[\frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\bar\theta)\partial_\mu\right]$

chiral: $\bar D_{\dot\alpha}\Phi = 0 \Rightarrow \Phi(X^\mu, \theta) \rightarrow$ ch. sup.

$D_\alpha \bar\Phi^* = 0 \Rightarrow \Phi^*(X^\mu, \bar\theta)$

OXSP. 29
1/5/01 → 7/5/01

$$\Phi(x,\theta) = \underbrace{A(x^\mu)}_{\text{complex scalar field}} + \underbrace{\sqrt{2}\,\theta\,\psi(x^\mu)}_{\text{Left-handed Weyl fermion}} + \underbrace{\theta\theta\,F}_{\text{Auxiliary field}}$$

Vector multiplet : gauge: $V = V^+$

g (spin 1)
$\tilde{g}$ (spin ½)

Wess-Zumino gauge $V = \Phi\,\Phi^* \rightarrow (\frac{1}{2},0)\otimes($

$$V_{WZ} = (\theta\sigma^\mu\bar{\theta})V_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}$$

gauge boson (V_μ) gaugino ($\bar{\lambda}$) auxili[ary] field.

graviton / gravition G : $\begin{pmatrix} 2 \\ 3/2 \end{pmatrix}$

N=2

hyper-multiplet. $\begin{pmatrix} & \psi_{1/2} & \\ \phi & & \tilde{\phi}^+ \\ & \psi^+_{\tilde{q}} & \end{pmatrix}$

Vector $\begin{pmatrix} & A^\mu & \\ \lambda & & \psi \\ & \phi & \end{pmatrix}$

only N=1 gives chiral matter !!!

OXSP. 40

1/5/01 → 7/5/01

Supersymmetric Lagrangian (Wess & Ba

I will only discuss the main points relevant for the string models:

string: two aspects:

1) superpotential
2) SUSY breaking.

1) superpotential → matter sector interactions → chir super

a) chiral superfield: (eg. $\binom{\nu}{e}$ SUSY, Loren, gaug.

gauge trans.: $\phi \to e^{ig\Lambda_\alpha T_\alpha}\phi$ $\qquad \Lambda_\alpha T_\alpha = \Lambda$

b) $\phi^+ e^{gV}\phi$ is gauge invariant if $e^{gV} \to e^{i\Lambda^+} e^{-i\Lambda}$

c) Define $W_\alpha = \frac{1}{4} i \bar{D}\bar{D}\left[e^{-gV} D_\alpha e^{gV}\right]$

This term contains fi strength, since D_α also contai space deriva

W_α is a chiral multiplet since another $\bar{D}$ acting on W_α gives zero.

The most general $N=1$ renormalizable gauge invariant Lagrangi

$$\mathcal{L} = \left[\frac{1}{4g^2}(\mathrm{tr}\, W^\alpha W_\alpha)_{\theta\theta} + h.c.\right] + \phi^+ e^{gV}\phi\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \left[f(\Phi\right.$$

analytic func

OXSP. 41

7/5/01

(24)

$f(\Phi) \to$ super potential.

$$f(\Phi_i) = C_i \phi_i + \mu_{ij} \phi_i \phi_j + \frac{\lambda_{ijk}}{3!} \phi_i \phi_j \phi_k$$

$\to$ 1. must be gauge invariant.

2. higher term $\to$ non-renormalizable.

In terms of components

$\mathrm{tr}(W^\alpha W_\alpha)$ term gives (gauge kinetic term

$$-\frac{1}{4}(G^\alpha_{mn})^2 + \left(\frac{i}{2} \bar{\lambda}^\alpha \bar{\sigma}^m D_m \lambda^\alpha + h.c.\right)$$

covariant Derivative.

$\phi^+ e^{gV} \phi$ term gives.

$$\underbrace{(D_m^+ A^i)(D^m A_i)}_{\text{Scalar Kinetic term}} + \underbrace{\left(\frac{i}{2} \bar{\psi}_i \bar{\sigma}^m D_m \psi_i + h.c.\right.}_{\text{Weyl spinor}}$$

from $\underbrace{f(\phi_i)_{\theta\theta} + h.c.}$ terms :

$$-\frac{1}{2} \frac{\partial^2 f}{\partial A_i \partial A_j} \psi_i \psi_j + i\sqrt{2} \frac{\partial D_\alpha}{\partial A_i} \psi_i \lambda^\alpha + h.c. - V(\ldots$$

fermion of chiral multiplet

fermion of vector mu[ltiplet]

mix

from $\frac{\partial \mathcal{L}}{\partial F} = 0 \Rightarrow F = \frac{\partial W}{\partial A_i}$

from gauge Lagrangian $\to \frac{1}{2} D^2$

from chiral kinetic term $\phi D^\mu \phi \to \phi T^\alpha \phi$

$$V = F_i F_i^+ + D_\alpha D^{\alpha +} \leftarrow \text{scalar potent[ial]}$$

$$F_i = \frac{\partial f}{\partial A_i} \qquad D_\alpha = \sum_i A_i g T^\alpha A_i^+$$

OX SP. 42 Observations: 1) All particles within a super multiplet have the same mass.

2) $V \geqslant 0$.

$\Rightarrow$ ground state is zero $F_i = 0$ $D_\alpha = 0$

$\Rightarrow$ SUSY is unbroken

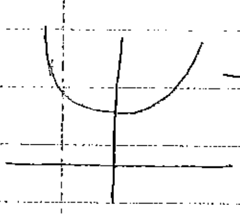
$V = 0 \Rightarrow$ SUSY is unbroken $V_0 > 0 \Rightarrow$ SUSY is brok

Also follows from the Algebra

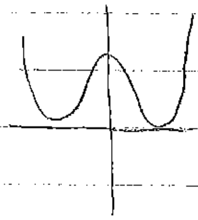
$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\gamma^\mu_{\alpha\dot\alpha} P_\mu \Rightarrow H = \frac{1}{4}\sum_{\alpha=1}^{2} Q_\alpha^2 \geqslant 0$$

$\Rightarrow Q_\alpha|0\rangle = 0$ $\bar{Q}_\alpha|0\rangle = 0$

F_i ; $D_\alpha \neq 0 \Rightarrow V_0 > 0 \Rightarrow$ SUSY is brok



$\rightarrow$ SUSY broken
gauge unbroken



SUSY unbroken
gauge broken

$$V = \sum_i |F_i|^2 + \sum_\alpha |D_\alpha|^2 \geqslant 0.$$

SUSY breaking $\exists$ $F_i \neq 0$ and/or $D_\alpha \neq 0$

① F- breaking. A, B, C chiral superfields.

$$f = m AB + \lambda C (B^2 - \mu^2)$$

$$\frac{\partial f}{\partial A} = 0 \Rightarrow B = 0$$

$$\frac{\partial f}{\partial C} = \lambda (B^2 - \mu^2) = -\lambda\mu^2 \neq 0 \Rightarrow \text{SUSY broke}$$

OXS p. 43

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D - breaking.

$D_\alpha = \sum A^{i\dagger} T^\alpha A_i$

for $U(1) \Rightarrow T^\alpha = Q^\alpha$

Fayet - Iliopoulos term. - Linear term.

$\mathcal{L}_{FI} = \xi [V_1]_{\theta\theta\bar{\theta}\bar{\theta}} \rightarrow$ D-component of $U(1)$ vector field not renormalized, gauge and SU invariant.

string $\rightarrow$ $\xi \propto \mathrm{tr}\, Q^i \neq 0$ over the spectrum. $\rightarrow$ anomalous U

Fayet-Iliopoulos D-term is generated in many string models

$\Rightarrow D_A = \xi + \sum Q_i |\phi^i|^2 = 0 \rightarrow$ for SUSY va

SUSY phenomenology

Particle spectrum is doubled.

Particle	spin		sparticle	spin
$q_{L,R}$	$1/2$		$\tilde{q}_{L,R}$	0
$\ell_{L,R}$	$1/2$		$\tilde{\ell}_{L,R}$	0
Photon γ	+1	gaugino (majorana) because A^μ is in adjoint-real Rep.	$\tilde{\gamma}$	$1/2$
gluon g			$\tilde{g}$	$1/2$
W			$\tilde{W}$	$1/2$
Z			$\tilde{Z}$	$1/2$
Higgs	0	Higgsino	$\tilde{H}$	$1/2$

two Higgs doublets h^* not allowed.

OXSP. 44

Recall $d Q h + U Q \cancel{h^*}$ → not analytic

Instead $d Q h_1 + U Q h_2$

R-parity $(-1)^{2J} (-1)^{3(B-L)}$

B - baryon # L - lepton # J - spin.

MSSM

gauge $SU(3) \times SU(2) \times U(1)$ vector superf.

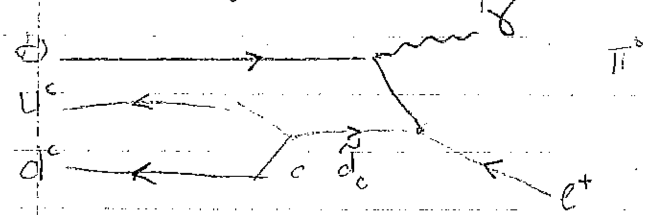
matter $U_L^c \, d_L^c \, Q_L \, L, \, e_L^c$ → chiral "

Higgs h_1 , h_2

$$W = \underbrace{\lambda_1^{ij} d_L^{c\,i} Q^j h_1 + \lambda_2^{ij} U_L^{c\,i} Q^j h_2 + \lambda_3^{ij} e_i L_j h}_{\text{fermion masses.}}$$

$+ \mu h_1 h_2$ → Higgs mass-term

R - parity violating term → Violate baryon/lepton #



$\lambda_4 d^c d^c U^c + \lambda_5 Q L d^c$

instanteneous proton dec

$\lambda_6 LLE + \lambda_7 Lh.$

End lecture 5.

R - parity forbids $\lambda_{4,5,6,7}$

→ LSP is stable → Dark matter cand

SUSY phenomenology

consider $f = \lambda \Phi \Phi \Phi$ in the Lagrangian

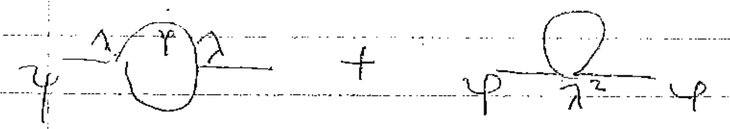
$\Phi = \varphi + \theta \psi + \theta\theta F$

$m_\varphi = m_\psi$

1) yukawa $\rightarrow \frac{\partial^2 f}{\partial \varphi^2} \psi\psi = \lambda \varphi \psi \psi$

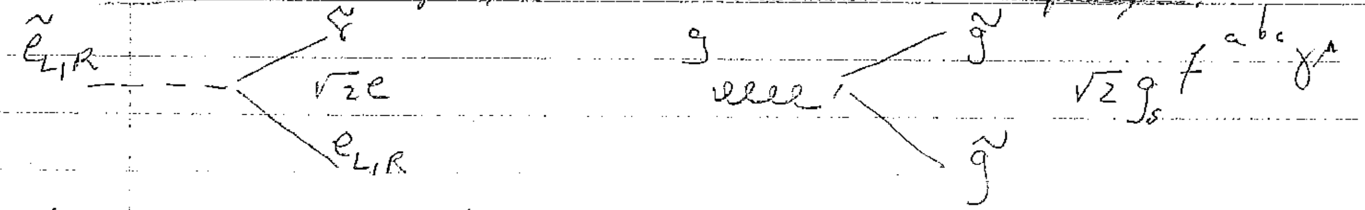
2) $V = F^* F = \left|\frac{\partial f}{\partial \varphi}\right|^2 = \lambda^2 (\varphi\varphi^*)^2$

from here we get



$\rightarrow$ boson–fermion.

from the Lagrangian we extract couplings



etc. summarized in Phys. Rep. of Haber & Kane 198?

R-parity, $\rightarrow$ LSP conserved

SUSY mass spectrum.

m_0, $m_{1/2}$, A_0, B $\rightarrow$ soft susy breaking parameters.

$m^2 = m_0^2 + \alpha_i^2 m_{1/2}^2 \rightarrow$ if yukawas are neglected.

local supersymmetry $\rightarrow$ supergravity.
supergravity breaking $\rightarrow$ global supersymmetry + soft breaking

OXSP. 96

$(\phi^i, \psi^i, F^i) \to$ chiral superfield

Supergravity

$V(\phi, \phi^*) = e^G [G_i (G^{-1})^i{}_j G^j - 3] + \frac{1}{g^2} |D|^2 \to$ scalar potential

F^2 of global SUSY

K.E. $G_i^j \partial_\mu \phi^i \partial^\mu \phi_j^*$

$G(\phi^i, \phi_i^*) \to$ Kähler potential.

$G = \frac{\varphi_i \varphi_i^*}{M_p^2} + \ln \left| \frac{f}{M_p^3} \right|^2$

Planck mass.

$m^{3/2} = e^{G/2}$

can have broken SUSY with $V_{min} = 0$.

special case - no-scale models.

$G_0 = -3 \ln (f(z) + f^*(z))$

$V(z, z^*) = 0 \quad \forall z.$

OSU- lectures on String Phenomenology

motivation: - First and Formost, why bother?

First I would like to motivate why this is an interesting and important avenue for research.

At present renormalizable second quantized gauge theories are the tools to model observations at the highest accesible energies. The successful model is the Standard Model of elementary particles. The structure of the standard Model tells us something about its origin.

Spectrum		$SU(3)$	$SU(2)$	$U(1)_Y$
2 component fields	e_L^c	1	1	$+1$
	L	1	2	$+1/2$
	Q	3	2	$1/6$
	$\bar{U}$	$\bar{3}$	1	$-\frac{2}{3}$
	D	$\bar{3}$	1	$+1/3$
Add	N_L^c	1	1	0

These fit into 16 of $SO(10)$

3 gen $\to$ $3 \cdot 16$ of $SO(10)$

$$SU(3) \times SU(2) \times U(1)_Y \to SO(10)$$

amazing coincidense !!!

Accident ?!

However $M_U \sim 10^{16-17}$ GeV! why?

slow evolution

$\sin^2\theta_W$, m_b/m_τ, $\tau_p \gtrsim 10^{32}$ years $m_\nu \ll m_{e,u,d}$

OSU.2

$\underbrace{\sin^2\theta_W \quad , \quad m_b/m_\tau}_{\text{slow evolution}}$ $\qquad \tau_p \gtrsim 10^{32}$ years $\qquad m_\nu << m_{e,u,d}$

$M_U \to M_P$ $\qquad m_\nu \sim \frac{1}{M_U} M^2$

high scale suppression. $\qquad$ see-saw.

All this is well known.

However,
$$M_U \sim (M_P) \cdot 10^{-(1-2)}$$

Gravity still not included.

Gravitational effect may start playing an important role.

Example: SUSY.

D=4 operators. $\lambda_1 Q L D \quad + \lambda_2 U D D$

mediate rapid proton decay.

$$(\lambda_1 \cdot \lambda_2) \lesssim 10^{-26}$$

in MSSM R-parity $\quad \lambda_1 = \lambda_2 = 0 \quad \to$ global symm.

but global symmetries $\to$ violated by quantum gravity

Gravity is important.

SUPERSTRING: consistent unification of gravity with gauge interactions.

$\Rightarrow$ The aim is twofold $\to$ Unification: Gravity + Gauge interactions

S.M. (standard model) $\qquad$ String dynamics

OSU3

where should we look in string theory ?
Answer not known → solve string theory ?

However → Highest symmetry
minimal embedding of S. M.

S.M. → SO(10)

but SO(10) broken at M_{string}

construct string models that partialy realize the SO(10) structure

$SO(10) \rightarrow SU(3) \times SU(2) \times U(1)_Y \times U(1)'$

but $(e, u, d, N, Q, L) \rightarrow 16$ of $SO(10)$

Highest symmetry → freefermionic construction ?

in 4d the maximal gauge symmetry is realized.

I will start by sketching the tools of the free fermionic construction and how they are derived.

Aim: Using the free fermionic model building rules construct a realistic superstring model.

Remark: You often here the statement that there are infinite number of string vacua and we have no way of selecting among them.
This statement is partialy true, but in my view is somewhat academic

OSU.4

it is of course true that we would like to know how the vacuum is selected dynamically in string theory. That's everyone's eventual goal. The question is how to go about it. In my view is that we have to use the abundance of low energy data as a guide to selecting phenomenological promising string vacua.

Our aim is therefore to construct string vacua which are compatible with many of the phenomenological constraints imposed by the low energy data.

At present the most advanced models are those constructed in the free fermionic formulation that I will describe in these lectures.

The list of phenomenological constraints include

1. gauge group $\longrightarrow SU(3)\times SU(2)\times U(1)^2 \times$ hidden.
2. three chiral generations.
3. $\sin^2\theta_W(M_Z) \Rightarrow U(1)_Y \in SO(10) \rightarrow$ excludes all orbifold models but the $Z_2\times Z_2$
4. $\tau_P \geq 10^{32}$ years.
5. light left handed neutrinos $\nu_e, \nu_\mu, \nu_\tau \lesssim$ 1-10
6. Higgs doublets + potentially realistic Yukawa couplings.
7. $N=1$ SUSY ($N=0$).
8. No FCNC.
9. Family mixing + weak CP violation.
10. no strong CP violation.

The string models, constructed to date, that can potentially satisfy all of these criteria are the free fermionic models.

OSU.5

Plan of the lectures

The aim of the lectures is to describe the **construction of the fermionic models**. The focus of the lectures will therefore be on the construction of these models rather than on the preliminary construction of the superstring theories which in itself requires a two semester course.

However, the construction of superstring theories have cumulated in a set of rules, derived by Antoniadis, Bachas and Kounnas and by Kawai, Lewellen and Tye. This set of rules is our tool for constructing the realistic free fermionic models. To fully understand the derivation of these rules and the full theoretical structure behind them will require more than the short time of my lectures. Therefore, I will try to give the flavor of the derivation of these rules rather than a full derivation, and the main part of the lectures will be devoted to application of these rules in the construction of the fermionic models.

The Plan is therefore

1) The ABK & KLT free fermionic model build
2) construction of consistent vacua.
3) construction of realistic vacua - relation with orbi
4) derivation of the superpotential
5) Phenomenology and future directions.

OSU.6

metadology

The ABK(KLT construction

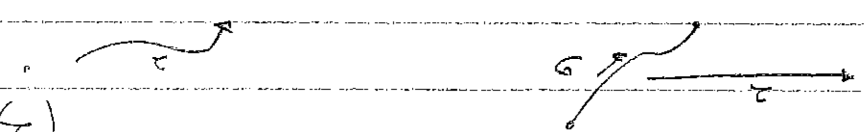
String model building follows the following methadology.

1. construct string vacua, consistent with the string consistency cr
2. extract the massless spectrum.
   for some purposes the massive spectrum is also of interes and can be extracted similarly.
3. analyze the cubic level + higher order terms in the superpotential.
4. Study flat F and D directions, while imposin phenomenological criteria.
5. Study the stringy characteristics that fix the phenomenological properties of the models.
   $\Rightarrow$ Extract general properties.

1. construction of consistent string vacua in FFF.

Let me briefly remind you the basics on strings.

Strings are one dimensional objects (rather than 0-dimensional objects) propogating in space and time.



$X(\tau)$ $\qquad\qquad\qquad\qquad$ $X(\tau, \sigma)$

world-line

$$S = -m \int ds$$
$$= -m \int d\tau \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}$$
$$= \frac{1}{2}\int d\tau \left(\frac{1}{e}\dot{x}^2 - e m^2\right)$$

$$S = -T \int d\sigma d\tau \sqrt{-\det\left|\frac{\partial X^\mu}{\partial \sigma^\alpha}\frac{\partial X^\nu}{\partial \sigma^\beta}\eta_{\mu\nu}\right|}$$
$$= -T \int d^2\sigma \left[(\dot{X}\cdot X')^2 - (\dot{X}^2 X'^2)\right]$$

where $\dot{X} = \frac{\partial X}{\partial \tau}$ $X' = \frac{\partial X}{\partial \sigma}$

OR in a linear form

$$S = \frac{1}{4\pi\alpha'} \int_{\Lambda_g} d^2\sigma \sqrt{-\det g_{\alpha\beta}}\; g^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

WS-metric — flat minkows metric.

$g_{\alpha\beta} \to$ auxiliary field $\to$ integrate out $\to$ NG-action.

$$S = -\frac{1}{2\pi\alpha'} \text{(area of the WS)}.$$

Constraints on -S-

1) a) Reparametrization invariance $\tau \to \tilde{\tau}(\tau)$

$$\left.\begin{matrix}\sigma \\ \tau\end{matrix}\right\} \to \begin{matrix}\tilde{\sigma}(\sigma,\tau) \\ \tilde{\tau}(\sigma,\tau)\end{matrix}$$

b) Weyl rescaling.

$$g_{\alpha\beta}(\sigma,\tau) \to e^{f(\sigma,\tau)} \cdot g_{\alpha\beta}(\sigma,\tau).$$

we can therefore fix three independent components of $g_{\alpha\beta}$.

$\Rightarrow$ we can reduce the 2d metric $g_{\alpha\beta} \to \eta_{\alpha\beta}$.

if we go over to complex coordinates.

$$z = \tau + i\sigma \qquad \bar{z} = \tau - i\sigma$$

we can write:

$$g_{zz} = g_{\bar{z}\bar{z}} = 0 \qquad g_{z\bar{z}} = g_{\bar{z}z} = \frac{1}{2} e^{\phi(z,\bar{z})}.$$

$$g = \begin{pmatrix} 0 & \frac{1}{2}e^{\phi} \\ \frac{1}{2}e^{\phi} & 0 \end{pmatrix} \qquad \det g = -\frac{1}{4} e^{2\phi}.$$

OSU.7 where $\dot{X} = \frac{\partial X}{\partial \tau}$ $X' = \frac{\partial X}{\partial \sigma}$.

OR in a linear form

$$S = \frac{1}{4\pi\alpha'} \int_{\Lambda g} d^2\sigma \, \underbrace{\sqrt{-\det g_{\alpha\beta}}}_{\text{WS-metric}} \, g^{\alpha\beta} \, \partial_\alpha X^\mu \, \partial_\beta X^\nu \, \underbrace{\eta_{\mu\nu}}_{\text{flat minkowsk metric.}}$$

$g_{\alpha\beta} \to$ auxiliary field $\to$ integrate out $\to$ NG-action.

$$S = -\frac{1}{2\pi\alpha'} \,(\text{area of the WS}).$$

Constraints on $-S-$

1) a) Reparametrization invariance $\tau \to \tilde{\tau}(\tau)$

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$$g = \begin{pmatrix} 0 & \frac{1}{2}e^{\phi} \\ \frac{1}{2}e^{\phi} & 0 \end{pmatrix} \qquad \det g = -\tfrac{1}{4} e^{2\phi}.$$

$$\Rightarrow \sqrt{-\det g_{\alpha\beta}} = \sqrt{+\frac{1}{4}e^{2\phi}} = \frac{1}{2}e^{\phi}$$

$$g^{\alpha\beta} = 2e^{-\phi}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sqrt{-\det g_{\alpha\beta}}\, g^{\alpha\beta} = \frac{1}{2}e^{\phi}\, 2e^{-\phi}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and the Louiville mode drops from the action. end of lecture

$\Rightarrow$ the classical action is invariant under the conformal transfo.

$$z \to f(z) \qquad \bar{z} \to f(\bar{z})$$

The conformal invariance of the classical action is one of the important properties of the perturbative string!

$\Rightarrow$ String theories described in terms of 2d - conformal field theories

The "flat-gauge" action is a two dimensional conformal field theory of D - free bosons.

$$S = \frac{1}{4\pi\alpha'}\int_{M} dz\, d\bar{z}\, \partial_z X^{\mu}\partial_{\bar{z}} X_{\mu} =$$

The Eq. of motion is a 2 dimensional wave Eq.

$$\left(\partial_\sigma^2 - \partial_\tau^2\right) X^{\mu}(\sigma,\tau) = 0$$

OR,

$$\partial_z \partial_{\bar{z}} X^{\mu}(z,\bar{z}) = 0$$

OSU, 9

which indicates that $(\partial_z X^\mu)$ is an analytic function of z.

This simply reflects the fact that the solution of the 2d wave Eq. can split into left and right-moving solutions.

(**)

$$X^\mu(\sigma,\tau) = X_R^\mu(\tau+\sigma) + X_L^\mu(\tau-\sigma)$$

OR $$X^\mu(z,\bar z) = X_R^\mu(z) + X_L^\mu(\bar z).$$

we can expand the solutions in Fourier modes

$$X_{L,R}^\mu = \frac{1}{2}x^\mu + \frac{1}{2}l^2 p^\mu(\tau\mp\sigma) + \frac{i}{2} l \sum \alpha_n^{(\sim)\mu} e^{-2in(\tau\mp\sigma)}$$

OR $$\partial_z X^\mu \sim \sum_n z^{-n-1}\alpha_n^\mu$$

$$\partial_{\bar z} X^\mu \sim \sum_n \bar z^{-n-1}\tilde\alpha_n^\mu$$

classically $\alpha_n^\mu, \tilde\alpha^\mu \rightarrow$ n^{th} oscilation mode amplitud.

Quantum $\alpha_n\ \tilde\alpha_n^\mu \rightarrow$ creation-annihilation operators.

$\rightarrow$ define a Hilbert space.

(**) closed string: $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$. periodicity

[open string $X'^\mu(\sigma=0;\pi) = 0$]

added 3/5/02

OSU.10

We now come to quantization of the bosonic string. To properly quantize the theory with local symmetries we must either introduce Fadeev-Popov ghosts or perform canonical quantization with first-class constraints. Let me sketch the canonical approach.

(14/6/03). impose equal time commutation relations: $[X^\mu(\sigma,\tau), \dot{X}^\nu(\sigma',\tau)] = \eta^{\mu\nu}\delta(\sigma-\sigma')T^{-1}$

Variation of the action with respect to the world-sheet metric gives the 2D energy-momentum tensor. Since $g_{\alpha\beta}$ is an auxiliary parameter we have the constraint Eqs.

$$\frac{\delta S}{\delta g_{\alpha\beta}} = 0 \Rightarrow T_{\alpha\beta} = 0.$$

in the "Light-cone" coordinates. world-sheet.

we have $T_{z\bar{z}} = 0$. which follows from tracelessness of the E-M-tensor.

We therefore have the two constraints operator

$$T(z) = T_{zz} = -\frac{1}{2}\partial_z X^\mu \partial_z X_\mu$$

$$\bar{T}(\bar{z}) = T_{\bar{z}\bar{z}} = -\frac{1}{2}\partial_{\bar{z}} X^\mu \partial_{\bar{z}} X_\mu$$

The E-M tensor is expanded in terms of Fourier modes.

$$T(z) = \sum_n z^{-n-2} L_n$$

classically the Fourier modes L_n satisfy the Virasoro algebra

$$[L_n, L_m] = (n-m) L_{m+n}$$

OSV.11)

quantum-mechanically $\alpha_n, \tilde{\alpha}_n$ became harmonic-oscillator creation-annihilation operators. we then pick an anomaly

$$[L_n, L_m] = (n-m) L_{m+n} + \frac{c}{12}(n(n^2-1)) \delta_{n+m}.$$

This anomaly arises due to the fact that the Louiville mode does not decouple when we quantize the theory. It signals the break down of conformal invariance. For the bosonic string we get that in $D=26$ $c=0$.

The constraint Eq. on the E-M tensor become in terms of the fourier modes. we have a normal ordering ambiguity in L_0.
The constraint Eq. become the condition that the L_n $\bar{L}_n$ annihilate the physical state. we then have

$$L_n |phys\rangle = 0 \qquad \text{for } n > 0$$
$$(L_0 - a)|phys\rangle = 0 \qquad (L_0 - \bar{L}_0)|phys\rangle =$$

with similar conditions for the right-movers.

The spectrum at this level contains states with negative norms due to the time-like commutation relation and the residual ~~for~~ gauge freedom
~~This reflect the fact that the longitudinal~~
~~can~~ we can fix the remaining freedom by going to the light-cone gauge

$$X_+ = (X^0 + X^{D-1})/\sqrt{2}$$

OSU.12

In the Light-cone gauge the spectrum is free of negative norm states. However, it is not manifestly Lorentz invariant. It turns out that it is Lorentz invariant

for $D = 26$, $a = 1$.

Alternatively we can use the Fadeev-Popov formalism to quantize the theory in the covariant path integral formalism. There we get a contribution from the ghost fields which is independent of the space-time dimensions.

C anomaly

$$C_{gh} = -26 \quad \text{for the bosonic string}$$

to give a total

$$C_B = D \cdot 1 + (-26).$$

The mass of the string is given in terms of

constraint equation $\rightarrow$ $L_0 = \frac{1}{2} \sum_{-\infty}^{\infty} \alpha_{-n} \alpha_n = \frac{1}{2} \alpha_0^2 + \frac{1}{2} \sum_{n \neq 0} \alpha_{-n} \alpha_n = 0$

OR $M^2 = 2 \sum_{n=1}^{\infty} (\alpha_{-n} \alpha_n + \tilde{\alpha}_{-n} \tilde{\alpha}_n)$

where $\alpha_0^2 = \tilde{\alpha}_0^2 = \frac{1}{4} p^\mu p_\mu = -\frac{1}{4} M^2$

For the closed string $L_0 = \tilde{L}_0$ by the constraint Eq.

In the quantized theory we have a normal ordering ambiguity. Then we get

$$M^2 = -8a + 8 \sum_{n=1}^{\infty} \alpha_{-n} \alpha_n = -8a + 8 \sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \tilde{\alpha}_n$$

where the equality is known as the Virasoro constraint relating the mass of the left and right movers.

OSU.13

The Hilbert-space is obtained by acting with the bosonic oscilators on the vacuum labeled by c.o.M momentum ~~for A~~

$$|0, p^\mu\rangle = :e^{ip_\mu X^\mu}: |0\rangle$$

for $n=0$ we see that $M^2 < 0 \Rightarrow$ tachyon

~~Then~~ At the next level we have

$$\alpha_{-1}^{\mu} \tilde{\alpha}_{-1}^{\nu} |0 \, p^\mu\rangle.$$

in this way we can build the entire spectrum of physical states.

So far I have been discussing the bosonic string. As we saw it contains
1) tachyons.
2) no space-time fermions.

~~we then~~

$\Rightarrow$ Superstring

introduce superpartners for $X^\mu \longrightarrow X^\mu, \psi^\mu$

$\psi^\mu(z, \bar{z})$ are D-majorana WS-fermions.

The Flat-gauge action is Then

$$\int d^2\sigma \left[\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu\right]$$

where ρ^α are 2d gamma matrices.

OSU.14

This gauge fixed action exhibits global world-sheet supersymmetry. Local world-sheet supersymmetry can be exhibited with the ungauged fixed action.

We can write the Majorana fermion in terms of its two Weyl components

$$\psi^\mu(z,\bar z) = \begin{pmatrix} \psi^\mu(z) \\ \bar\psi(\bar z) \end{pmatrix}$$

and in the light cone WS coordinates the action becomes.

$$S = \frac{1}{\pi} i \int d^2 z \left(\psi_z \partial_{\bar z} \psi_z + \bar\psi_{\bar z} \partial_z \bar\psi_{\bar z} \right).$$

Reflecting the fact that the left and right-moving degrees of freedom are decoupled.

Similarly to the bosonic case the fermionic modes give rise to negative norm states. In the covariant path integral formalism the total fermionic ghost contribution is

$$C_{fg} = +11$$

while each Majoran fermion gives $C = 1/2$

Then for the superstring.

$$C_{total} = -26 + 11 + D + \frac{D}{2} = -15 + \frac{D}{2}$$

and to get $C_{total} = 0$ we need $D =$

OSW 15

similarly to the bosonic case ~~the~~ to construct the theory in which ~~is~~ the negative norm states are decoupled we work in the Light cone-gauge in which only transverse excitations act on the nondegenerate vacuum.

For the bosonic coordinates of a ~~cl~~ closed string we have the boundary conditions.

$$X(\sigma,\tau) = X(\sigma+\pi,\tau)$$

for the fermionic Majorana coordinates we can have either periodic or antiperiodic boundary conditions. for the left or right movers seperately.

$$\psi_\pm(\sigma,\tau) = \pm\, \psi_\pm(\sigma+\pi,\tau) \;:\; \begin{matrix} R \\ NS \end{matrix}$$

which gives the mode expansion

$\psi^\mu_-(\pi) = \psi^\mu_-(0)$ $\quad R: \; \psi^\mu_- = \sum d^\mu_n \, e^{-2in(\tau-\sigma)}$ ← periodic

$\psi^\mu_-(\pi) = -\psi^\mu_-(0)$ $\quad NS: \; \psi^\mu_- = \sum b^\mu_r \, e^{-2ir(\tau-\sigma)}$ ← Antiperiodic

and similarly for the left movers ψ^μ_+

with $n \in \mathbb{Z}$ and $r \in \mathbb{Z}+\frac{1}{2}$.

The equal-time anticommutation relation for the fermionic coordinates gives the anticommutation relation for the fermionic R/NS oscilation modes

$$\{b^\mu_r, b^\nu_s\} = \eta^{\mu\nu}\delta_{r+s} \qquad \{d^\mu_m, d^\nu_n\} = \eta^{\mu\nu}\delta_{m+n}$$

OSV 16

For the NS we have that the non-degenerate Fock space is ~~null~~ unique because with ~~and~~ the lowest lying ~~state~~ state

$$b^{\mu}_{-1/2}|0\rangle$$

transforming as a vector of the Lorentz group. and

$$\{b^{\mu}_{-1/2}, b^{\nu}_{+1/2}\} = \eta^{\mu\nu}$$

However, for the Ramond sector the zero modes obey the dirac algebra.

$$\{d_0^{\mu}, d_0^{\nu}\} = \eta^{\mu\nu}$$

and commute with the mass operator

Therefore the zero mode operators are proportional to Dirac gamma states and the states

$$d_0^{\mu}|0\rangle$$

transform in the spinorial representation of the Lorentz group, thus generating space-time fermions.

For a single real R-fermion we can think of this as a doubly degenerate state

$$d_0|0\rangle; \quad |0\rangle$$

OSU.17

There are some other point that we should note:

1) similar to the E.M. tensor we have the WS supercurrent, obtained by varying the action WRT the 2d gravitino

$$T_F(z) = \psi^{\mu} \partial_z X_{\mu}(z)$$

$$\bar{T}_F(\bar{z}) = \bar{\psi}^{\mu} \partial_{\bar{z}} X_{\mu}(\bar{z})$$

2) The mass shell condition is modified.

$$(L_0 - 1)\,|phys\rangle = 0 \longrightarrow (L_0 - \tfrac{1}{2})\,|phys\rangle = 0$$

3) we define a fermion # operator.

$$(-1)^{N_{NS}} \qquad (-1)^{N_R}$$

which roughly counts the number of fermions acting on the nondegenerate ground state

$$|0\rangle_{NS}$$

and the number of R-zero modes acting on the non-degenerate Ramond vacua.

OSU.18

The next important symmetry that we have to discuss is that of modular invariance. Let me however turn to start describing the construction of string vacua in the fermionic formulation and describe modular invariance.

Vanishing of the conformal anomaly gives the constraint:

$$C_{total} = C_{bg} + C_{fg} + C_{\chi^\mu} D + C_{\psi_M} \cdot D$$

$$= -26 + 11 + D + \frac{D}{2}$$

we then get that $C_{total} = 0 \Rightarrow D = 10$.

and we get the ten dimensional superstring theory.

However because of the decoupling of the left and right moving modes for the closed string we can cancel the left and right moving anomalies seperately.

In this case we can impose WS supersymmetry in the left-moving sector while the right moving sector is bosonic

$$C_L = -26 + 11 + D + \frac{D}{2} = 0 \Rightarrow D = 10$$

$$C_R = -26 + D \qquad\qquad \Rightarrow D = 26$$

05V.19

This is the heterotic string construction. anomaly cancelation requires that compactify. 16 right-moving coordinates on a flat torus with fixed Radia. Each compactified coordinate generates a vertex operator for a $U(1)$ space time current.
We the get that there are only two allowed choices with $E_8 \times E_8$ OR $SO(32)$. This will become clearer when discuss the fermionic construction.
We then compactify 6D on a CY manifold to preserve $N=1$ space-time supersym in 4D giving E_6 in 4D.

Alternatively we can use Narain construct. Narain treat all the bosonic coordinates on equal footing (punto doppeggio) and ~~requires~~ compactifies on a flat torus with a Lorenzian even self dual lattice.
This construction gives a wider range of gauge groups in 4D and $N=4$ supersymmetry. We then use the orbifold technique to get $N=4 \rightarrow N=1$
how I will not get into the details of that ~~but rather~~ as this is a set of different lectures but rather ~~to~~ turn to the fermionic construction.

Let me just remark that in Narain construction as in the CY case we have a set of moduli that control the shape and size of the 6D compactified space.

OSU.20

Let me now turn to the fermionic construction. Rather than identifying the degrees of freedom needed to cancel the conformal anomaly as of space-time dimensions we can interpret them as free fermions propagating on the string WS and formulate the theory directly in 4D.

We have

$$C_L = -26 + 11 + D + \frac{D}{2} + n_{f_L} \cdot \frac{1}{2} = 0$$

$$C_R = -26 + D + n_{f_R} \cdot \frac{1}{2} = 0$$

In $D=4$ we get

$$n_{f_L} = 18 \qquad n_{f_R} = 44$$

Thus we need 18 left-moving real M-fermions and 44 right-moving real M-Weyl fermions

to cancel the left and right moving conformal anomaly.

A remark is in order here the bosonic and fermionic constructions are entirely equivalent. In the fermionic construction we are working with free fields on the WS.

The equivalence follows from the equivalence of fermions and bosons in two dimensions.

by defining: $e^{+iX} = y + i\omega \qquad e^{-iX} = y - i\omega$

we can go back to the bosonic construction.

The fermionic construction is however formulated

OSV.21

at a fixed point in the moduli space. If we want to move away from the fixed point we have to include the fermionic WS interactions that preserve the conformal invariance. This is not well developed but although the equivalence between the fermionic and bosonic descriptions may not be known in the general case. They are entirely equivalent.

In polyakov picture string theory is formulated as a perturbative sum over path integral on the string world-sheet. The string W.S. then defines a genus - g - Riemman surface including the conformal invariance we can describe the string states as vertex operator on the genus - g - riemman surface. We have to make sure however that we are only integrating over physically inequivalent paths.

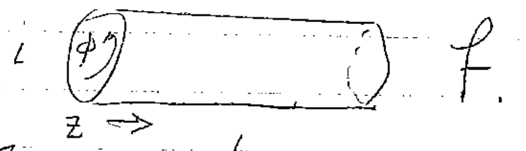
The partition function is defined as the one loop vacuum to vacuum to vacuum amplitude. Therefore we are integrating on a one dimensional riemman surface or on a torus.

The free fermions are world-sheet fermions that can be propaged around the non contractible loop of the torus and can pick up a phase

$$f \rightarrow -e^{-i\pi\alpha(f)} f$$

This form the boundary conditions of the WS fermions

To see how conformal invariance play a role consider the tree level scattering of a free closed string.



The metric of the cylinder is given by

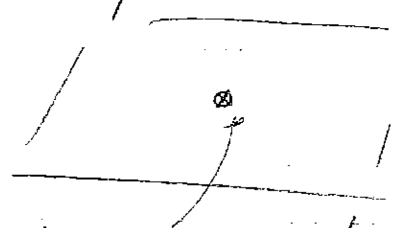
$$(ds)^2 = dz^2 + d\phi^2$$

take $z = \ln r$ $\quad r > 0$ → doesn't change shape.

$$dz = \frac{1}{r}dr \qquad ds^2 = \frac{1}{r^2}\left(dr^2 + r^2 d\phi^2\right)$$

conformal invariant → multiply by an analytic function.

$$\Rightarrow r^2 ds^2 \Rightarrow dr^2 + r^2 d\phi^2 \rightarrow \text{plane}$$

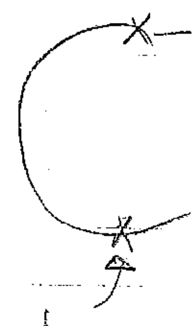


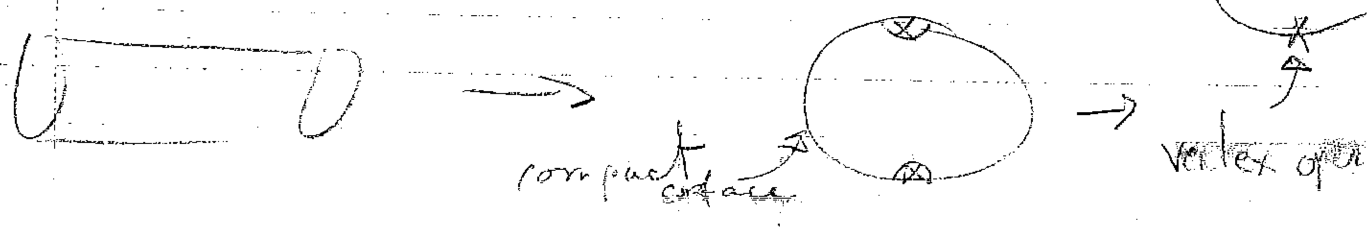
$r \to 0$ $\quad \otimes \quad$ $r \to \infty$

$\lim r \to 0 \sim \lim z \to -\infty$

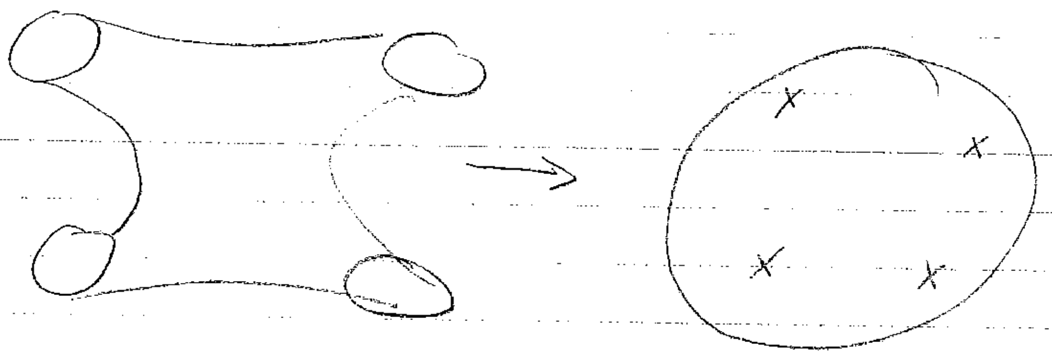
take $(d\hat{s})^2 = \left(1 + \frac{r^2}{a^2}\right)^{-2} (d\tilde{s})^2 =$

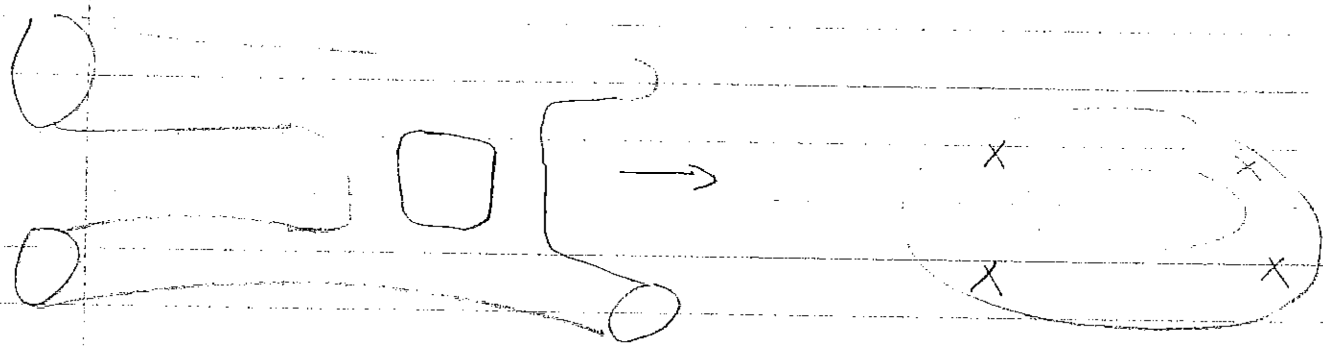
$$= \left(1 + \frac{r^2}{a^2}\right)^{-2}\left(dr^2 + r^2 d\phi^2\right) \rightarrow$$





So in general.





22.-1 → Second lecture

(39)

OSU lectures

Let me remind you that our goal is to construct superstring models or to identify a class of string compactifications that are as realistic as possible. In case that string theory is relevant our hope then is that such models will be relevant.

On the other hand we are developing the methodology needed to confront string theory with experimental data. If string theory is relevant in nature our models will then guide in trying to learn about the Planck scale dynamics

OSU2.2

In the light-cone gauge we have the following WS field content. I will use a notation that will become clearer later.

Left moving $X_L^\mu(z)$ $\mu = 1, 2$ the two transverse coordinates

ψ_L^μ $\mu = 1, 2$ their fermionic partners.

χ_i y_i w_i 18 internal real fermions

Right-moving $X_R^\mu(\bar{z})$ $\mu = 1, 2$ two transverse coordinates

$\phi^{1,\ldots,44}(\bar{z})$ 44 internal real fermions

we have to specify the boundary conditions of all the world-sheet fermions.

we specify these b.c. in basis vectors with 64 entries that will be used to expand the models and the partition function.

when we expand the partition function we have to include all possible b.c. ~~up to~~ ~~the~~ for all physically inequivalent path, or physically inequivalent tori.

we have then to integrate over all inequivalent tori which specify distinct path.

22.1 (40)

OSU lecture

In Polyakov approach to string theory

A string amplitude is calculated by the path integral

$$A_n = \sum_{g=0}^{\infty} A_n^{(g)}$$

$$= \sum_{g=0} \int Dh \, DX^\mu \int d^2 z_1 \cdots d^2 z_n \; V_1(z_1, \bar{z}_1) \cdots V_n(z_n, \bar{z}_n)$$

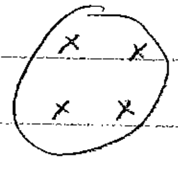
$$= \sum_{g=0} \int d^2 z_1 \cdots d^2 z_n \; \langle V(z_1, \bar{z}_1) \cdots V(z_n, \bar{z}_n) \rangle$$

where, due to the symmetries of the action we must insure that we only sum over physically inequivalent pa

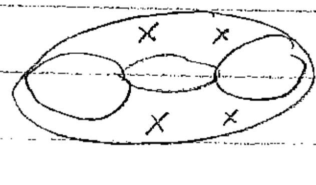
The V_i are vertex operators of external string states.

The string topologies, using the conformal invari are mapped to

tree level $\rightarrow$ sphere



one loop $\rightarrow$ torus



and so forth

At ~~tree~~ ~~tr~~ level all reparametrization are local.
At higher level further constraints will arise.

22.2 : modular invariance

OSU lectur

In order to find these new constraints it is instructive to consider the loop amplitude with no external states. This is ~~just~~ the one-loop partition function which is isomorphic to the torus. The analysis here is done in analogy to "quantum statistical mechanics" where the partition function is calculated by considering periodic temperature, here we are considering the time coordinate as a "complex temperature.

$\tau \rightarrow$ "analogy with quantum statistical mechanics" $\rightarrow$ periodic temperature.

Partition function $\rightarrow$ Sum over all physical states that can propogate around the loop and integrate over all inequivalent tori

22.3

OSU lecture

momentum generator, Hamiltonian

$$Z(\beta,\theta) = \sum_{s \in \mathcal{H}} \langle s | e^{i\theta P} e^{-2\pi\beta H} | s \rangle =$$

Hilbert space, spatial propagation, time propagation

state $\rightarrow$ created and annihilated from the Vacuum.

$$Z(\beta,\theta) = \mathrm{tr}_{\mathcal{H}}\, e^{i\theta P} e^{-2\pi\beta H}$$

Recall that translations are generated by the zero modes of the E-M tensor.

$$H = L_0 + \bar{L}_0 - \frac{1}{24}$$

normal ordering ambig.

$$P = L_0 - \bar{L}_0$$

and combining $\tau = i\beta - \frac{\theta}{2\pi}$ this can be rewritten as

$$Z(\tau) = q^{-1/48} \bar{q}^{-1/48} \mathrm{tr}_{\mathcal{H}}\, q^{L_0} \bar{q}^{\bar{L}_0}$$

where $q = e^{i2\pi\tau}$.

22,4

OSU lecture

We know how L_0 acts on the the Fock space and we can calculate this we still have to specify the boundary conditions for each WS fermion.

The total partition function will then be the product of

$$Z_F(\tau) = \prod_{i=1}^{64} Z_i \begin{bmatrix} \theta \\ \beta \end{bmatrix}(\tau).$$

where the individual $Z_i \begin{bmatrix} \theta \\ \beta \end{bmatrix}$ depend on the boundary conditions in the "time" and "spatial" directions.

space-fermion

The space-choice fixes $\Psi(z)$ to be R/NS.

FOR NS-"time" boundary conditions.

mode expansion

$$Z_{NS}^{NS}(\tau) = \mathrm{tr}_{NS}\, q^{L_0 - 1/48} \qquad Z_{NS}^{R}(\tau) = \mathrm{tr}_R\, q^{L_0 - 1/48}$$

mode expansion.

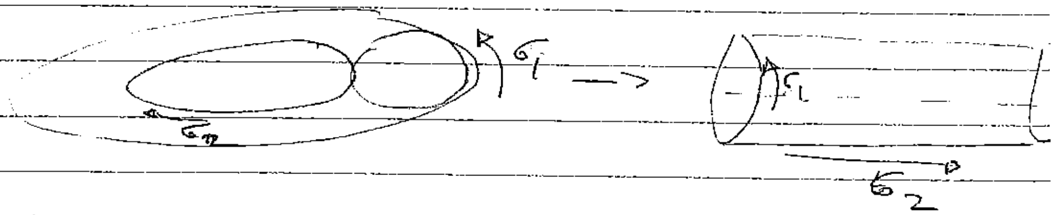
FOR R-"time"

$$Z_R^{NS}(\tau) = \mathrm{tr}_{NS}(-1)^F q^{L_0 - 1/48} \qquad Z_R^R(\tau) = \mathrm{tr}_R(-1)^F q^{L_0 - 1/48}$$

OSU.23

We then come to modular invariance.

To understand what is modular invariance we first note that we can map the torus to the complex plane. This is done by cutting the torus along it's two noncontractible loops

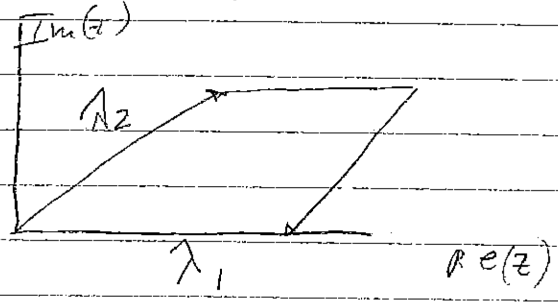


we can then define a complex parameter

$$z = \sigma_1 + i\,\sigma_2$$

where the two coordinates are periodic with length λ_1 & λ_2.

The Torus is then specifyed in the complex plane by



with

$$z \Longleftrightarrow z + n_1\lambda_1 + n_2\lambda_2 \qquad n_i \in \mathbb{Z}.$$

which specified the torus Alternatively we see that the torus is R_2 moded by the lattice.

It is clear that if we take $\lambda_1' = a\lambda_1$

$\lambda_2' = a\lambda_2$

OSU.24

we get the same torus.

The parameter that specifies inequivalent tori is then

$$\tau = \frac{\lambda_2}{\lambda_1}$$

still complex parameter then embodies the two real ~~parameters~~ ~~which~~ degrees of freedom inherent in torodial geometry. However there are still tori which are conformaly equivalent

suppose we take

$$\begin{pmatrix} \lambda_2' \\ \lambda_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \lambda_2 \\ \lambda_1 \end{pmatrix}$$

with arbitrary a, b, c, d.

Then.

$$\tau \to \tau' = \frac{a\tau + b}{c\tau + d}$$

we then ask what are the condition on τ' such that it describes the same tori as τ.

The new torus is defined by the identification

$$z \Leftrightarrow z + n_1' \lambda_1' + n_2' \lambda_2' \qquad (n_1', n_2') \in Z$$

we get

$$z \Leftrightarrow z + (n_1' d + n_2' b)\lambda_1 + (n_1' c + n_2' a)\lambda_2$$

OSV.25

The two tori will be the same provided that

$$\begin{pmatrix} n_2 \\ n_1 \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} n_2' \\ n_1' \end{pmatrix} \qquad \begin{pmatrix} n_2' \\ n_1' \end{pmatrix} = \frac{1}{(ad-bc)} \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \begin{pmatrix} n_2 \\ n_1 \end{pmatrix}$$

with $n_2, n_1 \in \mathbb{Z}$

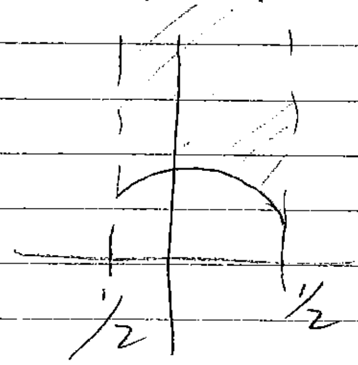
Thus the two tori are identical if

$$a\, b, c, d \in SL(2\mathbb{Z}).$$

Therefore we have to integrate over all conformally inequivalent tori.

The modular group fundamental domain is given by

$$\mathcal{F} = \{ \tau \quad |\tau| \geq 1 \quad |\tau_1| \leq 1/2 \quad \tau_2 > 0$$



and to get the contribution of all inequivalent tori we have to integrate over this fundamental domain.

and require that the partition function is invariant under modular transformations.

OSV.26 In addition we have to require that the partition function does not depend on the parametrization of the tori. This is related to the fact that the string action is invariant under reparametrization of the string world sheet, ~~Hor~~ and to the fact that in two dimensions the metric ~~as~~ is conformally equivalent to the flat metric. However, in the one loop amplitude not all transformations are continuously connected to the identity. Then we have to require that the partition function is invariant under the modular transformations which are spanned by

$$\tau \to -\frac{1}{\tau}$$

$$\tau \to \tau + 1$$

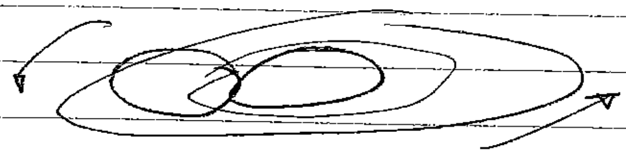
These are the important transformations and requiring invariance under them will lead to a set of constraints on the allowed boundary conditions

These are the consistency constraints derived by ABK and KLT.

OSV.27

Let's see how they are implemented.

First for every world sheet fermion we have to specify the transformation ~~under~~ around the two noncontractible loop



for real fermions these can only be periodic / Antiperiodic R/NS

so we have four possibilites $f \to -e^{i\pi}$

$0 \to NS$
$1 \to R$

$$Z_f\begin{bmatrix}0\\0\end{bmatrix} \quad Z_f\begin{bmatrix}1\\0\end{bmatrix} \quad Z_f\begin{bmatrix}0\\1\end{bmatrix} \quad Z_f\begin{bmatrix}1\\1\end{bmatrix}$$

These are the so called spin-structures of the fermion on the torus.

More generally we can combine two real fermions to form a complex fermion

$$f = \frac{1}{\sqrt{2}} f_1 + i f_2 \qquad \bar{f} = \frac{1}{\sqrt{2}} f_1 - i f_2$$

in this case the boundary conditions may be complex

$$f \to -e^{i\pi\alpha(f)} f$$

with $\alpha(f) \in (-1, 1)$

However I will only sketch the derivation for real fermions.

OSU,28

The partition function is then given by

$$Z = \int \frac{d\tau d\bar{\tau}}{[Im(\tau)]^2} Z_B^2(\tau,\bar{\tau}) \sum_{\substack{\text{spin}\\ \text{Structure}}} C\begin{pmatrix} a \\ b \end{pmatrix} \underbrace{Z_{long,1/2}}_{\substack{\| \\ 1 \text{ for } g=1}} \begin{bmatrix} a_\psi \\ b_\psi \end{bmatrix} \prod_{f=1}^{6^4} Z$$

and the rules are derived by requiring invariance under

$$\tau \to -\frac{1}{\tau}$$

$$\tau \to \tau + 1.$$

The measure $\frac{d\tau d\bar{\tau}}{\tau_2^2}$ is modular invariant by itself,

so is $Z_B = \frac{1}{|\tau_2|^2 \, |\eta(\tau)|^2}$

which is the contribution of the transverse coordinates

$\eta(\tau)$ is the Dedekind ~~et~~ eta function

$$\eta(\tau) = q^{\frac{1}{12}} \prod_n (1 - q^{2n}) \qquad q = e^{i}$$

The partition function is simply the sum over all the string states, massless and massive, ~~that propagate on~~ we have to include when taking the string around a closed loop.

OSV.29

So for example for the $Z\begin{bmatrix}0\\0\end{bmatrix}$ it is given by

$$Z\binom{0}{0} = C\binom{0}{0} \operatorname{Tr} e^{2\pi i \tau H_{NS}}$$

Phase

$$= C\binom{0}{0} \operatorname{Tr} q^{H_{NS}}$$

$$= C\binom{0}{0} q^{-1/24} \operatorname{Tr} q^{\sum_{r=1/2} r b_{-r} b}$$

$$= C\binom{0}{0} q^{-1/24} \prod_{r} \sum_{N_r} q^{r N_r}$$

$$= C\binom{0}{0} q^{-1/24} \prod_{n=1}^{\infty} \left(1 + q^{2n-1/2}\right)$$

(This is the grand partition function for an ideal fermi gas with energy levels $E_r = r$).

We can now write

$$\left(q^{-1/12} \prod_{n=1}^{\infty} (1-q^{2n})^{-1}\right)^{1/2} \cdot \left(\prod_{n=1}^{\infty} (1-q^{2n}) (1+q^{2n-1/2})\right)$$

$$= \frac{\theta_3^{1/2}(\tau)}{\eta^{1/2}(\tau)}.$$

OSU.30

Similarly for the others

$$Z_F\begin{bmatrix}0\\1\end{bmatrix} = \Theta_4^{1/2}(\tau) = \mathrm{tr}\left[(-1)^F e^{i\tau H_{NS}}\right]$$

$$Z_F\begin{bmatrix}1\\0\end{bmatrix} = \frac{\Theta_2^{1/2}(\tau)}{\eta^{1/2}(\tau)} = \mathrm{tr}\left[e^{i\tau H_R}\right]$$

$$Z\begin{bmatrix}1\\1\end{bmatrix} = \frac{\Theta_1^{1/2}(\tau)}{\eta^{1/2}(\tau)} = \mathrm{tr}\left[e^{i\tau H_R}(-1)^F\right]$$

The fermionic partition function is then just a product of left and right moving θ functions (depend on z $\bar{z}$).

we can now determine the transformation under modular transformations.

The effect of $\tau \to -\frac{1}{\tau} = -\frac{\lambda_1}{\lambda_2}$ $\quad \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}\lambda_1\\\lambda_2\end{pmatrix}$

is just to interchange $\lambda_1 \leftrightarrow \lambda_2$

Let me just state the result which is obtained with some gymnastic with poisson resummation and theta functions.

OSU.31 $\tau \to \tau+1$

$$\eta \to e^{i\pi/12}\eta \qquad \theta_1 \to e^{i\pi/4}\theta_1$$

$$\theta_2 \to e^{i\pi/4}\theta_2$$

$$\theta_3 \leftrightarrow \theta_4$$

$$\tau \to -\frac{1}{\tau}$$

$$\eta \to (-i\tau)^{1/2}\eta \qquad \frac{\theta_1}{\eta} \to e^{\frac{i\pi}{2}}\frac{\theta_1}{\eta} \qquad \frac{\theta_2}{\eta} \leftrightarrow$$

$$\frac{\theta_3}{\eta} \to \frac{\theta_3}{\eta}.$$

the crucial point is that now.

the fermionic partition function is a product of the spin structure of 14 fermions.

performing the modular transformations $\tau \to -\frac{1}{\tau}$ $\tau \to \tau$ will take us from one spin struct to another will take us from one product of θ_i functio to another.

Invariance therefore requires that both spin structures related by modular transformations be present in the partition function with equal weight.

$$C\left(\frac{\vec{\alpha}}{\beta}\right)$$

OSU,32

The one loop fermionic partition function is then

$$\sum_{\substack{\text{spin} \\ \text{structures}}} C\begin{pmatrix}\vec{\alpha}\\ \vec{\beta}\end{pmatrix} Z\begin{pmatrix}\vec{\alpha}\\ \vec{\beta}\end{pmatrix}$$

on all spin-structures allowed by modular invariance

For example say we have

$$\begin{matrix}a=0\\b=0\end{matrix}\left(\Theta_3^{1/2}(\tau)\right)^n \quad \begin{matrix}a=0\\b=1\end{matrix}\left(\Theta_4^{1/2}(\tau)\right)^m \quad \begin{matrix}a=1\\b=0\end{matrix}\left(\Theta_2^{1/2}(\tau)\right)^\ell$$

(and complex conjugate for the right-movers).

$\Theta_1 \equiv C$ at one

$\tau \to \tau+1$

$$\begin{matrix}a=0\\b=1\end{matrix}\left(\Theta_4^{1/2}\right)^n. \quad \begin{matrix}a=0\\b=0\end{matrix}\left(\Theta_3^{1/2}\right)^m \quad \begin{matrix}a=1\\b=0\end{matrix}\left(\Theta_2^{1/2}\right)^\ell \cdot \left(e^{\frac{i\pi}{4}}\right)^{1/2\,\ell}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = \left(\Theta_2^{1/2}\right)^\ell e^{\frac{i\pi}{8}\sum a_f}$$

$$\begin{pmatrix}0\\0\end{pmatrix} \to \begin{pmatrix}0\\1\end{pmatrix} \qquad \begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}0\\0\end{pmatrix} \qquad \begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}1\\0\end{pmatrix}$$

$\Rightarrow$ we have that the sector with $C\begin{pmatrix}a\\a+b+1\end{pmatrix}$ also has to be included in the sum with

$$C\begin{pmatrix}a\\a+b+1\end{pmatrix} = - e^{\frac{i\pi}{8}\sum a_f} C\begin{pmatrix}a\\b\end{pmatrix}.$$

from the η's.

and $\sum a_f^2 = 0 \bmod 8.$

OSO.32

$$\Theta(z, \tau) = 2e^{i\pi\tau/4} \sin \pi z \prod_{n=1}^{\infty} f(\tau, z, n).$$

OSV.33

under $\tau \to -\frac{1}{\tau}$

$$\begin{pmatrix}0\\0\end{pmatrix}^n : \left(\frac{\theta_3^{1/2}(\tau)}{\eta^{1/2}}\right)^n \qquad \begin{pmatrix}0\\1\end{pmatrix}^m : \left(\frac{\theta_4^{1/2}}{\eta^{1/2}}\right)^m \qquad \begin{pmatrix}1\\0\end{pmatrix}^l : \left(\frac{\theta_2^{1/2}}{\eta^{1/2}}\right)^l \qquad \begin{pmatrix}1\\1\end{pmatrix}^p : \left(\frac{\theta_1^{1/2}}{\eta^{1/2}}\right.$$

$\tau \to -\frac{1}{\tau}$

$$\begin{pmatrix}0\\0\end{pmatrix}^n : \left(\frac{\theta_3^{1/2}}{\eta^{1/2}}\right)^n \qquad \begin{pmatrix}1\\0\end{pmatrix}^m : \left(\frac{\theta_2^{1/2}}{\eta^{1/2}}\right)^m \qquad \begin{pmatrix}0\\1\end{pmatrix} : \left(\frac{\theta_4^{1/2}}{\eta^{1/2}}\right)^l \qquad \begin{pmatrix}1\\1\end{pmatrix}^p : e^{-\frac{i\pi}{4}} \cdot \left(\right.$$

we get
$$C\begin{pmatrix}a\\b\end{pmatrix} = e^{\frac{i\pi \sum a_f \cdot b_f}{4}} \cdot C\begin{pmatrix}b\\a\end{pmatrix}$$

and
$$\vec{a} \cdot \vec{b} = 0 \bmod 4.$$

where $\vec{a} \cdot \vec{b}$ is a lorentzian product
$$\sum_{\text{left}} a_f \cdot b_f - \sum_{\text{Right}} a_f b_f.$$

This is how the ABK rules are derived.

In Addition the modular invariance imposes the GSO projection on the physical spectrum.

OSU.34

The entire partition function can then be generated by specifying a set of boundary condition basis vectors

$$B = \{ \vec{b}_1 \cdots \vec{b}_n \}.$$

and all the terms in the partition functi are of the form.

$$Z \begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \end{pmatrix}$$

where

$$\vec{\alpha} = \alpha_1 \vec{b}_1 + \cdots + \alpha_n \vec{b}_n$$

$$\vec{\beta} = \beta_1 \vec{b}_1 + \cdots + \beta_n \vec{b}_n$$

~~where~~ and for models with only NS/R boundary conditions.

$$\alpha_i = 0, 1 \qquad \beta_i = 0, 1.$$

With the basis vectors b_i and the coefficients $C \begin{pmatrix} b_i \\ b_j \end{pmatrix}$ subject to the consistency constraint.

There is one more constraint that I should mention. This important constraint has in fact lead to some confusion in the construction of SO(10) GUT models.

Important

The constraint is that world-sheet supersymmetry be preserved.

This means that the supercurrent T_F must be uniquely defined, up to a sign, under the transformation of the world-sheet fermions under the specified boundary conditions in all the sectors. Again it is sufficient to insure this for the basis vectors.

Recall that in the heterotic string the supercurrent has the form

$$T_F = \psi^\mu \partial X_\mu \qquad \mu = 1, 8.$$

in the fermionic construction it will take the form

$$T_F = \psi^\mu \partial X_\mu + f_{abc}\, \psi^a \psi^b \psi^c$$

OSV.37

$$x_i y_i w_i \rightarrow - x_i y_i w_i$$

so we can have the following boundary condition for each x, y, w

(x, y, w): $(1,1,0)$ $(0,1,1)$ $(1,0,1)$ $(0,0,0)$

if $\psi^\mu \rightarrow + \psi^\mu$ then (ST-fermion)

$(x, y, w) \rightarrow (1,0,0)$ $(0,1,0)$ $(0,0,1)$ $(1,1,1)$

and $xyw \rightarrow + xyw$.

This ensures that we have a well defined WS - supercurrent.

It would seem that we are done with the presequisit. Not yet, but don't despair we are almost there. The fun only begins in constructing real models. fortunately for us, ABK summerized their rules very nicely, so we are almost ready to get the cream,

we could as well have started from the ABK rules and forget about all the prlimeneries, but I am sure you'd rather know where they come from.

OSU36

where f_{abc} are the structure functions of some semi-simple group of dimension 18.

In the models that I describe the group is

$$SU(2)^6 .$$

The 18 internal left moving fermions are then grouped into

6 representations in the adjoint of $SU(2)$. The supercurrent then takes the form.

$$T_F = \psi^\mu \partial X_\mu + \sum_{i=1}^{6} x_i y_i w_i$$

Requiring that the supercurrent is well defined Then imposes that every product

$x_i y_i w_i$ transforms with the same sign as $\psi^\mu \partial X_\mu$

This means that if $\psi^\mu \to -\psi^\mu$ (NS) (ST bosons)

we have

OSU.38

The next thing we should know is that every complex fermion generates a world-sheet current. This world-sheet curren[t] produce the cartan generators of the 4 dimensio[nal] gauge group. The charges with respect to these are given by

$$Q(f) = \frac{1}{2}\alpha(f) + F(f)$$

where $\alpha(f)$ is the boundary condition of a complex fermion and $F(f)$ is the fermion number.

The fermion number is given by (± 1) for a fermion acting on the ~~$|0\rangle$~~ nondegenerate vacuum and (-1) for its complex conjugate

For the Ramond vacua there are the two degenerate vacua denoted by $|\pm\rangle$ with fermion numbers

$$F: |+\rangle = (0)|+\rangle$$
$$F: |-\rangle = -1\ |+\rangle$$

OSU, 39

There is one more type of world-sheet opera in the models that I will describe

These are obtained by combinig a real-left moving fermion with a real right-moving fermion. They generate the conformal field theory of an Ising operators with the field operator of the Ising model. This will become clearer when I discuss specific examples.

OSU.40

Let me now summarize the ABK rules and start with ~~the~~ the real fun of constructing real models.

A model is defined by specifying two ingredi-

1) A set of boundary condition basis vectors

2) The one-loop phases $C\binom{b_i}{b_j}$ for All intersection of basis vectors.

I will now write down the explicit rules. This is our starting point for the construction of actual models. Pay close attention now. Even if you snoozed until now, if you know the rules you can construct models.

1) Basis vectors $\{b_1 \cdots b_n\}$

Additive group $\equiv \sum_{i=1}^{n} m_i \, b_i$

$m_i = 0, \ldots, N_i - 1$ where $N_i b_i = 0 \bmod 2$.

OSV.41

1 -> periodic (Ramond) | 0 -> Aperiodic (NS)

In ABA

$f \to -e^{i\pi}$

Rules on the basis vectors.

1) $\sum m_i b_i = 0$ iff $\forall\, m_i = 0 \bmod N_i$

2) $N_{ij}\, b_i \cdot b_j = 0 \bmod 4$ — N_{ij} the least common multiplier of b_i and b_j

3) $N_i\, b_i\, b_i = 0 \bmod 8$

4) # of Real fermions -> even.

5) $b_1 = \vec{1}$ (more generaly $\vec{1} \in \Xi$)

where $b_i \cdot b_j = \left\{ \sum_{c.l} + \frac{1}{2} \sum_{R.l} - \left(\sum_{c.R.} + \frac{1}{2} \sum_{R.R} \right\} b_i(f)\, b_j(\right.$

(52)

OSV.42

Rules on one-loop phases

1) $c\binom{b_i}{b_j} = \delta_{b_i}\, e^{\frac{i2\pi n}{N_j}} = \delta_{b_j}\, e^{\frac{i2\pi m}{N_i}}\, e^{i\pi\frac{b_i\cdot b_j}{2}}$

2) $c\binom{b_i}{b_i} = -\, e^{i\pi\frac{b_i\cdot b_i}{4}}\, c\binom{b_i}{1}$

3) $c\binom{b_i}{b_j} = e^{i\frac{b_i\cdot b_j}{2}}\, c\binom{b_j}{b_i}^*$

4) $c\binom{b_i}{b_j+b_k} = \delta_{b_i}\, c\binom{b_i}{b_j}\, c\binom{b_i}{b_k}$

where $\delta_{b_i} = e^{i b_i(\psi^\mu)\pi} = \begin{cases} -1 & b_i(\psi^\mu) = 1 \\ +1 & b_i(\psi^\mu) = 0 \end{cases}$

This index insures the correct space-time statistics for space-time fermions and bosons

basis: $\rightarrow$ spans a finite additive group $\Xi = m_i b_i$

$m_i = 0, \ldots,$

for every $\alpha \in \Xi$ there is a Hilbert space, obtained by acting with the fermionic oscilators on the vacuum and for peridic fermions $\rightarrow |\pm\rangle$ doubly degenerate spinorial vacua

$\nu_\alpha |0\rangle_\alpha \;;\; |\pm\rangle$

OSU, 43

$$\nu_f = \frac{1+\alpha(f)}{2} \qquad \nu_{f^*} = \frac{1-\alpha(f)}{2}.$$

for NS vacuum $\alpha(f)=0 \Rightarrow \nu_f = \nu_{f^*} = 1/2$

matching (Virasoro) condition:

$$M_L^2 = -1/2 + \frac{\alpha_L \cdot \alpha_L}{8} + N_L = -1 + \frac{\alpha_R \cdot \alpha_R}{8} + N_R = M_R^2$$

$$N_L = \sum \nu_L \qquad N_L = \sum \nu_R$$

oscilators that act on the vacuum.

Fermion number: $F(f)$

$$F = \begin{cases} +1 & f \\ -1 & f^* \end{cases}$$

U(1) charges

$$Q(f) = \frac{1}{2}\alpha(f) + F(f)$$

OSV.44

GSO Projection

$$e^{i\pi b_j \cdot F_\alpha} |S\rangle_\alpha = \delta_\alpha C\binom{\alpha}{b_j}^* |S\rangle_\alpha$$

$\alpha \in \Xi$ $\qquad$ $b_j \in B$asis.

$|S\rangle_\alpha$ is a state in the sector $\alpha \in \Xi$

$$b_j \cdot F_\alpha = \left(\sum_{\text{Left}} - \sum_{\text{Right}} \right) b_j(f) F_\alpha(f)$$

A remark on the GSO projections

The GSO projection also arises by modular invariance.
The partition function simply counts the spectrum at all mass
when we expand the patition function for A sector α we can take as a sum over its intersection with other sectors.

$$\sum_\alpha \sum_\beta C\binom{\alpha}{\beta} Z\binom{\alpha}{\beta}$$

This means that in the sum there may be cancelation between different parts. This cancelations will be reflected in the spectrum. For example $N=4$ susy

$$\Rightarrow \underbrace{\left[\theta_3^4 - \theta_2^4 - \theta_4^4\right]}_{\substack{\| \\ 0 \equiv N=4 \text{ susy}}} \theta_3^6 \, \bar{\theta}_3^{22}$$

OSV,45

(Hilbertspace)

When we construct the models by using the basis vectors, the GSO projections incorporate those cancelations. Thus, they are just a result of the modular invariance of the partition function. Nothing fancy.

Let us finally start to construct models.

Let start with the simplest one with a single basis vector.

$B = \{\vec{1}\}$ → Required by consistency

we have only two sectors $\{\vec{1}, 2 \cdot \vec{1} = \vec{0}\}$
$\vec{0} = NS$

The rules are trivialy satisfied.

$N_1 = 2 \quad \vec{1} \cdot \vec{1} = \frac{1}{2} \cdot 20 - \frac{1}{2} \cdot 44 = -12$

$N_1 \cdot \vec{1} \cdot \vec{1} = -24 = 0 \bmod 8$

The number of real fermions is even.

In general the ~~num~~ even # of real fermions means that ~~for~~ ~~a~~ if we pair two real fermions they will have the same B.C. in all vectors (basis vectors)

For phenomenology we are only interested in ~~zero~~ massless states and must ensure that there are no states with $M < 0$.

In this model we have

$\alpha = I$:

$$M_L^2 = -\frac{1}{2} + \frac{10 \cdot \cancel{10}}{8} + N_L = -\frac{1}{2} + \frac{5}{4} + N_L = \frac{3}{4} + N_L >$$

Therefore this sector contains no massless states.

For the NS sector:

$$M_L^2 = -\frac{1}{2} + \frac{0}{8} + N_L = -1 + \frac{0}{8} + N_R = M_R^2$$

The possible oscilators of the NS fermions are

$$\gamma_{f, f^*} = \frac{1 \pm 0}{2} = \frac{1}{2}$$

So we need one left-moving fermionic oscilat[or]

two right moving fermionic oscilat[ors]

one right-moving bosonic oscilator

to get a massless state.

Or one fermionic right-moving oscilator to get a tachyonic state.

OSV.47

$$C\binom{NS}{NS} = +1 = \delta_{NS}\, e^{i\frac{2\pi\, 0}{0}} = \delta_{NS} = 1.$$

$$C\binom{NS}{b_j} = \delta_{b_j}\, \underbrace{e^{i\frac{2\pi m_{NS}}{N_{NS}}}}_{=1}\, \underbrace{e^{i\pi\frac{b_j\cdot NS}{2}}}_{=1} = \delta_{b_j} = \delta_{NS}\, e^{i\frac{2\pi n_j}{N_{b_j}}}$$

we have $\quad C\binom{NS}{b_j} = \delta_{b_j}.$ $\quad$ important!

Remark:

In the following I will use the following notation which you often find in my papers.

Left movers

$$\underbrace{\psi^\mu_{1,2}}_{\text{complex}}\ \underbrace{(\chi_1 y_1 \omega_1)(\chi_2 y_2 \omega_2)}_{\text{complex}}\underbrace{(\chi_3 y_3 \omega_3)}_{\text{complex}}\ \underbrace{(\chi_4 y_4 \omega_4)}_{\text{Real}}\ \underbrace{(\chi_5 y_5 \omega_5)\ (\chi_6 y_6 \omega_6)}_{\text{complex}}$$

Right movers

$$\underbrace{\bar y_1 \bar\omega_1\, \bar y_2 \bar\omega_2\, \bar y_3 \bar\omega_3\, \bar y_4 \bar\omega_4\, \bar y_5 \bar\omega_5\, \bar y_6 \bar\omega_6}_{\text{Real}}\ \ \underbrace{\bar\psi_{1\ldots5}\ \bar\eta_1\ \bar\eta_2\ \bar\eta_3\ \ \bar\phi_{1\ldots8}}_{\text{complex.}}$$

The use of this notation will become obvious in the following

we therefore have the following states

$$\psi^\mu_{\frac12}\, \partial\bar X^\nu_1\, |0\rangle_{NS} \quad \begin{cases}\text{graviton}\\ \text{dilaton}\\ \text{antisymmetric tensor}\end{cases}$$

$$\psi^\mu_{1/2}\, \bar\phi^a_{1/2}\, \bar\phi^b_{1/2}\, |0\rangle_{NS} \to \{a,b\} = 1,\ldots,44$$
$$\to \text{gauge bosons of } SO(44).$$

$$\{\chi^i_{1/2};\, y^i_{1/2},\, \omega^i_{1/2}\}\, \bar\partial X^\mu_1\, |0\rangle_{NS} \to \{i = 1\ldots6\}$$
$$\to \text{gauge bosons of } SU(2)^6$$

OSU. 48

$\{\chi^i_{1/2}, y^i_{1/2}, \omega^i_{1/2}\}\bar{\phi}^a_{1/2}\,\bar{\phi}^b_{1/2}|0\rangle_{NS}$ scalars in the adjoint of $SU(2)^6 \times SO(44)$

These states have $M^2 = 0$.
we also have

$$\bar{\phi}^a_{1/2}|0\rangle_{NS} \quad \text{with } M^2 = -\tfrac{1}{2} \to \text{tachyon}$$

We have to perform the GSO projection to see which states remain in the massless spectrum

$$e^{i\pi 1\cdot F_{NS}} |S\rangle_{NS} = \delta_{b_1} |S\rangle_{NS} = -|S\rangle_{NS}$$

$$e^{i\pi 1\cdot F_{NS}}\left(\psi^\mu_{1/2}\,\partial X^\nu_{\frac{1}{2}}|0\rangle_{NS}\right) = \underset{-1}{-}\,\psi^\mu_{1/2}\,\partial\bar{X}^\nu_{\frac{1}{2}}|0\rangle_{NS} = \delta_{b_1}(\psi^\mu_{0}$$

in general we have that $\psi^\mu_{1/2}\,\partial\bar{X}^\nu_{1/2}|0\rangle_{NS}$ always survives because $C\binom{NS}{b_j} = \delta_{b_j}$

then $e^{i\pi b_j(\psi^\mu) F_{NS}(\psi^\mu)} = e^{i\pi b_j(\psi^\mu)} = \delta_{b_j}$
and we have an equality.
This is the statement that the graviton multiplet is always in the spectrum of a string theory. This is also true in the

bosonic $\quad \partial X^\nu_1\,\partial\bar{X}^\mu_1\,|0\rangle_{NS}$

type II $\quad \psi^\nu_{1/2}\,\psi^\mu_{1/2}\,|0\rangle_{NS}$

Which also always excist in the massless spectrum
Gravity is always there.

OSU 49

for the other states

$$\psi^{\mu} \quad \phi \quad \phi \quad |0\rangle$$
$$e^{i\pi F_{NS}}: \quad -1 \quad -1 \quad -1 \qquad = \delta_1 = -1 \quad \checkmark$$

$$\begin{Bmatrix} x \\ y \\ w \end{Bmatrix} \quad \phi \quad \phi \quad |0\rangle_{NS}$$
$$e^{i\pi F_{NS}} \quad -1 \quad -1 \; -1 \quad |0\rangle_{NS} = \delta_1 = -1 \quad \checkmark$$

$$\begin{Bmatrix} x \\ y \\ w \end{Bmatrix} \bar{\partial} X_1^{\mu} \, |0\rangle_{NS}$$
$$e^{i\pi F_{NS}} \quad -1 \qquad |0\rangle_{NS} = \delta_1 = -1 \quad \checkmark$$

How about the tachyons?

$$\bar{\phi}^{a}_{1/2} \, |0\rangle_{NS}$$
$$e^{i\pi F_{NS}} = -1 \qquad = \delta_1 = -1 \quad \checkmark$$

Therefore the tachyon survives the GSO projection in this model.

The partition function in this case is composed three sectors.

$$C\binom{0}{0} Z\binom{0}{0} + C\binom{0}{1} Z\binom{0}{1} + C\binom{1}{0} Z\binom{1}{0}.$$

$$C\binom{1}{0} = e^{i\pi\frac{0\cdot 1}{2}} \, C\binom{0}{1}^{*} = \delta_1$$

$$\Rightarrow Z = Z\binom{0}{0} - Z\binom{0}{1} - Z\binom{1}{0}$$

Is this modular invariant?

OSV.50

$$Z\binom{0}{0} = \frac{\Theta_3^{10}\bar{\Theta}_3^{22}}{\eta^{10}\bar{\eta}^{22}} \quad Z\binom{0}{1} = \frac{\Theta_4^{10}\bar{\Theta}_4^{22}}{\eta^{10}\bar{\eta}^{22}} \quad Z\binom{1}{0} = \frac{\Theta}{\eta}$$

$$Z = \frac{1}{\eta^{10}\bar{\eta}^{22}}\left[\Theta_3^{10}\bar{\Theta}_3^{22} - \Theta_4^{10}\bar{\Theta}_4^{22} - \Theta_2^{10}\bar{\Theta}_2^{22}\right]$$

under

$\tau \to \tau+1$ $\quad \eta \to e^{i\frac{\pi}{12}}\eta \quad \Theta_2 \to e^{i\pi/4}\Theta_2 \quad \Theta_3 \longleftrightarrow \Theta_4$

we get

$$\frac{1}{e^{i\pi(10-22)/12}\,\eta^{10}\bar{\eta}^{22}}\left[\Theta_4^{10}\bar{\Theta}_4^{22} - \Theta_3^{10}\bar{\Theta}_3^{22} - e^{i\pi\left(\frac{10-22}{4}\right)}\Theta_2^{10}\bar{\Theta}_2^{22}\right]$$

$$- \qquad + \qquad - \qquad -(-)$$

$$\Rightarrow \qquad - \qquad + \qquad -$$

invariant ✓

under

$\tau \to -\frac{1}{\tau}$ $\quad \frac{\Theta_2}{\eta} \longleftrightarrow \frac{\Theta_4}{\eta} \qquad \frac{\Theta_3}{\eta} \longrightarrow \frac{\Theta_3}{\eta}$

so trivialy it is invariant under this transform

This was of course expected, but is a nice check that we know what is going on.

OSV.51

As we saw this model contains a tachyon.

Let's add another basis vector $b_2 = \vec{S}$ $\{\vec{1}, \ldots$

$\vec{S}: S\{\psi^{\mu}_{1,2}, \chi_{1,2}, \chi_{3,4}, \chi_{5,6}\} = 1 \rightarrow$ Periodic (Ramond)

all the other fermions are antiperiodic (Neveu-Schwarz).

Rules: $N_S = 2$.

$\vec{S}\cdot\vec{S} = 4 - 0 = 4 \qquad N_S \cdot \vec{S}\cdot\vec{S} = 0 \bmod 8$ ✓

$S\cdot\mathbb{1} = 4-0 = 4 \qquad N_{S,1}\, S\cdot\mathbb{1} = 0 \bmod 4$ ✓

We have four sectors.

$\{\mathbb{1},\ S,\ 1+S,\ NS\}$

$\{1,\ 1+S\} \rightarrow$ do not give massless states.

$(1+S)_L\cdot(1+S)_L = 6 \qquad (1+S)_R(1+S)_R = 22$

$-\frac{1}{2} + \frac{6}{8} = +\frac{1}{4} > 0 \qquad -1 + \frac{22}{8} > 0.$

The Neveu-Schwarz sector gives the same states as before. We have to perform the GSO projections of the S vector.

$$e^{i\pi\vec{S}\cdot\vec{F}_{NS}}\,|S\rangle_{NS} = \delta_S\,|S\rangle_{NS} = -\,|S\rangle_{NS}.$$

$$e^{i\pi(1-0)}\left\{\psi^{\mu}_{1/2}\,\partial X^{\nu}_{+1}|0\rangle_{NS}\right\} = -\left\{\psi^{\mu}_{1/2}\,\partial X^{\nu}_{+1}|0\rangle_{NS}\right\}.$$ ✓

OSU.52

$$\overbrace{\{\chi_{1/2};\ y_{1/2};\ w_{1/2}\}}^{\text{graviphotons.}}\ \partial \bar{X}^{\mu}_{+1}\ |0\rangle_{NS}$$

$$\underset{\text{in}}{-1}\quad \underbrace{+1\quad +1}_{\text{out}} \longrightarrow SU(2)^6 \text{ is broken.}$$

$$e^{i\pi(1\cdot 1-0\cdot 1-0\cdot 1)}\ \psi^{\mu}_{1/2}\ \bar\phi^{a}_{1/2}\ \bar\phi^{b}_{1/2}\ |0\rangle_{NS} = e^{i\pi(S(\psi^\mu)\cdot F_{NS}(\psi^\mu) - S(\bar\phi^a)\cdot F(\phi^a) - S(\bar\phi^b) F}$$

$$-1 \quad + \quad + \qquad = -1 \quad \checkmark \quad \text{adjoint of SO}$$

$$\{\chi_{1/2};\ y_{1/2};\ \omega_{1/2}\}\ \bar\phi^{a}_{1/2}\ \bar\phi^{b}_{1/2}\ |0\rangle$$

$$\{-1;\ +1;\ +1\}\quad +1\quad +1$$

$$\checkmark \quad \times \quad \times$$

scalar in the adj of SO(44).

2: 1 ↓ 3/2, 2 ↓ 1, 3 ↓ 1/2, 4 ↓ 0

we also had the tachyon

$$e^{i\pi(0-0\cdot 1)}\left\{\bar\phi^{a}_{1/2}\ |0\rangle_{NS}\right\} = +\left\{\bar\phi^{a}_{1/2}\ |0\rangle_{NS}\right\} \to \text{ou}$$

⇒ The tachyon is projected! OUR A⁴

How about the $\vec{S}$ sector.

$$S_L\cdot S_L = 4 \qquad M_L^2 = -\tfrac{1}{2} + \tfrac{4}{8} = 0 \Rightarrow \text{no oscilators}$$

$$S_R\cdot S_R = 0 \qquad\qquad \text{purely Ramond va}$$

$$M_R^2 = -1 + \tfrac{0}{8} = -1 \Rightarrow \begin{array}{l} 2\ \nu_f = 1/2 \\ 1\ \nu_X = 1. \end{array}$$

complexify: $\psi^{\mu}_{12} = \frac{1}{\sqrt2}(\psi^{\mu}_1 + i\,\psi^{\mu}_2)$

$$\chi_{12} = \tfrac{1}{\sqrt2}(\chi_1 + i\chi_2)$$

$$\chi_{34} = \tfrac{1}{\sqrt2}(\chi_3 + i\chi_4)$$

$$\chi_{56} = \tfrac{1}{\sqrt2}(\chi_5 + i\chi_6).$$

→ This will be convenient for later so let's use it for now

OSV.53

The S-vacuum

$$|S\rangle_L = \underset{\psi^\mu}{|\pm\rangle} \underset{\chi_{12}}{|\pm\rangle} \underset{\chi_{34}}{|\pm\rangle} \underset{\chi_{56}}{|\pm\rangle} |0\rangle_L = 2^4 = 16 \text{ ch}$$

The states in the S-vacuum are

$$|S\rangle_L \, \partial \bar{X}^\mu_{+1} \, |0\rangle_R.$$

The states in the Left-moving sector are in the Ramond vacuum of the space-time fermions

$\Rightarrow$ these states are in the spinorial (spin 1/2) representation of the Lorentz group.

$\Rightarrow$ $|S\rangle_L \partial \bar{X}^\mu_{+1} |0\rangle_R \rightarrow$ spin 3/2 $\rightarrow$ gravitinos

The other states are

$$|S\rangle_L \, \bar{\Phi}^a_{1/2} \, \Phi^b_{1/2} \, |0\rangle_R \rightarrow \text{spin } 1/2.$$

we still have to apply the GSO projections.

OSV, 54)

I use a combinatorial notation.

$$S:\quad \psi^{\mu}\quad \chi_{12}\quad \chi_{34}\quad \chi_{56}$$
$$|\pm\rangle\quad |\pm\rangle\quad |\pm\rangle\quad |\pm\rangle$$

$$\left[\binom{4}{0}+\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4}\right]$$

where $\binom{4}{i}$ counts the number of $|-\rangle$ in a given state.

$$\binom{4}{0} = |+\rangle|+\rangle|+\rangle|+\rangle$$
$$\binom{4}{2} = |-\rangle|-\rangle|+\rangle|+\rangle \qquad \text{etc.}$$

Recall that

$$F_i|\pm\rangle = \begin{cases} 0 & |+\rangle \\ -1 & |-\rangle \end{cases}$$

we have the GSO projections of two vectors $\{1, S\}$

$$1:\ e^{i\pi\, 1\cdot F_s}|S\rangle_s = \delta_s\, C\binom{S}{1}^*$$

$$C\binom{S}{1} = \delta_1\, e^{i\pi\frac{1\cdot S}{2}}\, e^{i\frac{2\pi m_s}{N_s}} = \delta_s\, e^{\frac{i2\pi n_1}{N_1}}$$
$$= (-1)(+1)(\pm 1) = (-1)(\pm 1) \rightarrow \text{two choices}$$
$$1\cdot S = 4 - 0 = 4$$

we can take $C\binom{S}{1} = -1$ then also $C\binom{1}{S} = e^{i\pi\frac{S\cdot 1}{2}}\, C\binom{S}{1}$

$$C\binom{S}{S} = -e^{i\pi\frac{S\cdot S}{4}}\, C\binom{S}{1} = (-)(-)(-) = -1$$

OSV.55

$$C\binom{\alpha}{\beta} \quad \begin{array}{c|cc} & 1 & S \\ \hline 1 & -1 & -1 \\ S & -1 & -1 \end{array} \quad \text{OR} \quad \begin{pmatrix} -1 & +1 \\ +1 & +1 \end{pmatrix}$$

choice.

Therefore we have.

only $C\binom{S}{1}$ OR $C\binom{1}{S}$ are independent the rest are fixed by modular invariance. This become important when the models become more complicated.

We therefore have.

$$e^{i\pi \mathbb{1}\cdot F_s} |S\rangle_S = + |S\rangle_S$$

$$e^{i\pi 1\cdot F_s} |S\rangle_L \bar{\phi}^a \bar{\phi}^b |0\rangle_R = e^{i\pi(1*(\#\text{ of } |-\rangle) - 1\cdot 1 - 1\cdot 1)} |S\rangle_S =$$

$$= e^{i\pi(1\cdot\#\text{ of } |-\rangle)} |S\rangle$$

if ($\#$ of $|-\rangle \Rightarrow$ odd) $\rightarrow$ Projected)

if ($\#$ of $|-\rangle \Rightarrow$ even) $\rightarrow$ stays

So we have that only even states remain OK.

$$|S\rangle_S = \left[\binom{4}{0} + \binom{4}{2} + \binom{4}{4}\right] \bar{\Phi}^a_{\frac{1}{2}} \bar{\Phi}^b_{\frac{1}{2}} \rightarrow \text{spin } 1/2 \rightarrow \text{gaugino}$$

The same is true for

$$\left[\binom{4}{0} + \binom{4}{2} + \binom{4}{4}\right] \partial \bar{X}^\mu_{\frac{1}{2}} \rightarrow \text{spin } 3/2 \rightarrow \text{Gravitino}$$

how many gravitinos are there.

$$\overset{\psi^\mu}{\binom{1}{0}} \left[\binom{3}{0} + \binom{3}{2}\right]^{\chi_{12}\, \chi_{34}\, \chi_{56}} + \binom{1}{1}\left[\binom{3}{1} + \binom{3}{3}\right] \Rightarrow$$ These are the two components of a space time Weyl spinor.

$\Rightarrow$ 4 gravitinos $\Rightarrow$ $N=4$ SUSY.

QSU56 (59)

we still have to check the S projection.

$$e^{i\pi S\cdot F_S}\,|S\rangle_S = \delta_S\, C\binom{S}{S}^* |S\rangle_S = +|S\rangle_S.$$

This projection acts the same as for the $\mathbb{1}$ vector because $S_R \equiv 0$.

So the spectrum remains intact also after the S projection.

Therefore the model contains:

$N=4$ SUSY $\quad SO(44)$ gauge group.

what about the partition function.

$\{1, S, 1+S, NS\}$.

Recall $\quad \binom{1}{1} \to \theta_1(\tau) \equiv 0.$

therefore whenever we have two intersecting periodic form this term vanishes in the Partition function.

for example $\binom{1}{S}; \binom{1}{1}; \binom{1+S}{1} \equiv 0$ but $\binom{1+S}{S} \neq 0$

$$\binom{NS}{S}\ \binom{S}{NS}\ \binom{NS}{NS}\ \binom{1}{NS}\ \binom{1+S}{NS}\ \binom{1+S}{S}\ \binom{NS}{1}\ \binom{NS}{1+S}\ \binom{S}{1+S}$$

9 sectors.

OSU.57

$\binom{0}{0} \to \theta_3 \quad \binom{0}{1} \to \theta_4 \quad \binom{1}{0} \to \theta_2$

$$-\binom{NS}{S} \to \theta_4^4\,\theta_3^6\,\bar\theta_3^{22} \quad -\binom{S}{NS} \to \theta_2^4\,\theta_3^6\,\bar\theta_3^{22} \quad +\binom{NS}{NS} \to \theta_3^4\,\theta_3^6\,\bar\theta_3^{22}$$

$$-\binom{1}{NS} \Rightarrow \theta_2^4\,\theta_2^6\,\bar\theta_2^{22} \quad +\binom{1+S}{NS} \to \theta_3^4\,\theta_2^6\,\bar\theta_2^{22} \quad -\binom{1+S}{S} \to \theta_4^4\,\theta_2^6\,\bar\theta_2^{22}$$

$$-\binom{NS}{1} = \theta_4^4\,\theta_4^6\,\bar\theta_4^{22} \quad +\binom{NS}{1+S} \to \theta_3^4\,\theta_4^6\,\bar\theta_4^{22} \quad -\binom{S}{1+S} \to \theta_2^4\,\theta_4^6\,\bar\theta_4^{22}$$

$$C\binom{NS}{S} = \delta_S = -1 \qquad C\binom{S}{NS} = e^{i\pi\frac{NS\cdot S}{2}}\, C\binom{NS}{S}^* = -1$$

$C\binom{NS}{b_j} = C\binom{b_j}{NS}$

$$C\binom{S}{1+S} = \delta_S\, C\binom{S}{1}\, C\binom{S}{S} = - - - = (-1)$$

$$C\binom{1+S}{S} = e^{\frac{[1+S]\cdot[S]}{2}}\, C\binom{S}{1+S}^* = C\binom{S}{1+S} = -1$$

$$Z = \left[\theta_3^4 - \theta_2^4 - \theta_4^4\right]\left[\theta_3^6\,\bar\theta_3^{22} + \theta_2^6\,\bar\theta_2^{22} + \theta_4^6\,\bar\theta_4^{22}\right]$$

This is the partition function of the $N=4$ model.

$$\left[\theta_3^4 - \theta_2^4 - \theta_4^4\right] = 0 \longleftrightarrow \text{Jacobi identity.}$$

$$\Rightarrow \quad \Lambda_{\text{cosmological}} = 0$$

OSU.58

we have a model with no tachyons.
But 1) the GG. $SO(44)$
2) no matter.
3) $N=4$ ~~et~~ SUSY

$\Rightarrow$ want: 1) matter 2) $GG \rightarrow GG \subset SU(3)\times SU(2)\times U(1)$. 3) N: SUSY

1) $SO(44)$ too large $\rightarrow$ observable $\otimes$ hidden.
2) matter $\rightarrow$ 3 gen.
3) $N=1$ susy.

The game now is to add more basis vectors.
Each added vector gives rise to new sectors and ~~and~~ at the same time imposes GSO projections on the previous sectors

The process then goes as follows.
1) choose intelligently a basis vector (This is the creative part
2) Checks compatibility with the rules.
3) Check for massless states.
4) check type of massless states, i.e. # of oscillator acting on the vacuum? Purely Ramond?
5) Analyze Hilbert space from the new sectors $\{b_1, \dots, b_n$
impose GSO projections on the states from these sectors
6) impose the GSO of b_{n+1} on $H\{b_1, \dots, b_n\}$.

Qu. 59

Add A vector b_1

$$b_1 = \{\psi^\mu\ \chi_{12}\ y_3 y_4\ y_5 y_6 \mid \underset{1}{\overline{y_3 y_4}}\ \underset{1}{\overline{y_5 y_6}}\ \underset{5}{\overline{\psi}_{1,\dots,5}}\ \underset{1}{\overline{\eta}_1}\} \equiv 1$$

check rules:

$N_{1b_1}\, 1\cdot b_1 = 2(4-8) = 0 \bmod 4$ ✓ $N_{Sb_1}\, S\cdot b_1 = 2(2-0) = 0 \bmod 4$ ✓

$N_{b_1}\, b_1 \cdot b_1 = 2\cdot(4-8) = 0 \bmod 8$ ✓

$b_{1L}\cdot b_{1L} = 4 \qquad b_{1R}\cdot b_{1R} = 8$

$$M_L^2 = -\frac{1}{2} + \frac{4}{8} + \underset{\text{no oscilators}}{} = -1 + \frac{8}{8} + \underset{\text{no oscilators}}{} = M_R^2$$

vacuum purely Ramond => no oscilators.

we have to calculate the one loop GSO coefficients.

$$C \quad \begin{array}{c|ccc} & 1 & S & b_1 \\ \hline 1 & -1 & -1 & \vdots \\ S & -1 & -1 & \vdots \\ b_1 & & & \end{array}$$

two new independent GSO: $C\binom{1}{b_1}$ $C\binom{S}{b_1}$

$$C\binom{1}{b_1} = \delta_1 e^{i\frac{2\pi m}{N_{b_1}}} = \delta_{b_1} e^{i\pi\frac{b_1\cdot 1}{2}} e^{i\frac{2\pi n}{N_1}} = \pm 1 \qquad \text{choose } (+1)$$

$$C\binom{S}{b_1} = \delta_S e^{i\frac{2\pi m}{N_{b_1}}} = \delta_{b_1} e^{i\pi\frac{S\cdot b_1}{2}} e^{i\frac{2\pi m}{N_S}} = \pm 1 \qquad \text{choose } (+1)$$

$$C\binom{b_1}{1} = e^{i\pi\frac{b_1\cdot 1}{2}} C\binom{1}{b_1}^* = C\binom{1}{b_1}$$

$$C\binom{b_1}{S} = e^{i\pi\frac{b_1\cdot S}{2}} C\binom{S}{b_1}^* = (-1)(+1) = (-1)$$

OSU.60

First, let's analyze the GSO of b_1 on NS,

$\chi^{1,2}_{1/2}\, \partial X^\mu_+ |0\rangle_{NS}$ $\quad \chi^{3,4}\, \partial X^\mu_+ |0\rangle_{NS}$ $\quad \chi^{5,6}\, \partial X^\mu_+ |0\rangle$

$e^{i\pi b_1 F}$: -1 ✓ $\quad +1$ X $\quad +1$ X $\quad =$

$\psi^\mu \quad \bar\phi^a_{1/2}\, \bar\phi^b_{1/2}$ $\qquad \phi\phi : \{1,1\}\ \{1,0\}\ \{0,1\}\ \{0,0\}$

$e^{i\pi b_1 F}$: $\qquad -1\ -1 \quad -1\ +1 \quad +1\ -1 \quad +1\ +1$

$\psi^\mu \{\bar\psi^{1..5}, \bar\eta_1, \bar y_{34}, \bar y_{56}\}\{\bar\psi^{1..5}, \bar\eta_1, \bar y_{34}, \bar y_{56}\}$ $\quad \psi^\mu \{\bar w^{1,..,6}, \bar y^{1,2}, \bar\eta_2, \bar\eta_3, \bar\phi^{1..8}\}\{\bar w^{1..6} \ldots$

$e^{i\pi b_1 F}$: $-1 \quad -1 \quad -1$ ✓ $\qquad -1 \quad +1 \quad +1$ ✓

$\psi^\mu \{\bar\psi^{1..5}, \bar\eta_1, \bar y_{34}, \bar y_{56}\}\{\bar w^{1..6}, \bar y^{1,2}, \bar\eta_2, \bar\eta_3, \bar\phi^{1..8}\}$

$-1 \quad -1 \quad +1$ X $\qquad$ X

we get: $SO(44) \to SO(16) \times SO(28)$

we also have the states $\chi^{1..6}\, \bar\phi^a_{1/2}\, \bar\phi^b_{1/2} |0\rangle_{NS}$.

similarly we'll get.

$\chi^{1,2}\{\bar w^{1..6}, \bar\eta^2 \bar\eta^3 \bar\phi^{1..8}\}\{\}$ $\quad \chi^{1,2}\{\bar\psi^{1..5}, \bar\eta_1, \bar y_3 \bar y_4, \bar y^5 \bar y_6\}\{\bar\psi^{1,5}, \bar\eta, \bar y^{3..6}\}$,

$- \quad +$ $\quad$ GSO $-1 \ldots \quad -1 \quad -1$ ✓

$\chi^{34,56}\{\bar\psi^{1..5}, \bar\eta_1, \bar y^{3..6}\}\{\bar w^{1..6}, \bar y^{1,2}, \bar\eta^2 \bar\eta^3\, \bar\phi^{1..8}\}$

$+1 \quad -1 \quad +1$ ✓

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next we would analyze the S- sector.

$$S \quad \{\psi^\mu \quad \chi_{12} \quad \chi_{34} \quad \chi_{56}\} \; : \; 1$$

$$\quad |\pm\rangle \quad |\pm\rangle \quad |\pm\rangle \quad |\pm\rangle$$

$|S\rangle_L \partial X^\mu \quad \left[\binom{4}{0} + \binom{4}{2} + \binom{4}{4}\right] \quad \rightarrow$ gravitinos

$b_1 \quad 1 \quad 1 \quad 0 \quad 0 \qquad GSO = \delta_S \, C\binom{S}{b_1}^* = -1$

$$\left[\binom{2}{1}\right]\left[\binom{2}{1}\right] = \left[\binom{1}{0}\binom{1}{1}\right]\binom{2}{1} + \left[\binom{1}{1}\binom{1}{0}\binom{2}{1}\right]$$

2 - spin 3/2 states.

$N=4 \rightarrow N=2$

how about the gauginos

$$\left[\binom{4}{0} + \binom{4}{2} + \binom{4}{4}\right] \qquad \bar{\phi}^a_{\frac{1}{2}} \; \bar{\phi}^b_{1/2} \; |0\rangle_R.$$

$\quad \psi^\mu_{1,2} \quad \chi_{12} \quad \chi_{34} \quad \chi_{56}$

$b_1 \quad 1 \quad 1 \quad 0 \quad 0$

$e^{i\pi b_i F_S} \rightarrow$ acting on the left-side of $|S\rangle_S$

$$\left[\binom{2}{0} + \binom{2}{2}\right]_+ \quad OR \quad \left[\binom{2}{1}\binom{2}{1}\right]_-$$

$e^{i\pi b_1 F_S} \rightarrow$ acting on the right

$A: \quad \left[\{\bar{\psi}^{1..5}\,\bar{\eta}_1\,\bar{y}^{3..6}\}\{\bar{\psi}^{1..5}\,\bar{\eta}_1\,\bar{y}^{3..6}\}\right]_+$ and $\{\bar{w}^{1..6}, y^{1,2}\,\bar{\eta}_2\,\bar{\eta}_3\,\bar{\phi}^{1..8}\}\{\bar{w}^{1..6}\ldots$

OR $B_1 \left[\{\bar{\psi}^{1..5}\,\bar{\eta}_1\,\bar{y}^{3..6}\}_-\{\bar{w}^{1..6}\,y^{1,2}\,\bar{\eta}_2\,\bar{\eta}_3\,\bar{\phi}^{1..8}\}_+\right]_-$

So the invariant state are

$$\left[\binom{2}{0} + \binom{2}{2}\right] \otimes B \quad \text{and} \quad \left[\binom{2}{1}\binom{2}{1}\right] \otimes A$$

These produce the states that complete the $N=2$ supersymmetric representations.

The sector S is the SUSY generator

To know how many supersymmetries are left it is enough to calculate the # of gravitinos that are left in the massless spectrum.

$$|S\rangle_{Left}\ \partial X^{\mu}|0\rangle_R \rightarrow \text{gravitinos.}$$

$|S\rangle_{Left} \rightarrow$ count the number of gravitinos.

if we have at least $N=1$ it is enough to check the spectrum from

$$\{1, , b_1 \cdots b_n\}$$

the sectors $S + \{1, , b_1, \cdots b_n\}$ will give the superpartners.

Two sector $\vec{\alpha}, \vec{\beta} \in \Xi$ related by $\vec{\alpha} = \vec{\beta} + S$ are related by the space-time susy generator.

A convenient procedure which is easily implemented on a computer is to analyse all the sectors that give space-time fermion i.e. $\alpha(\psi^{\mu}) = 1$. Then all the sectors with $\alpha(\psi^{\mu}) = 0$ which give space-time scalars are related by the susy generator S.

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To know the massless spectrum content of a model it's therefore sufficient to analyze the massless fermions.

We have to find the new states from the vector b_1.

b_1 ψ_{12} χ_{12} $y_3 y_4$ $y_5 y_6$ $\bar{y}_3 \bar{y}_4$ $\bar{y}_5 \bar{y}_6$ $\bar{\psi}^{1..5}$ $\bar{\eta}_1$ G.S.O

$|\pm\rangle$ $|\pm\rangle$ $|\pm\rangle$ $|\pm\rangle$ $|\pm\rangle$ $|\pm\rangle$ $|\pm\rangle$ $|\pm\rangle$

$\left[\binom{12}{0} + \binom{12}{1} + \ldots + \binom{12}{12}\right] \rightarrow 2^{12}$ states

$\underline{1}$ | | | | | | ||||| | $\delta_{b_1} C\binom{b_1}{1}^* = +$

$\left[\binom{12}{0} + \binom{12}{2} + \ldots \binom{12}{12}\right] \rightarrow 2^{11}$ states.

S: | | O O O O OOOOO O $\delta_{b_1} C\binom{b_1}{S}^* = +$

$\left[\binom{2}{0}+\binom{2}{2}\right] \left[\binom{10}{even}\right] \rightarrow 2^{10}$ states.

b_1 | | | | | | |||||| | $\delta_{b_1} C\binom{b_1}{b_1} = +$

$\left[\binom{2}{0}+\binom{2}{2}\right] \left[\binom{10}{even}\right]$

This are the states which are left after GSO projections we can write them down as representations of the 4D gauge group but this will not be very illuminating for now.

As we said we also have the sector $S+b_1$.

$S+b_1$: χ_{34} χ_{56} $y_3 y_4$ $y_5 y_6$ $\bar{y}_3 \bar{y}_4$ $\bar{y}_5 \bar{y}_6$ $\bar{\psi}^{1..5}$ $\bar{\eta}^1$

This gives the SUSY partners of $b_1 \rightarrow$ space-time bosons.

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with the addition of more basis vectors the analysis procedes similarly.

new vectors: $\Rightarrow$ new GSO projections on previous spec

new vectors: $\Rightarrow$ new massless sectors.

off course in practice since we gained some experience and intuition with the construction of these models we first specify the

1) B.C
2) GSO phases.

then analyze the spectrum in one scoop.

Let me therefore proceed to add two more basis vectors b_2, b_3

Remark: at this point the analysis of the partition function by hand become very tedious though of course not impossible. In any case as we are mainly interested in the massless spectrum the partition function is not very illuminating for us. The massive spectrum is of interest for example for threshold corrections, etc.

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The new model has $\{1, S, b_1, b_2, b_3\}$

	$\psi^\mu_{1,2}$	$(\chi_1 y_1 \omega_1)$	$(\chi_2 y_2 \omega_2)$	$(\chi_3 y_3 \omega_3)$	$(\chi_4 y_4 \omega_4)$	$(\chi_5 y_5 \omega_5)$	$(\chi_6 y_6 \omega_6)$	$\bar y_1 \bar\omega_1 \bar y_2 \bar\omega_2 \bar y_3 \bar\omega_3 \bar y_4 \bar\omega_4 \bar y_5 \bar\omega_5 \bar y_6 \bar\omega_6$	$\bar\psi_1 \bar\psi_2 \bar\psi_3 \bar\psi_4 \bar\psi_5 \bar\eta_1 \bar\eta_2 \bar\eta_3$
$\mathbf{1}$							$\vec{1}$	$\vec{1}$	
S	11	(100)	(100)	(100)	(100)	(100)	(100)	$\vec{0}$	
b_1	11	(100)	(100)	(010)	(010)	(010)	(010)	000010101010	11111100
b_2	11	(010)	(010)	(100)	(100)	(001)	(001)	101000000101	11111010
b_3	11	(001)	(001)	(001)	(001)	(100)	(100)	010101010000	11111001

This is the so called NAHE set

Rules: we already checked for $\{1, S, b_1\}$

$b_2 \cdot 1 = 4 - 8 = -4 = 0 \bmod 4$. $\quad b_2 \cdot b_1 = 1 - 5 = -4$ ✓ $\quad N_{b_2} b_2 \cdot b_2 = (4-8) \cdot 2 = -8$

$N_{b_2 S}\, b_2 \cdot S = 2 \cdot 2 = 4$ ✓

$b_3 \cdot 1 = 4 - 8 = -4$ ✓ $\quad b_3 \cdot S = 2$ ✓ $\quad b_3 \cdot b_1 = 1 - 5 = -4$ ✓ $\quad b_3 \cdot b_2 = 1 - 5$ ✓ $\quad b_3 \cdot b_3 = -$

G.S.O coefficients

	1	S	b_1	b_2	b_3
1	-1	$+1$	-1	± 1	± 1
S	$+1$	$+1$	$+1$	$\oplus 1$	$\oplus 1$
b_1	-1	$+1$	-1	± 1	± 1
b_2	± 1	$\mp$	± 1	± 1	± 1
b_3	± 1	∓ 1	± 1	± 1	± 1

note that $\mathcal{I}$ changed

← 6 new phas

The next step is to analyse the massless spectrum.

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NS sector

vector bosons

$\psi^\mu \{\bar\psi^{1..5}\bar\psi^{1..5}\}\{\bar\eta^{1}\bar y^{3..6}\}\{\bar\eta^{2}\bar y^{1,2}\bar\omega^{5,6}\}\{\bar\eta^{3}\bar\omega^{1..4}\}\{\bar\phi^{1..8}\}$

$SO(10) \quad SO(6)_1 \quad SO(6)_2 \quad SO(6)_3 \quad SO(16).$

$(\chi^i, y^i, \omega^i)\, \partial\bar X^\mu |0\rangle \rightarrow$ all are out.

Scalars

After b_1 we had

$\chi^{1,2}\{\bar\eta^2\bar\eta^3\, \omega^{1..6}\, y^{1,2}\,\bar\phi^{1..8}\}\{\ \}$ $\quad \chi^{12}\{\bar\psi^{1..5}\,\bar\eta^1\bar y^{3..6}\}\{\}$ $\quad \chi^{3,4,5,6}\{\bar\psi^{1..5}\,\bar\eta^1\bar y^{3..6}\}\{\omega^{1..6}\, y \ldots$

b_2 $\chi^{12}\{\bar\eta^2\bar\omega^{56}\bar y^{1,2}\}\{\bar\eta^3\bar\omega^{1..4}\bar\phi^{1..8}\}$ $\quad \chi^{1,2}\{\psi^{1..5}\}\{\bar\eta^1\bar y^{3..6}\}.$

$\chi^{34}\{\bar\psi^{1..5}\}\{\omega^{56}y^{12}\eta^2\}$

$\chi^{34}\{\bar\eta^1\bar y^{3..6}\}\{\omega^{1..4}\bar\eta^3\,\phi^{1..8}$

$\chi^{56}\{\psi^{1..5}\}\{\omega^{1..4}\eta^3\}$

$\chi^{56}\{\bar\eta^1 y^{3..6}\}\{\omega^{56}y^{1,2}\bar\eta^2\}.$

b_3 $\chi^{1,2}\{\bar\eta^2\bar y^{1,2}\omega^{5,6}\}\{\bar\eta^3\bar\omega^{1..4}\}$ $\quad \chi^{1,2}\psi^{1..5}\{\bar\eta^1 y^{3..6}\}$

$\chi^{3,4}\,\bar\psi^{1..5}\{\bar\omega^{5,6}\bar y^{1,2}\bar\eta^2\}$

$\chi^{3,4}\{\bar\eta^1\bar y^{3..6}\}\{\bar\eta^3\omega^{1..4}\}$

$\chi^{5,6}\{\bar\psi^{1..5}\}\{\bar\eta^3\,\omega^{1..4}\}$

$\chi^{5,6}\{\bar\eta^1\bar y^{3..6}\}\{\bar y^{1,2}\,\omega^{5,6}\,\bar\eta^2$

This is the massless spectrum from the NS sector.

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S- sector how many gravitinos are left?

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\partial \bar{X}^\mu_H$	G.S.O
S	$\|\pm\rangle$	$\|\pm\rangle$	$\|\pm\rangle$	$\|\pm\rangle$		
$\mathbb{1}$	1	1	1	1		$\delta_s\, C\binom{s}{1}^* = -1$
	$\left[\binom{4}{1} + \binom{4}{3}\right]$					
S →	same					
b_1	1	1	0	0		$\delta_s\, C\binom{s}{b_1} = -1$, $\overset{\shortparallel}{1}$
	$\binom{2}{1}$	$\left[\binom{2}{0} + \binom{2}{2}\right]$				
b_2	1	0	1	0		$\delta_s\, C\binom{s}{b_2} = -1$, $\overset{\shortparallel}{\pm 1}$
	$\binom{1}{1}\binom{1}{0}\binom{2}{0} + \binom{1}{0}\binom{1}{1}\binom{2}{2}$					

only 1 gravitino is left.

b_3	1	0	0	1	$\delta_s C\binom{s}{b_3}^* = \pm 1$

$\binom{1}{1}\binom{1}{0}\binom{2}{0} + \binom{1}{0}\binom{1}{1}\binom{2}{2} \Rightarrow C\binom{s}{b_3} = +1$

out $\Rightarrow C\binom{s}{b_3} = -1$

depending on the relative phase of $C\binom{s}{b_2}$ and $C\binom{s}{b_3}$

we can either project or keep the remaining gravitino.

we can construct tachyon free $N=0$ models,

but $\Lambda \neq 0$ $(Z \neq 0)$

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b_1 ψ^μ χ_{12} $\underbrace{y^{3.4}\ y^{56}\ |\ \bar{y}^{34}\ \bar{y}^{56}}$ $\bar{\psi}^{1..5}$ $\bar{\eta}_1$ G.S. 0

1 $\left[\binom{12}{\text{even}}\right]$ $+1$

S $\left[\binom{2}{0}+\binom{2}{2}\right]\left[\binom{10}{\text{even}}\right]$ $+1$

b_2 1 0 0 0 0 0 1 0 $+1$ $\delta_b, C\binom{b_1}{b_2}$

$$\left[\binom{2}{0}\left[\binom{4}{0}+\binom{4}{2}+\binom{4}{4}\right]\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right]\binom{1}{0}+\binom{2}{0}\left[\binom{4}{1}+\binom{4}{3}\right]\left[\binom{4}{1}+\binom{4}{3}\right]\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right]\right.$$

+ the other components by CPT invariance.

$\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right] \Rightarrow$ 16 of SO(10)

$\left[\binom{5}{1}+\binom{5}{3}+\binom{5}{5}\right] \rightarrow$ $\overline{16}$ of SO(10).

$\Rightarrow$ The spectrum is chiral only 16 of SO(10).

b_3 1 0 0 0 0 0 1 0 $\delta_{b_1}\ C\binom{b_1}{b_3} \overset{*}{=}$

once again: we can either keep all the generations from b_1 or project all of them depending on the relative phase of $C\binom{b_1}{b_2}$ $C\binom{b_1}{b_3}$.

if $C\binom{b_1}{b_2} = C\binom{b_1}{b_3}$ we have

16 generations from b_1.

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b_2 ψ^μ χ_{34} $y^{1,2}$ $\omega^{5,6}$ | $\bar{y}^{1,2}$ $\bar{\omega}^{5,6}$ $\bar{\psi}^{1\cdots5}$ $\bar{\eta}^2$

similarly we'll get 16 generations from b_2

b_3 ψ^μ χ_{56} $\omega^{1,2}$ $\omega^{3,4}$ | $\bar{\omega}^{1,2}$, $\bar{\omega}^{3,4}$ $\bar{\psi}^{1\cdots5}$ $\bar{\eta}^3$

similarly we'll get 16 generation from b_3

note the permutation symmetry.

$b_1 \to b_2 \to b_3 \to b_1$

$\begin{Bmatrix} \bar{\eta}_1 \; \bar{y}^{3\cdots6} \\ \chi^{12} \; y^{3\cdots6} \end{Bmatrix} \to \begin{Bmatrix} \bar{\eta}_2 \; \bar{y}^{1,2} \bar{\omega}^{56} \\ \chi^{34} \; y^{1,2} \; \omega^{5,6} \end{Bmatrix} \to \begin{Bmatrix} \bar{\eta}_3 \; \bar{\omega}^{1\cdots4} \\ \chi^{56} \; \omega^{1\cdots4} \end{Bmatrix}$

This permutation symmetry play an important role

we have 48 generations.

we have also the sectors

$S+b_1$; $S+b_2$ $S+b_3$

$\to$ These are scalars

and $1+b_1+b_2+b_3 \to \{\bar{\phi}^{1\cdots8}\}$ periodic

all other Antiperiodic

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$$\xi = 1 + b_1 + b_2 + b_3$$

$$C\begin{pmatrix}\xi\\1\end{pmatrix} \quad C\begin{pmatrix}\xi\\\xi\end{pmatrix} \quad C\begin{pmatrix}\xi\\b_1\end{pmatrix} \quad C\begin{pmatrix}\xi\\b_2\end{pmatrix} \quad C\begin{pmatrix}\xi\\b_3\end{pmatrix}$$

$$C\begin{pmatrix}\xi\\1\end{pmatrix} = C\begin{pmatrix}1+b_1+b_2+b_3\\1\end{pmatrix} = e^{i\pi\frac{1\cdot\xi}{2}} C\begin{pmatrix}1\\\xi\end{pmatrix}^*$$

$$\boxed{1\cdot\xi = 0-8=-8} = C\begin{pmatrix}1\\\xi\end{pmatrix} = \delta_1 C\begin{pmatrix}1\\1\end{pmatrix} C\begin{pmatrix}1\\b_1+b_2+b_3\end{pmatrix} = \delta_1^2 C\begin{pmatrix}1\\1\end{pmatrix} C\begin{pmatrix}1\\b_1\end{pmatrix} C\begin{pmatrix}1\\b_{\cdot}\end{pmatrix}$$

$$= \delta_1^3 C\begin{pmatrix}1\\1\end{pmatrix} C\begin{pmatrix}1\\b_1\end{pmatrix} C\begin{pmatrix}1\\b_2\end{pmatrix} C\begin{pmatrix}1\\b_3\end{pmatrix} =$$

$$= (-) \quad - \quad - \quad - \quad - \quad = -1$$

In general $C\begin{pmatrix}\vec{\alpha}\\\beta\end{pmatrix} = C\begin{pmatrix}m_1 a_1 + \cdots + m_n a_n\\n_1 b_1 + \cdots + n_n b_n\end{pmatrix}$.

$$C\begin{pmatrix}\alpha\\n_1 b_1 + \cdots + n_n b_n\end{pmatrix} = \delta_\alpha C\begin{pmatrix}\alpha\\n_1 b_1\end{pmatrix} C\begin{pmatrix}\alpha\\n_2 b_2 + \cdots + n_n b_n\end{pmatrix} = \delta_\alpha C\begin{pmatrix}\alpha\\\underbrace{b_1 + \cdots + b_1}_{n_1}\end{pmatrix} C\begin{pmatrix}\alpha\\n_2 b_2 + \cdots + n_n b_n\end{pmatrix}$$

$$C\begin{pmatrix}\alpha\\n_1 b_1\end{pmatrix} = \delta_\alpha C\begin{pmatrix}\alpha\\b_1\end{pmatrix} C\begin{pmatrix}\alpha\\(n_1-1)b_1\end{pmatrix} = \delta_\alpha^2 C\begin{pmatrix}\alpha\\b_1\end{pmatrix}^2 C\begin{pmatrix}\alpha\\(n_1-2)b_1\end{pmatrix} = \cdots = \delta_\alpha^{n_1-1} C\begin{pmatrix}\alpha\\b_1\end{pmatrix}^{n_1-1} C\begin{pmatrix}\alpha\\n_1-(n_1\cdots\end{pmatrix}$$

$$= \delta_\alpha^{n_1-1} C\begin{pmatrix}\alpha\\b_1\end{pmatrix}^{n_1}$$

$$C\begin{pmatrix}\alpha\\\underbrace{n_1 b_1 + \cdots + n_n b_n}_{k}\end{pmatrix} = \delta_\alpha^{k-1} \underbrace{\delta_\alpha^{n_1-1} \cdots \delta_\alpha^{n_n-1}}_{k} C\begin{pmatrix}\alpha\\b_1\end{pmatrix}^{n_1} C\begin{pmatrix}\alpha\\b_2\end{pmatrix}^{n_2} \cdots C\begin{pmatrix}\alpha\\b_n\end{pmatrix}^{n_n}$$

$$= \delta_\alpha^{k-k-1+n_1+\cdots+n_n} C\begin{pmatrix}\alpha\\b_1\end{pmatrix}^{n_1} \cdots C\begin{pmatrix}\alpha\\b_n\end{pmatrix}^{n_n}$$

general formula

$$C\begin{pmatrix}\alpha\\n_1 b_1 + \cdots + n_n b_n\end{pmatrix} = \delta_\alpha^{-1+n_1+\cdots+n_n} C\begin{pmatrix}\alpha\\b_1\end{pmatrix}^{n_1} \cdots C\begin{pmatrix}\alpha\\b_n\end{pmatrix}^{n_n}$$

$$C\begin{pmatrix}\alpha\\b_j\end{pmatrix} = e^{i\pi\frac{\alpha\cdot b_j}{2}} C\begin{pmatrix}b_j\\\alpha\end{pmatrix}^* = e^{i\pi\frac{\alpha\cdot b_j}{2}} \delta_{b_j}^{-1+m_1+\cdots+m_n} \prod_{i=1}^{n} C\begin{pmatrix}b_j\\b_i\end{pmatrix}^{*\,m_i}$$

$$* \quad C\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \delta_\alpha^{-1+n_1+\cdots+n_n} \prod_{j=1}^{n} e^{i\pi\frac{\alpha\cdot b_j}{2} n_j} \delta_{b_j}^{(-1+m_1+m_2+\cdots+m_n)n_j} \prod_{i=1}^{n} C\begin{pmatrix}b_j\\b_i\end{pmatrix}^{*\,m_i \cdot n_j}$$

Again: a simple formula to put in a computer.

OSU.71

$C\begin{pmatrix}\xi_3\\1\end{pmatrix} = -1$

$C\begin{pmatrix}\xi_3\\\xi_3\end{pmatrix} = e^{i\pi\frac{\xi_3\cdot\xi_3}{2}} C\begin{pmatrix}\xi_3\\1+b_1+b_2+b_3\end{pmatrix} = \delta_{\xi_3}^{3} C\begin{pmatrix}\xi_3\\1\end{pmatrix} C\begin{pmatrix}\xi_3\\b_1\end{pmatrix} C\begin{pmatrix}\xi_3\\b_2\end{pmatrix} C\begin{pmatrix}\xi_3\\b_3\end{pmatrix}$

$-1 \quad +1 \quad + \quad + \quad + \quad = -1$

$C\begin{pmatrix}\xi_3\\b_1\end{pmatrix} = e^{i\pi\frac{\xi_3\cdot b_1}{2}} C\begin{pmatrix}b_1\\1+b_1+b_2+b_3\end{pmatrix} = \delta_{b_1}^{3} C\begin{pmatrix}b_1\\1\end{pmatrix} C\begin{pmatrix}b_1\\b_1\end{pmatrix} C\begin{pmatrix}b_1\\b_2\end{pmatrix} \begin{pmatrix}b_1\\b_3\end{pmatrix} = -1$

$C\begin{pmatrix}\xi_3\\b_2\end{pmatrix} = -1 \qquad C\begin{pmatrix}\xi_3\\b_3\end{pmatrix} = -1$

ξ_3		$\bar\phi_1 \cdots$	$\bar\phi^8$	GSO
	ψ^μ	$\lvert\pm\rangle$	$\lvert\pm\rangle$	
$\underline{1}$	$\underline{1}$	$1 \cdots$	1	$\delta_{\xi_3} C\begin{pmatrix}\xi_3\\1\end{pmatrix} = -1$
	ψ^μ	$\left[\binom{8}{0}+\binom{8}{2}\right.$	$\left.+\binom{8}{8}\right]$	
S	1	$\vec{0}$ ✓		$\delta_{\xi_3} C\begin{pmatrix}\xi_3\\S\end{pmatrix} = -1$
$b_{1,2,3}$	1	0 ✓		$\delta_{\xi_3} C\begin{pmatrix}\xi_3\\b_1\end{pmatrix} = -1$

These states transform as $2^7 = 128$ of SO(16)

$120 + 128 = 248 \to SO(16) \to E_8$

Scalars $\begin{Bmatrix}x\\y\\w\end{Bmatrix}$ $\lvert\pm\rangle \cdots \lvert\pm\rangle$

1				-1
	1	$\left[\binom{8}{0}+\binom{8}{2}\right. \cdots$	$\left.\binom{8}{8}\right]$	
S	$\chi^{1..6}$	$[\quad$	$\quad]$	-1
b_1	$\chi^{1,2}$	$[\quad$	$\quad]$	-1

$b_{2,3}$ all are out.

Before moving on to the case of the three generation models, I would to mention a simple extention of the NAHE which is not directly relevant to the realistic models but illustrates the structure of this models and also illustrates the relation between (2,2) and (2,0) models

This extention is obtained by adding the basis vector

$$X = \{\bar{\psi}^{1..5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$$

and all the other are antiperiodic
we can fix the phases appropriately but I'll spair you the details.

With this additional basis vector and appropriate choice of phases.
The gauge group is

$$E_6 \times U(1)^2 \times SO(4)^3 \times E_8$$

The vector X produces the $16 + \overline{16}$

in $78 = 45 + 16 + \overline{16} + 1$

and enhances the gauge group to E_6

~~64.2~~ 71.2

The $SO(4)^3$ groups are produced by

$\{\bar{y}^{3\cdots6}\}$, $\{\bar{y}^{1,2}\,\bar{w}^{5,6}\}$ $\{\bar{w}^{1\cdots4}\}$

$SO(4)_1$ $SO(4)_2$ $SO(4)_3$

The three world-sheet complex fermions $\bar{\eta}_1, \bar{\eta}_{2,}$ produce three $U(1)$ currents.

one combination is embedded in E_6.

$$U(1)_{E_6} = U(1)_{\bar{\eta}_1} + U(1)_{\bar{\eta}_2} + U(1)_{\bar{\eta}_3}$$

and we have two orthogonal combinations.

$$U(1)_{\bar{\eta}_1} - U(1)_{\bar{\eta}_2}$$

$$U(1)_{\bar{\eta}_1} + U(1)_{\bar{\eta}_2} - 2\,U(1)_{\bar{\eta}_3}$$

The important thing to note is that the X basis vector splits $\{\bar{y}\,\bar{w}\}$ from $\bar{\eta}_1, \bar{\eta}_2$

There is a symmetry here between the left and right movers. we have $N=2$ in the right and left moving sector

$$T_{F_L} = e^{i X_{i,i+1}}(y_i w_i \pm i y_{i+1} w_{i+1}) \qquad T_{F_R} = e^{i\bar{\eta}_i}(\bar{y}_i \bar{w}_i + i \bar{y}_{i+1}\bar{w}_{i+1})$$

and the $U(1)$ currents are generated by in N:

$$U(1)_L = \chi_{12} + \chi_{34} + \chi_{56}$$

$$U(1)_R = \bar\eta_1 + \bar\eta_2 + \bar\eta_3$$

and for any choice of B.C. the $N=2$ T_F is well defined.

However we can generate this model in a different way.
we can start with

$$\{1, S, X, \xi = 1 + b_1 + b_2 + b_3\}.$$

with appropriate choice of phases this gives

$$N=4 \qquad SO(12) \times E_8 \times E_8.$$

now twist by b_1 and b_2. This gives

$$N=1 \qquad SO(4)^3 \times E_6 \times U(1)^2 \times E_8$$

The same group. Indeed it is the same m
we can choose any set of basis vectors as long as they produce the same additive group and are careful with the phases

~~79~~ 134

OSU lectures

just to be complete, I should mention also check what happens with the matter sector.

Let's examine for example b_1 after the projection of the NAHE set

The vacuum was

$$\left[\binom{2}{0}\right]\left\{\left[\binom{4}{0}+\binom{4}{2}+\binom{4}{4}\right]\underbrace{\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right]}_{\bar\psi^{1\cdots5}}\binom{1}{0}_{\bar\eta_1}+\left[\binom{4}{1}+\binom{4}{3}\right]\underbrace{\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right]}_{\bar\psi^{1\cdots5}}\left(\right.\right.$$

+ the CPT components.

Since $X = \{\vec{O}_L \mid O_R \; \underbrace{\bar\psi_1^{\cdots5}\;\bar\eta_1\;\bar\eta_2\;\bar\eta_3}\;\; \vec{O}_R\}$

we see that in the b_1 vacuum we have either odd or even number of $|->$ among the ~~state~~ fermions which are periodic in X. Then for any choice of GSO ½ of those are projected out

now I will not go into detail but the spectrum is easily worked out.

F1.5

we find that

$$b_j + X \longrightarrow 8 \times (10_j + 1_j + 1_j) + (E_8 \times E_8)$$

The 10 + 1 combine with the 16 of SO(10) to form 27 of E_6.

so we have

$8 * 3\ (27)$ of $E_6 = 24$ generations

The other 1_j are singlets of E_6 and are the twisted moduli with an equal number to the number of generations. Also we get that the scalar reps. from the NS sector are completed to $3(27 + \overline{27})$ of E_6. and ~~and~~ equal number of untwisted moduli

There are ~~also~~ also ~~E~~ $E_8 \times E_8$ singlets charged under $SO(4)^3$

69.6

The final remark that I wanted to make is that we can project the E_6 generators by
The choice of the GSO projection

$$C\begin{pmatrix}\xi\\ X\end{pmatrix} = \pm 1$$

One choice will produce the model that I just described
The other choice will project.
The generators in X and ξ that produce the 128 in $E_8 \times E_8$
This is easy to see

$$\begin{array}{lcr} X & \psi^{1..5}\ \bar\eta^1\ \bar\eta^2\ \eta^3 & \text{GSO} \\ \xi & \vec{\sigma} & \pm 1 \end{array}$$

$$\{\underbrace{e^{i\pi\xi\cdot F_X}}_{=\,+1} = \delta_X\, C\begin{pmatrix}X\\ \xi\end{pmatrix}\}|S\rangle_X$$

$-1 \Rightarrow$ all out.

Then the gauge group that we get is $N=4$ $SO(12) \times SO(16) \times SO(16)$.

19.7

20

When we now apply the b_1, b_2 projections we get:

$N=1$ $\quad SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$.

with $\quad 24 * 16$ of $SO(10)$.

and $\quad 3(10+10) + (1, 4)$

from the NS sector.

A similar thing operates in the fermionic three generation models.

The important thing is that the X projection splits the $\{\bar{y}, \bar{\omega}\}$ from the $\{\bar{z}\}$

compactified space

The degeneracy of the Ramond vacua under the $\{y, \omega \mid \bar{y}, \bar{\omega}\}$ produces the multiplicity of generations.

19.8

The models remember some of their (2,2) structure

The important for extentions is the symmetry between $\{y w \mid \bar{y} \bar{w}\}$

The structure that I just elaborated is the key to the Gepner construction that could also split the gauge degrees of freedom from the internal CFT degrees of freedom and find modular invariant solutions for more complica minimal models. now we see that also in the (2,0) This structure can be preserved

OSU.72 (71)

we have analyzed the full massless spectrum of the NAHE set.

we have: $N=1$ SUSY
: $SO(10)$ observable G.G.
48 generations

The construction of the realistic models is really only starting after the NAHE set.

$\bar\psi^{1..5} \to SO(10)$

The states are

$\psi^\mu \bar\Psi^{1.5} \bar\Psi^{1.5} |0\rangle_{NS}$ $\qquad \bar\psi^1 \dots \bar\psi^5$

$SO(6) \leftarrow \psi^\mu \bar\psi^{1..3} \bar\psi^{1..3} |0\rangle$ $\qquad \leftarrow 1\,1\,1\,0\,0$;

$SO(4)$, $\psi^\mu \bar\psi^{4,5} \bar\psi^{4,5}$

$\Rightarrow SO(10) \to SO(6) \times SO(4)$

$\bar\psi^{1..5}$

$\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}$;

$e^{i\pi\alpha \cdot F} |S\rangle_{NS} = \delta_\alpha$.

Suppose $\delta_\alpha = -1 \Leftrightarrow \alpha(\psi^\mu) = 1$.

$\psi^\mu \; \bar\psi^{1..5*} \; \bar\psi^{1..5*}$
$\bar\psi^{1..5*} \; \bar\psi^{1..5}$
$\bar\psi^{1..5} \; \bar\psi^{1..5}$

OSU, 73)

$$e^{i\pi\left(1-\left\{\frac{1}{2}(-1)+\frac{1}{2}(-1)\right)\right)} = e^{i\pi(1+1)} = +1 \times \quad X \neq \delta_\alpha$$

$$e^{i\pi\left(1-\left\{\frac{1}{2}(1)+\frac{1}{2}(1)\right)\right)} = e^{i\pi(1-1)} = +1 \times \neq \delta_\alpha$$

$$e^{i\pi\left(1-\left(\frac{1}{2}(-1)+\frac{1}{2}(+1)\right)\right)} = e^{i\pi} = -1 \checkmark = \delta_\alpha.$$

$\Rightarrow$ only $\psi^\mu \, \bar{\psi}^{*1..5} \, \bar{\psi}^{1..5} |0\rangle$ SURVIVE.

$\Rightarrow$ $SO(10) \rightarrow U(5) = SU(5) \times U(1)$.

Suppose we have.

$$\begin{array}{lccccc} & \bar{\psi}_1 & \bar{\psi}_2 & \bar{\psi}_3 & \bar{\psi}_4 & \bar{\psi}_5 \\ \alpha: & 1 & 1 & 1 & 0 & 0 \\ \beta: & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}$$

$\alpha \rightarrow SO(10) \rightarrow SO(6) \times SO(4)$; β

$\beta \rightarrow SO(6) \times SO(4) \longrightarrow SU(3) \times U(1)_{B-L} \times SU(2) \times U(1)_{5R}$

This is how the gauge groups are obtained in the FF[...]

Thus we can construct models with

$$SU(5) \times U(1)$$
$$SO(6) \times SO(4)$$
$$SU(3) \times SU(2) \times U(1)^2$$

gauge groups.

05.4.74 (72)

how about the number of generations.

I will analyze only one of the vectors b_1 & b_2 to see how the generations are ~~to a~~ obtained

The number of generations is controlled by the boundary conditions of $\{y\,\omega\,|\,y\,\omega\}$

1) The boundary conditions of $\{\psi^\mu\ \chi_{12}\ \chi_{34}\ \chi_{56}\}$ are fixed by requiring $N=1$ SUSY.

2) we donot want to break $SU(3) * SU(2) \to \bar\psi^{1\ldots5} = \begin{cases} 1\ 1\ 1\ 0\ 0 \\ \text{or/and.} \\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2} \end{cases}$

3) The number of generation is reduced by adding three additional basis vectors.

Remark with the type of boundary condition that we are allowing 12 - right movers can be real or complexified to form six complex fermions so far this choices has no significance. however in the construction beyond the NAHE set these pairings ~~are~~ play an important role in the phenomenology of the models.

So we can choose the pairing.

$y^3\bar y^3\ \ y^4\bar y^4\ \ y^5\bar y^5\ \ y^6\bar y^6\ |\ y^1\bar y^1\ \ y^2\bar y^2\ \ \omega^5\bar\omega^5\ \ \omega^6\bar\omega^6\ |\ \omega_1\bar\omega_1\ \ \omega_2\bar\omega_2\ \ \omega_3\bar\omega_3\ \ \omega$

OSU.75

Let's now construct a three generation model with standard model gauge group.

We need three additional basis vectors

As we saw in the discussion on the E_6 model half of the generations are projected by the vector X.

In the three generation model we have something similar.

we have a basis vector with

$$\gamma \rightarrow \gamma(\psi^{1..5}\ \bar\eta^1\ \bar\eta^2\ \bar\eta\ \bar\phi^{1..4}) = 1/2$$

gamma.

$$\gamma\{\text{others}\} = \{0,1\}.$$

Then $2\gamma \rightarrow 2\gamma(\bar\psi^{1..5}\ \bar\eta^{1,2,3}\ \bar\phi^{1..4}) = 1$

$$2\gamma(\text{others}) = 0.$$

The effect of $\overrightarrow{2\gamma}$ vector is to project ½ of the generations ~~just~~ in the same way that the vector X did. This will choose the sign under $U(1)_{\bar\eta_i}$ to be $+1/2$ or $-1/2$ for all

73

$SU(2)_4$ / Q_{UH}

The remaining degeneracy is due to the degeneracy of the zero modes under $\{y, \omega \mid \bar{y}, \bar{\omega}\}$

The NAHE splits those into three groups

$$\{y^{3\cdots6} \mid \bar{y}^{3\cdots6}\} \quad \{y^1 y^2 \omega^{5,6} \mid \bar{y}^{1,2} \bar{\omega}^{5,6}\} \quad \{\omega^{1\cdots4} \mid \bar{\omega}\ldots$$

$$\begin{array}{lcccccccccccccc}
b_1 & 1 & 1 & 1 & 1 & & 0 & 0 & & 0 & 0 & & 0 & 0 & 0 \\
b_2 & 0 & 0 & 0 & 0 & & 1 & 1 & & 1 & 1 & & 0 & 0 & 0 \\
b_3 & 0 & 0 & 0 & 0 & & 0 & 0 & & 0 & 0 & & 1 & 1 & 1
\end{array}$$

$$\left[\binom{4}{0}+\binom{4}{2}+\binom{4}{4}\right]_{b_1} \quad \left[\binom{4}{0}+\binom{4}{2}+\binom{4}{4}\right]_{b_2} \quad \left[\binom{4}{0}+\binom{4}{2}+\binom{4}{4}\ldots\right.$$

To reduce the number of generations we have to reduce this degeneracy by using additional b.c. basis vectors.

Projected by

$$b_1 \quad \psi^\mu \ \chi_{1,2} \ \left[\binom{2}{0}\right] \quad y_3 \bar{y}_3 \ y_4 \bar{y}_4 \ y_5 \bar{y}_5 \ y_6 \bar{y}_6 \ \left[\binom{4}{0}+\binom{4}{2}+\binom{4}{4}\right] \quad \psi^{1\cdots5} \ \left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right] \quad \bar{\eta} \ \left[\binom{1}{0}\right] + \left[\binom{4}{1}+\binom{4}{3}\right]\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\ldots\right.$$

$$\alpha = (11(100)(100)(010)(001)(001)(010)\ 00100101 \mid 0\ \underbrace{111\ 00}_{SO(6)\times SO(4)}\ \underbrace{1}_{\bar{\eta}_1}\ 00\ 00000$$

$\alpha\cdot b_1 = 2-4=-2 \qquad \alpha\cdot b_2 = 1-3=-2 \qquad \alpha\cdot b_3 = 1-3=-2 \qquad \alpha\cdot\alpha = 2-4-2=-4$

$$1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1\ 1\ 1 \quad 1 \qquad GSO=+1.$$

$$\left[\binom{2}{0}\right]\left[\binom{4}{0}+\binom{1}{1}\binom{2}{0}\binom{1}{1}+\binom{1}{0}\binom{2}{2}\binom{1}{0}+\binom{4}{4}\right]_+\left[\binom{3}{0}\binom{2}{0}+\binom{3}{0}\binom{2}{2}+\binom{3}{2}\binom{2}{2}+\binom{3}{2}\binom{2}{0}\right]_+\binom{1}{0}_+ +$$

$$\left[\binom{1}{0}\binom{2}{1}\binom{1}{1}+\binom{1}{1}\binom{2}{1}\binom{1}{0}\right]_-\left[\binom{3}{1}\binom{2}{1}+\binom{3}{3}\binom{2}{1}\right]_-\binom{1}{0}_+$$

OSU,77 $\Rightarrow$ we have 4 full 16 of $so(10)$.

Add another β

$\beta\,(11(001)(010)(100)(100)(001)(010)\,01100000110\;11100010\;00000\ldots$

$\beta\cdot b_1 = 1-3=-2\quad \beta\cdot b_2 = 2-4=-2\quad \beta\cdot b_3 = 1-3=-2\quad \beta\cdot\alpha = 1-3-2=-4\quad \beta\cdot\beta = 2-6=-4.$

$b_1\quad \psi^\mu\quad \chi_{12}\quad y_3\bar{y}_3\quad y_4\bar{y}_4\quad y_5\bar{y}_5\quad y_6\bar{y}_6\quad \bar{\psi}_1\quad \bar{\psi}_2\quad \bar{\psi}_3\quad \bar{\psi}_4\quad \bar{\psi}_5\quad \bar{\eta}_1$

$\beta:\quad 1\quad 1\quad 0\quad 0\quad 0\quad 1\quad 1\quad 1\quad 1\quad 0\quad 0\quad 0\qquad GSO = +1$

$$\left[\binom{2}{6}\right]_+ \left[\binom{4}{0}+\binom{1}{0}\binom{2}{2}\binom{1}{0}\right]_+ \left[\binom{3}{0}\binom{2}{2}+\binom{3}{0}\binom{2}{0}+\binom{3}{2}\binom{2}{2}+\binom{3}{2}\binom{2}{0}\right]_+ \binom{1}{0}_+ +$$

$$\left[\binom{}{6}\binom{2}{1}\binom{1}{1}\right]_- \left[\binom{3}{1}\binom{2}{1}+\binom{3}{3}\binom{2}{1}\right]_- \binom{1}{6}_+$$

$\Rightarrow$ we have 2 full 16 of $so(10)$.

Add another.

$\gamma:\,(11(010)(010)(001)(010)(100)(100)\;10100110\,0000\;\tfrac12\tfrac12\tfrac12\tfrac12\;\tfrac12\tfrac12\tfrac12\tfrac12\;\tfrac12\tfrac12\tfrac12$

$N_\gamma=4$

$\gamma\cdot 1 = 2-\frac{1}{2}\cdot 14 = 2-7=-5\quad \gamma\cdot b_1 = 1-\frac12\cdot 6=-2\quad \gamma\cdot b_2 = 1-\frac12\cdot 6=-2\quad \gamma\cdot b_3 = 2-3=-1$

$\gamma\cdot\alpha = 1-\frac12\cdot 4 = -1\quad \gamma\cdot\beta = 1-\frac12 4 = -1\quad \gamma\cdot\gamma = 2-\frac14\cdot 12 = -1 = -2$

$$C\binom{b_1}{\gamma} = \delta_{b_1} e^{i\frac{\pi 2\cdot m}{N_\gamma}} = \delta_\gamma e^{i\frac{\pi b_1\cdot\gamma}{2}} e^{i\frac{2\pi n}{N_{b_1}}} = \pm 1.$$

$$C\binom{b_3}{\gamma} = \delta_{b_3} e^{i\frac{2\pi m}{N_\gamma}} = \delta_\gamma e^{i\frac{\pi b_3\cdot\gamma}{2}} e^{i\frac{2\pi n}{N_{b_3}}} = \pm i.$$

the sign of under U(1) is fixed by $C\binom{b_j}{\gamma}$

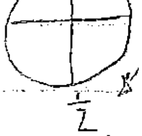
$b_1\quad \psi^\mu\quad \chi_{12}\quad y_3\bar{y}_3\quad y_4\bar{y}_4\quad y_5\bar{y}_5\quad y_6\bar{y}_6\quad \bar{\psi}_1\quad \bar{\psi}_2\quad \bar{\psi}_3\quad \bar{\psi}_4\quad \bar{\psi}_5\quad \bar{\eta}_1\qquad GSO_\gamma = +1,$

$\gamma\quad 1\quad 1\quad 0\quad 1\quad 0\quad 0\quad \tfrac12\quad \tfrac12\quad \tfrac12\quad \tfrac12\quad \tfrac12\quad \tfrac12$

$$e^{i\pi\gamma\cdot F_{b_1}}|S\rangle_{b_1} = +1\,|S\rangle_{b_1} = e^{i\pi(\underbrace{\gamma_L\cdot F_{b_{1L}}}_{(\pm 1)} - \gamma_R\cdot F_{b_{1R}})}|S\rangle_{b_1}.$$

$(-1)\,|-\rangle$

$0\,|+\rangle$



$\frac12$.

OSV.78

$$b_1: \left[\binom{2}{0}\right]\left\{\left[\binom{4}{0}\right]_+ \left[\binom{3}{0}\binom{2}{0}+\binom{3}{2}\binom{2}{2}\right]_+\binom{1}{0}\right. +$$

$$+ \left[\binom{1}{0}\binom{2}{2}\binom{1}{0}\right]_- \left[\binom{3}{0}\binom{2}{2}+\binom{3}{2}\binom{2}{0}\right]_- \binom{1}{0}_+ +$$

$$+ \left[\binom{1}{0}\binom{1}{0}\binom{1}{1}\binom{1}{1}\right]_+ \left[\binom{3}{3}\binom{2}{1}\right]_+ \binom{1}{0}_+ + \left[\binom{1}{0}\binom{1}{1}\binom{1}{0}\binom{1}{1}\right]_- \left[\binom{3}{1}\binom{2}{1}\right]\binom{1}{0}$$

$\Rightarrow$ we have one 16 of $SO(10)$

we can achieve this reduction for the three sectors b_1, b_2, b_3.

it is sufficient to focus on $\{y\omega|\bar{y}\bar{\omega}\}$.

	$y_3\bar{y}_3$	$y_4\bar{y}_4$	$y_5\bar{y}_5$	$y_6\bar{y}_6$	$y_1\bar{y}_1$	$y_2\bar{y}_2$	$\omega_5\bar{\omega}_5$	$\omega_6\bar{\omega}_6$	$\omega_1\bar{\omega}_1$	$\omega_2\bar{\omega}_2$	$\omega_3\bar{\omega}_3$	$\omega_4\bar{\omega}_4$
α	1	0	0	1	0	0	1	0	0	0	0	1
β	0	0	0	1	0	1	1	0	1	0	0	0
γ	0	1	0	0	1	0	0	0	0	1	0	0

we get 3 generations under b_1, b_2, b_3.

what about the Higgs?

$$\chi_{12}\ \bar{\psi}^{1\cdots5}\ \bar{\eta}_1\ |0\rangle_{NS} \to \psi^\mu\bar{\psi}^{1\cdot3}\bar{\eta}_1|0\rangle_{NS} \to D, \quad \psi^\mu\bar{\psi}^{45}\bar{\eta}_1|0\rangle_{NS} \to h \qquad GSO = \delta_\alpha = -1.$$

α: 1 11100 1

$$e^{i\pi\alpha\cdot F_{NS}}\{\chi_{12}\ \bar{\psi}^{1\cdot3}\ \bar{\eta}_1\}|0\rangle_{NS} = -1 \checkmark \Rightarrow D \text{ in}$$
$-1 \quad -1 \quad -1$

$$e^{i\pi\alpha\cdot F_{NS}}\{\chi_{12}\ \bar{\psi}^{45}\ \bar{\eta}_1\}|0\rangle_{NS} = +1 \times \Rightarrow h \text{ out}$$
$-1 \quad +1 \quad -1$

bad

OSV.79

Triplets in the spectrum, doublets are out } bad.

how can we fix that ?

$\alpha(\bar\eta_1) \to 0$

but then $\alpha \cdot b_1 = 2 - 3 = -1$ X $\alpha \cdot b_1 = 0 \bmod 2$.

The way to fix this is by the choice of left-right pairings.

$\overline{y_3 y_6} = \frac{1}{\sqrt{2}}(y_3 + i y_6)$ $y_3 y_6 = \frac{1}{\sqrt{2}}(y_3 +$

	$y_3 y_6$	$y_4 \bar y_4$	$y_5 \bar y_5$	$\bar y_3 \bar y_6$	$y_1 \omega_6$	$y_2 \bar y_2$	$\omega_5 \bar\omega_5$	$\bar y_1 \omega_6$	$\omega_1 \omega_3$	$\omega_2 \bar\omega_2$	$\omega_4 \bar\omega_4$
α $\psi^\mu \chi_{12}$	1	0	0	0	0	0	1	1	0	0	1
β $\psi^\mu \chi_{34}$	0	0	1	1	1	0	0	0	0	1	0
γ $\psi^\mu \chi_{56}$	0	1	0	1	0	1	0	1	1	0	0

in this case we still get 3 generations.

χ_{12} $\bar\psi^{1,\dots,3}$ $\bar\eta_1$

α : -1 -1 $+$ $= +1$ X $\to$ out.

χ_{12} $\bar\psi^{4,5}$ $\bar\eta_1$

α : -1 $+$ $+$ $= -1$ ✓ $\to$ in.

we projected the triplets and kept the doublet.

A stringy doublet-triplet splitting mechanism.

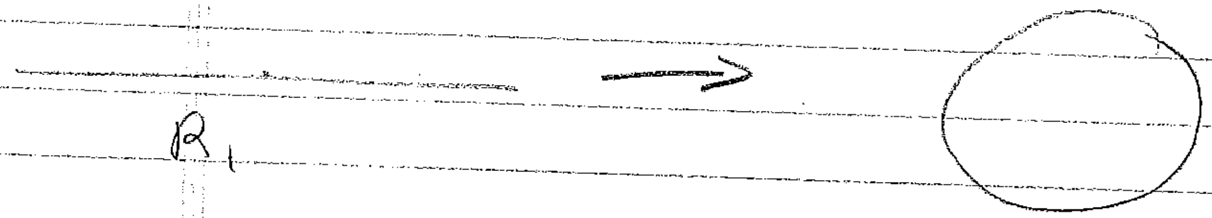
05 V 80 (75)

I now turn to discuss orbifold constructions which are similar to the fermionic construction

Toroidal compactifications

→ compactify on a flat torus

1D : $X \sim X + 2\pi R L \qquad L \in \mathbb{Z}$



$$S_1 = \frac{R_1}{2\pi R L}$$

on a string

$$X(\sigma, \tau) \qquad 0 \leq \sigma \leq 2\pi$$

periodicity: $X(\sigma + 2\pi, \tau) = X(\sigma, \tau) + 2\pi R L$

strings that close on a circle

mode expansion: $X(\sigma,\tau) = X_0 + 2P_0\tau + LR\sigma + i\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n e^{-in(\tau-\sigma)} + \tilde{\alpha}_n e^{-in}\right.$

and $[x^0, p^0] = i$.

$e^{ip\cdot X}$ → single valued → $p = \frac{M}{R} \qquad M \in \mathbb{Z}$

OSU.81

The solution can again be split into left and right movers.

$$X_L(\tau+\sigma) = \frac{1}{2}X + \left(P + \frac{1}{2}LR\right)(\tau+\sigma) + i\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n e^{-in(\tau}$$

$$X_R(\tau-\sigma) = \frac{1}{2}X + \left(P - \frac{1}{2}LR\right)(\tau-\sigma) + i\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in(\tau-}$$

The mass operator now becomes.

$$m_L^2 = \frac{1}{2}\left(\frac{M}{R} + \frac{1}{2}LR\right)^2 + N_L - 1$$

zero modes. momenta and winding number. oscilators. normal ordering

$$m_R^2 = \frac{1}{2}\left(\frac{M}{R} - \frac{1}{2}LR\right)^2 + N_R - 1.$$

$$m^2 = m_L^2 + m_R^2 = \frac{M^2}{R^2} + \frac{1}{4}L^2R^2 + N_L + N_R - 2.$$

momenta in the compact dimension. energy to wind the string on the compactified dimension.

<u>Reparametrization constraints on physical states</u>

$$m_L^2 = m_R^2 \qquad N_R - N_L = ML.$$

OSV.82

Lowest state → tachyon.

At the one loop order.

$$|G^{\mu\nu}\rangle = \alpha_{-1}^{\mu}\bar{\alpha}_{-1}^{\nu}|0\rangle \qquad \mu,\nu = 0,$$

also
$$|V_1^{\mu}\rangle = \alpha_{-1}^{\mu}\bar{\alpha}_{-1}^{25}|0\rangle \qquad U(1)$$
$$|V_2^{\mu}\rangle = \alpha_{-1}^{25}\bar{\alpha}_{-1}^{\mu}|0\rangle \qquad U(1)$$

$$R = \langle\phi\rangle \leftarrow |\phi\rangle = \alpha_{-1}^{25}\bar{\alpha}_{-1}^{25}|0\rangle \rightarrow \text{Scala}$$

This is what we will have also in field theory.

new feature in string theory.
→ non trivial internal momenta and winding number.

take $M = \pm L = \pm 1 \Rightarrow$

$M = L = \pm 1$

$$m_L^2 = \frac{1}{2}\left(\frac{1}{R} + \frac{1}{2}R\right)^2 + N_L - 1 = \frac{1}{2}\frac{1}{R^2} + \frac{1}{8}R^2 + N_L - \frac{1}{2}$$

$$m_R^2 = \qquad\qquad = \frac{1}{2R^2} + \frac{1}{8}R^2 + N_R - \frac{3}{2}$$

OSU, 8

$$m^2 = \frac{1}{R^2} + \frac{1}{4}R^2 + N_L + N_R - 2$$

The level matching conditions are satisfied with,

$$N_L = 0 \qquad N_R = 1$$

we have two vector states

$$|V_a^\mu\rangle = \alpha_{-1}^\mu \,|\overset{M}{\pm 1}\,;\,\overset{L}{\pm 1}\rangle \qquad \begin{matrix} a = \\ \mu = 1 \end{matrix}$$

and two scalar states.

$$|\phi_a\rangle = \alpha_{25}^\mu \,|\pm 1\,;\,\pm 1\rangle$$

The mass depend on the Radius of the compactified dimension.

$$m^2(R) = \frac{1}{R^2} + \frac{1}{4}R^2 - 1$$

we'll get similar states for

$$M = -L = \pm 1.$$

with

$$N_L = 1 \qquad N_R = 0$$

OSU,84

$$|V_a^{1\,\mu}\rangle = \bar{\alpha}^{\mu}_{-1}\;|\pm1;\ \mp1\rangle \quad a=1,2,\ \mu=1,\ldots$$

$$|\phi_a^{1}\rangle = \bar{\alpha}^{25}_{-1}\;|\pm1;\ \mp1\rangle$$

with

$$m^2(R) = \frac{1}{R^2} + \frac{1}{4}R^2 - 1.$$

we have

$$m^2(R) \geq 0$$

with $m^2(R) = 0$ for $R = \sqrt{2}$.

for $R=\sqrt{2}$ we have

$$U(1)_L \times U(1)_R \longrightarrow SU(2)_L \times SU(2)_R.$$

The symmetry is enhanced.

with $(M+L,\ M-L)$ the quantum numbers under the Cartan subalgebra $U(1)_L \times U(1)_R$.

(we also have tachyons for $M=\pm1\ \ L=0$
$M=0\ \ L=\pm1$)

OSU.85

and the massless scalars

$$\alpha_{-1}^{25}|\pm1,\pm\rangle \quad \bar{\alpha}_{-1}^{25}|\pm1\,\mp1\rangle \quad |\pm 2,0\rangle \quad |0$$

which together with $\alpha_{-1}^{25}\,\bar{\alpha}_{-1}^{25}|0,0\rangle$

forms the adjoint of

$$SU(2)_L \times SU(2)_R,$$

for $R \neq \sqrt{2}$

$$SU(2)_L \times SU(2)_R \to U(1)_L \times U(1)_R.$$

massless for any R.

moduli $\longrightarrow |\phi\rangle = \alpha_{-1}^{25}\,\bar{\alpha}_{-1}^{25}\,|0\rangle$

flat potential.

$\langle\phi\rangle \sim R$ - Radius of compactified dimension.

duality.

$$R \leftrightarrow \frac{2}{R}.$$

with.

$$M \longleftrightarrow L$$

The spectrum is invariant under this exchange

05.V.86

This is the target-space duality of string theory which can be generaliz to more compactified dimensions

Remark: in M-theory,

ϕ - dilaton - moduli of 11-dimension.

duality in the 11^{th} dimension correspond to duality of the gauge coupling of the string in 10D.

$R > \sqrt{2}$ is the same as $R < \sqrt{2}$

$R = \sqrt{2} \to$ self-dual point

This is generalized to D-compactified dimen.

compactify D-dimensions on a flat Torus T^D

$$T^D = \frac{\mathbb{R}^D}{\Lambda^D}$$

OSU.87

The torus coordinates are identified as coordinates in R^D up to a lattice translation.

$$X^I \sim X^I + \sqrt{2}\,\pi \sum_{i=1}^{D} n_i R_i e_i^I = X^I + 2\pi L$$

$n_i \in \mathbb{Z}$

$$L^I = \sqrt{\frac{1}{2}} \sum_{i=1}^{D} n_i R_i e_i^I$$

and $e_i = \{e_i^I\}$ $i = 1, \ldots, D$

D-dimensional - linearly independent vectors.

with $(e_i)^2 = 2$.

$L = \{L^I\}$ is a D-dimensional latt

center of mass position and momentum satisfy the usual commutation relations.

$$[X^I, P^J] = i\delta^{IJ}$$

$e^{iP^I X^I} \rightarrow$ single valued.

$\Rightarrow P^I$ are on the dual lattice (Λ^D)

OSU.88

(19)

$$P^I = \sqrt{2} \sum_{i=1}^{D} \frac{m_i}{R_i} e_i^{*I}$$ dual lattice

the e_i^{*I} satisfy.

$$\sum_{I=1}^{D} e_i^I e_j^{*I} = \delta_{ij}.$$

$$(e_i^*)^2 = \frac{1}{2} \Rightarrow \sum_{i=1}^{D} e_i^I e_i^{*J} = \delta^{IJ}$$

The basis vectors of $(\Lambda^D)^*$ are $\frac{\sqrt{2}}{R_i} e_i^*$

The boundary condition for the closed string is now.

$$X^I(\sigma + 2\pi, \tau) = X^I(\sigma, \tau) + 2\pi L^I$$

The L^I play the role of winding numb

The mode expansion becomes

$$X_L^I(\tau+\sigma) = \frac{1}{2} X^I + \left(P^I + \frac{1}{2} L^I\right)(\tau+\sigma) + i \sum_{n\neq 0} \frac{1}{n} \bar{\alpha}_n^I e^{-in(}$$

$$X_R^I(\tau-\sigma) = \frac{1}{2} X^I + \left(P^I - \frac{1}{2} L^I\right)(\tau-\sigma) + i \sum_{n\neq 0} \frac{1}{n} \alpha_n^I e^{in(\tau}$$

OSU89

and the mass formula p_L

$$m_L^2 = \frac{1}{2}\sum_{i=1}^{D}\left(p^I + \frac{1}{2}L^I\right)^2 + N_L - 1 = \frac{1}{2}p_L^2 + N_L - 1$$

$$m_R^2 = \frac{1}{2}\sum_{i=1}^{D}\left(p^I - \frac{1}{2}L^I\right)^2 + N_R - 1 = \frac{1}{2}p_R^2 + N_R - 1$$

p_R.

$$m^2 = m_L^2 + m_R^2 = \sum_{I=1}^{D}\left(p^I p^I + \frac{1}{4}L^I L^I\right) + N_R + N_L - 2$$

$$= \sum_{i,j=1}^{D}\left(m_i g^*_{ij} m_j + \frac{1}{4} n_i g_{ij} n_j\right) + N_L + N_R - 2$$

where $\quad g_{ij} = \frac{1}{2}\sum_{I=1}^{D} R_i e_i^I R_j e_j^I$

$$g^*_{ij} = \frac{1}{2}\frac{1}{R_i}e_i^{*I}\frac{1}{R_j}e_j^{*I}$$

$$g^*_{ij} = (g^{-1})_{ij}.$$

$\mathrm{Vol}(\Lambda^D) = \sqrt{\det g} \qquad \mathrm{Vol}(\Lambda^{D*}) = \frac{1}{\sqrt{\det g}}$ ← volume unit cell

matching condition

$$N_R - N_L = \vec{p}\cdot\vec{L} = \sum_{i=1}^{D} m_i n_i.$$

The 2D dimensional vectors $\vec{P}_L$; P_R.

Lorentzian even self-dual lattice.

OSU.90

similarly to the 1-D case we can now extract the massless spectrum

In particular we have the states.

Scalars: $|\phi^{IJ}\rangle = \alpha_{-1}^{I} \bar{\alpha}_{-1}^{J} |0\rangle$. $I,J=1$

These states are .

$\frac{D}{2}(D+1) \rightarrow$ internal graviton components.
$\rightarrow$ internal metric. $\rightarrow$ symmetric

$\langle \phi^{IJ} \rangle \longrightarrow g_{ij} \rightarrow$ shape of the T^D.

$\frac{D(D-1)}{2} \rightarrow$ internal components of the antisymmetric tensor field B_{ij}.

$\rightarrow$ also influences the string spectrum.

For special values of $\langle \phi^{IJ} \rangle$ we will have enhanced symmetry:

i.e. for special value of the metric and the antisymmetric tensor.

we get enhanced symmetries.

OSU.91

$$P^I_{L,R} = [m_i - \frac{1}{2}(B_{ij} \mp G_{ij}) n_j] e_i^{*I}$$

For the heterotic string.

$$m_L^2 = -\frac{1}{2} + \frac{P_L^2}{2} + N_L = -1 + \frac{P_R^2}{2} + N_R = m_R^2$$

For special values of B and G and at special R_I symmetry is enhanced. We get additional space-time vector bosons.

For example.

$$G = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad B = \begin{cases} G_{ij} & i > j \\ 0 & i = j \\ -G_{ij} & i < j \end{cases}$$

$R_I = \sqrt{2}$

$$U(1)^6 \rightarrow SO(12)$$

$SO(12)$ is the lattice at the free fermionic point i.e. when all internal dimensions are identified as free fermions

$$e^{iX_L^I} = \frac{1}{\sqrt{2}}(y_L^I + i w_L^I) \qquad e^{iX_R} = \frac{1}{\sqrt{2}}(y_R^I + i w$$

OSU.92

As we did before we can calculate the partition function.

D compactified scalars.

$$L_0 = \frac{p_R^2}{2} + N_R - \frac{D}{24}$$

$$\bar{L}_0 = \frac{p_L^2}{2} + N_L - \frac{D}{24}$$

$$Z(q,\bar{q}) = Tr\, q^{L_0} \bar{q}^{\bar{L}_0} = \frac{\Theta_\Gamma(\tau, \bar{\tau})}{\eta(q)^D \bar{\eta}(\bar{q})^D} \qquad q = e^{i2\pi\tau}$$

$T: \tau \leftrightarrow \tau + 1$
$S: \tau \to -1/\tau$

$$\eta(-1/\tau) = (-i\tau)^{1/2}\eta(\tau) \quad / \quad \eta(\tau+1) = e^{2\pi i/24}\eta(\tau)$$

$$\Theta_\Gamma(-1/\tau, -1/\bar{\tau}) = (\tau)^{D/2}(\bar{\tau})^{D/2}\,\Theta_\Gamma(\tau, \bar{\tau})$$

$\Rightarrow$ $Z(q,\bar{q})$ is modular invariant.

OSU.93

the lattice compactificati

In addition to translations we can impose invariance under rotations and reflection provided that it is a symmetry of the compactified lattice.

For a compactified manifold M.

with discrete symmetry G

$\to$ new manifold $\tilde{M} = M/G$.

if G freely acting $\tilde{M}$ smooth manifold.

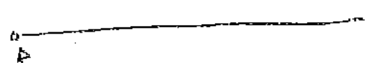
if G not freely acting $\to$ fixed points

$\tilde{M}$ is not smooth $\to$ orbifold

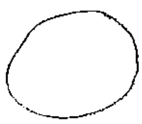
consider the real axis X.

it is symmetric under $Z_2 : X \to -X$.

with a fixed point at $X = 0$.

orbifold $R/Z_2 = \{x | x \geq 0\}$ 

$X = 0$ is an orbifold singular point.

$X \to X + 2\pi\lambda \to$  $\to$ smooth manifold

OSU.94 | 2^{nd} example

consider the circle $X \sim X + 2\pi L$.

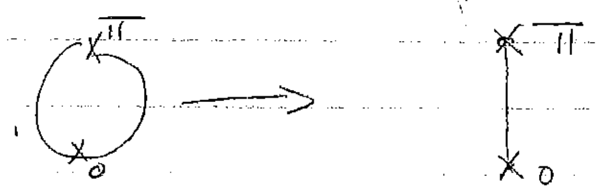
and take $O = \frac{S_1}{Z_2}$ $\quad X \to -X$

$$X = -X + m_1 e_1 \qquad e_1 = 2\pi$$

There are two fixed points

$$X_{fixed} = \frac{m_1 e_1}{2} \Rightarrow X_1 = 0 \quad X_2 = \pi.$$

$$e^{i\theta} \quad \theta = X + 2\pi L.$$



higher dimensional orbifolds are constructed in a similar way.

$$O_D = \frac{T_D}{G} \qquad G = \{I, g_1 \cdots g_n\}.$$

The Hilbert space is constructed for every $g \in$

i.e. we calculate the Hilbert space that close on the torus up to the transformation $g \in G$.

OSU.95

we have to solve the equation

$$X(\sigma+2\pi, \tau) = g\, X(\sigma, \tau) \mod \Lambda$$

say $g = Z_2$

then $X(\sigma+2\pi, \tau) = -X(\sigma, \tau) \mod \Lambda$.

we have to solve the wave equation with these boundary conditions.

which gives $X = \underset{\uparrow}{X_{cm}} + \frac{i}{2} \sum_{n \in Z+1/2} \left(\frac{\alpha_n}{n} e^{-in(\tau-\sigma)} + \frac{\tilde{\alpha}_n}{n} e^{-in(\tau+\sigma)} \right)$

fixed points

$X_{cm} = -X_{cm} \mod \Lambda \rightarrow$ fixed points of g

classically The string can sit at any one of this points. and we can build quantum states using the creation ~~annihilation~~ operators ~~around~~ on any one of these points.

we have to include all these states in the partition function.

and we have to project on states that

are invariant under the orbifold projectio

$$Z = \sum_{g\in G} \sum_{h\in G} \frac{1}{|G|} \mathrm{Tr}_{H_g}\, G\, q^{L_0} \bar{q}^{L_0}$$

$|G| \to$ order of the group $= 2$ for Z_2; 4 for $Z_2 \times$

For example for $\Theta = Z_2$: $X \to -X$

$$Z = \frac{1}{2}\left\{Z_{1,1} + Z_{1,\Theta}\right\} + \frac{1}{2}\left\{Z_{\Theta,1} + Z_{\Theta,\Theta}\right\}$$

The partition function can then be obtained using the eta, theta gymnastics similar to the previous lectures.

The partition function is modular invarian in particular it is modular invariant only by the inclusion of the twisted sectors.

OSU.97

The last comment that I want to make is that the partition function of two Majorana-Weyl fermions is the same as the partition function of a boson at a fixed Radius.

Taking two fermions ψ^1, ψ^2

define $\quad \psi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2) \quad \bar{\psi} = \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2)$.

The ψ generates a $U(1)$ current,

$$J(z) = :\psi\bar{\psi}:, \quad J(z)J(\omega) = \frac{1}{(z-\omega)^2} + \cdots$$

$$J(z)\psi(\omega) = \frac{\psi(\omega)}{z-\omega} + \cdots \quad J(z)\bar{\psi}(\omega) = -\frac{\bar{\psi}(\omega)}{z-\omega}$$

we can represent the same algebra using.

$$J(z) = i\partial X \quad \psi = :e^{iX}: \quad \bar{\psi} = :e^{-iX}:$$

Using poisson resummation gymnastics.

$$Z_{Dirac} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{\tau_2}|\eta|^2} \sum_{m,n\in Z} e^{\left[-\frac{\pi}{2\tau_2}|n+\tau m|^2\right]}$$

left and right components.

$$Z(R) = \frac{R}{\sqrt{\tau_2}\,|\eta|^2} \sum_{m,n\in Z} e^{\left[-\frac{\pi R^2}{\tau_2}|m-n\tau|^2\right]}$$

$R = 1/\sqrt{2} \Rightarrow Z$

Let's apply the orbifold construction to $Z_2 \times Z_2$ on

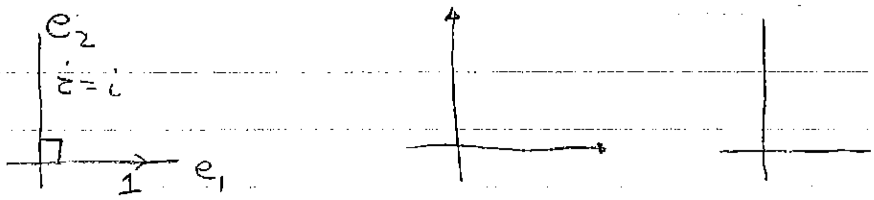
$$X(2\pi) = \theta X(0) + \sum m_i e_i$$

$$(1-\theta) X = \sum m_i e_i$$

fixed points are solutions of $(1-\theta^T) \sum e_i m_i = 0$

$$\sum m_i (1-\theta^T) e_i = 0$$

start with $\Lambda = R^6 / SO(4)^3$. $\Gamma = SO(4)^3$.



$$SO(4) = SU(2)_1 \times SU(2)_2$$

$T^2 \to X(2\pi) = -X(0) + m_1 e_1 + m_2 e_2$.

$$X = \frac{1}{2}(m_1 e_1 + m_2 e_2) \Rightarrow \frac{1}{2}\{(0,0)\ (1,0)\ (0,1)\ (1,1$$

4 fixed points in a unit Lattice cell.

on $T^2 \times T^2 \times T^2$

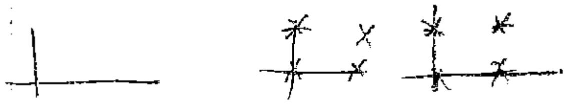
$Z_2 : \alpha$ $X_1 \to -X_1$ $X_2 \to -X_2$ $X_3 \to -X_3$ $X_4 \to X_4$ $X_5 \to X_5$

$\Rightarrow$ 16 fixed points.

$Z_2 : \beta$

$\Rightarrow$ 16 fixed points.

$\alpha\beta$



$\Rightarrow$ 16 fixed points

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at each one of the fixed points we can attach 27 of E_6.

<u>untwisted sector.</u>

$N=4$ SUSY $\quad E_8 \times E_8$

$\downarrow$

$P_L^2 = 0. \quad P_R^2 = 2 \qquad E_6 \times U(1)^2 \times E_8$

$$248 \rightarrow 78_A + 3(27_f + \overline{27}_f) + \cdots$$

$N=4 \rightarrow N=1$

$\left.\begin{matrix}\chi_{12}\\ \chi_{34}\\ \chi_{56}.\end{matrix}\right\} \rightarrow$ in the fermionic lang

on $SO(4)^3$ the spectrum that we get is

51×27 of E_6

$3 \times \overline{27}$ of E_6

+ singlets.

but in the fermionic model we had $(27, 3) \Rightarrow$ home

<u>but:</u> $\{1, S, X_1, X_2\} \Rightarrow SO(12) \times E_8 \times E_8.$

not $SO(4)^3$

we should mod the $SO(12)$ lattice by $Z_2 \times Z_2$.

$SO(12)$:
$e_1 \; (1, -1, 0, 0, 0, 0)$
$e_2 \; (0, 1, -1, 0, 0, 0)$
$e_3 \; (0, 0, 1, -1, 0, 0)$
$e_4 \; (0, 0, 0, 1, -1, 0)$
$e_5 \; (0, 0, 0, 0, 1, -1)$
$e_6 \; (0, 0, 0, 0, 1, 1)$

OSU,100 85

$\mathbb{Z}_2^A \quad \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & -1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -1 \end{pmatrix}$ $\qquad$ $\mathbb{Z}_2^B \quad \begin{pmatrix} -1 & & & & & \\ & -1 & & & & \\ & & -1 & & & \\ & & & -1 & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix}$

$A^T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$ $\qquad$ $B^T = \begin{pmatrix} \end{pmatrix}$

$(1-A^T)\vec{P} = 0$ $\qquad$ $(1-B^T)P = 0.$

$\Rightarrow$ 32 solutions.

$\chi = \frac{1}{4} \cdot 32 \cdot 6 = 48 \Rightarrow 24$ fixed points.

$\Rightarrow \mathbb{Z}_2 \times \mathbb{Z}_2$ on $SO(12)$ lattice $\rightarrow$ 24 fixed points.

$\mathbb{Z}_2 \times \mathbb{Z}_2$ on $SO(12) \Rightarrow (27, 3) + (27, \overline{27}) +$ sing

matches the spectrum of the fermionic model.

OSU.101/

top quark mass prediction:

fermionic model $\{1, S_1, b_1, b_2, b_3\} \to$ NA
$\{\alpha, \beta, \gamma\} \to$ beyond NA

we saw that the free fermionic models produce.
PLB 274 (1992) 47; NPB 487 (1997) 55.

gauge group:

$SO(10) \to SU(3) \times SU(2) \times U(1)_C \times U(1)_L$

$SO(6)^3 \to U(1)_{1,2,3} \quad U(1)_{4,5,6}$.

$E_8 \to SU(5) \times SU(3) \times U(1)^2$.

matter: $b_1, b_2, b_3 \to$ 3 generations.

$3 * 16 \to Q + L + U_L^C + D_L^C + e_L^C + N_L^C$.

with horizontal charges.

$b_1 \to U_1, U_4$

$b_2 \to U_2, U_5$

$b_3 \to U_3, U_6$.

Higgs: $\to$ untwisted sector & $b_1 + b_2 + \alpha + \beta$.

3 pairs.

$h_1 + \bar{h}_1 \to U_1$
$h_2 + \bar{h}_2 \to U_2$
$h_3 + \bar{h}_3 \to U_3$.

+ SO(10) singlets.

$b_1 + b_2 + \alpha + \beta$:

1 pair: $h_{\alpha\beta} + \bar{h}_{\alpha\beta} \to U_1, U_2$.

+ SO(10) singlets

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Fermion mass terms

$$\lambda^{ij} Q U \bar{h} \qquad \lambda_D^{ij} Q D h \qquad \lambda_E^{ij} L E h$$

we have to calculate the λ_{ij}

The superpotential terms are calculated from.

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle$$

on the sphere.

where each V_i correspond to a massless state in the string spectrum and is represented as a vertex operator

The allowed terms must be invariant under all the gauge symmetries and the string selection rules.

The world-sheet and consequently the space-time charges of the massless states are fixed by the free fermions boundary cond. and the GSO phases.

hence it will fix the terms which are invarian

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A general consequence of the $Z_2 \times Z_2$ orbifold structu[re]

Each generation is charged with respect to a different set of horizontal charges.

$$b_1 \to U_1, U_4 \qquad h_1, \bar{h}_1 \to U_1$$
$$b_2 \to U_2, U_5 \qquad h_2, \bar{h}_2 \to U_2$$
$$b_3 \to U_3, U_6. \qquad h_3, \bar{h}_3 \to U_3.$$

only the couplings,
$$b_1\, b_1\, (h_1\, ;\, \bar{h}_1)$$
$$b_2\, b_2\, (h_2\, ;\, \bar{h}_2)$$
$$b_3\, b_3\, (h_3\, ;\, \bar{h}_3)$$

are allowed.

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one can show that in the basis vector

γ $SO(10) \to SU(5) \times U(1)$.

Depending on the of b.c. to $\{y, \omega \mid \bar{y}, \bar{\omega}$

selects Yukawa coupling for $+2/3$ charged qu[ark]
or $-1/3$ charged quark. according to the difference

$$\Delta_j = |\gamma(L_j) - \gamma(R_j)| = 0, 1.$$

where

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$\gamma(L_j) / \gamma(R_j)$ are the boundary conditions γ for the world-sheet fermions from the set $\{y, \omega | \bar{y}, \bar{\omega}\}$ that are periodic in b_j.

Example

b_1	y_{36}	$y_4\bar{y}_4$	$y_5\bar{y}_5$	$\overline{y_3 y_6}$
γ:	1	0	0	0

$$\Delta_1 = |\gamma(y^3 y^6) - \gamma(\bar{y}^3\bar{y}^6)| = 1 \Rightarrow$$

$\lambda'_U Q_1 U_1 \bar{h}_1$ is invariant.

$\lambda'_D Q_1 U_1 h_1$ is not invariant.

and the opposite holds for $\Delta_1 = 0$.

and similarly for Δ_2, Δ_3.

so if we construct a model with

$$\Delta_{1,2,3} = 1$$

then only up-type Yukawa couplings are allowed at the cubic level.

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$$\lambda_1 U_1 Q_1 \bar{h}_1 + \lambda_2 U_2 Q_2 \bar{h}_2 + \lambda_3 U_3 Q_3 \bar{h}_3$$

The mass terms of the ligther quarks and leptons will arise from non-renormaliza terms.

<u>calculation of the Yukawa coupling:</u>

Each massless state → vertex operator V_i^f, V_i^b

<u>These have the generic form</u>

$$V^{\ell}_{(q)} = e^{(qc)} \mathcal{L}^{\ell} e^{i\alpha X_{12}} e^{i\beta X_{34}} e^{i\gamma X_{56}} \; *$$

$$\left(\prod_j e^{(i q_j \xi_j)} \{\sigma's\} \prod_j e^{(i \bar{q}_j \bar{\xi}_j)} \right)$$

$$* \; e^{(i \bar{\alpha} \bar{\eta}_1)} e^{(i \beta \bar{\eta}_2)} e^{(i \bar{\gamma} \bar{\eta}_3)} e^{i W_R \cdot \bar{J}} e^{i \frac{1}{2} K \cdot X} e^{i \frac{1}{2} K \cdot \bar{X}}$$

<u>where</u> * e^{qc} -> ghost charge with conformal dimension.

$$h = -\frac{q^2}{2} - q.$$

$q = -1/2$ for fermions.

$q = -1$ for bosons.

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* $\mathcal{L}^{\ell}$ is the Lorentz group factor and signals the space-time spin of a state, which are determined by the B.C. of ψ^{μ}.

$b(\psi^{\mu}) = 1 \rightarrow$ fermion, $\mathcal{L}^{\ell} = S_{\alpha} \rightarrow$ spinor index

$b(\psi^{\mu}) = 0 \Rightarrow$ boson $\mathcal{L}^{\ell} = \psi^{\mu} \rightarrow$ vectors
$\qquad\qquad\qquad\qquad\qquad I \rightarrow$ scalars.

<u>conformal dimensions</u>.

$$I = (0,0)$$
$$S_{\alpha} = (1/4, 0)$$
$$\psi^{\mu} = (1/2, 0)$$

<u>Respectively</u>.

(*) e^{iqf} ; $e^{i\bar{q}\bar{f}}$ are the factors that arise from complexified fermions. which produce global left-moving and local right moving currents.

complex f: $\quad f = \frac{1}{\sqrt{2}}(f_1 + i f_2) = e^{-iH} \qquad F(f) = +1$

$\qquad f^{*} = \frac{1}{\sqrt{2}}(f_1 - i f_2) = e^{iH}. \qquad F(f) = -1$.

U(1) charges. $\quad Q(f) = \frac{1}{2}\alpha(f) + F(f)$.

conformal dimensions: $\quad h = q^2/2 \qquad \bar{h} = \bar{q}^2/2$.

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* σ's : A left-moving real fermion f which is paired with a right-moving real fermion $\bar{f}$ produces an Ising model operator with the following conformal fields.

$$I = (0, 0)$$
$$\sigma_\pm(z, \bar{z}) = \left(\tfrac{1}{16}; \tfrac{1}{16}\right).$$
$$f(z) = (1/2, 0).$$
$$\bar{f}(\bar{z}) = (0, 1/2).$$
$$\epsilon = f(z)\bar{f}(\bar{z}) = (1/2, 1/2).$$

$\sigma_\pm$ = order/disorder operators. → when both f are periodic α.
ϵ energy operator.

$\sigma_\pm$ arise when both f, $\bar{f}$ are periodic in α.
The others arise when f or $\bar{f}$, both or none act on the vacuum.

* $e^{iW_R \cdot \bar{J}}$ → from NA gauge groups.

$$\bar{h} = \frac{W \cdot W}{2}$$

W - weight vector of representation R.

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$* \; e^{\frac{i}{2}K\cdot X} \quad f e^{\frac{i}{2}K\cdot \bar{X}} \rightarrow$ Poincary quantum #'s. Kinetic terms.

massless states $\quad h = 1 \quad \bar{h} = 1.$

calculation of Yukawa couplings

$$A_N = \frac{g^{N-2}}{(2\pi)^{N-3}} \mathcal{N} \int \prod_{i=1}^{3} d^2 z_i \langle V_1^f(z_\infty) V_2^f(1) V_3^b(z_1) \cdots V_{N-1}^b(z_{N-3}) V_N^b($$

where $\mathcal{N} = \sqrt{2}$ - normalization factor.

$SL(2,C)$ invariance - fixes the location of three vertex operators at

$$z = z_\infty, \; 1, \; 0.$$

total ghost charge. $q_g = -2.$

$$q_g^f = -1/2 \qquad q_g^b = -1.$$

$\Rightarrow$ Picture changing

$$V_{q+1}(z) = \lim_{w \to z} e^c(w) \, T_F(w) \, V_q(z)$$

where

$$T_F = \psi^\mu \partial_\mu X + i \sum_{i=1}^{6} \chi_i y_i w_i = T_F^0 + T_F^{-1} +$$

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with $T_F^{-1} = e^{-i\chi^{12}} \tau_{12} + e^{-i\chi^{34}} \tau_{34} + e^{-i\chi^{56}} \tau_{56}$ $\quad T_F^{-1} = (T_F$

where

$$\tau_{ij} = \frac{i}{\sqrt{2}} \left(y^i w^i + i y^j w^j \right)$$

$$e^{i\chi^{ij}} = \frac{1}{\sqrt{2}} \left(\chi^i + i \chi^j \right).$$

picture changing : changes the ghost charge by +1.

We use the OPE's to evaluate the fermion ~~mass terms.~~ $\quad Z_{ij} = |z_i - z_j|$,

* Ghosts.

$$\langle e^{-c/2}(z_1)\, e^{-c/2}(z_2)\, e^{-c}(z_3) \rangle = Z_{12}^{-1/4} Z_{13}^{-1/2} Z_{23}^{-1/2}$$

* Lorentz group.

$$\langle S_\alpha(z_1)\, S_\beta(z_2) \rangle = C_{\alpha\beta}\, Z_{12}^{-1/2}$$

* correlators of exponentials,

$$\langle \prod_j e^{i\alpha_j J} \rangle = \prod_{i<j} (Z_{ij})^{\alpha_i \alpha_j}$$

* Ising model correlators

$$\langle \sigma^{\pm}(z_1)\, \sigma^{\pm}(z_2) \rangle = Z_{12}^{-1/8}\, \bar{Z}_{12}^{1/8}$$

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$$\langle \sigma^+(z_1)\, \sigma(\bar z_2)\rangle = 0$$

$$\langle \sigma^+(z_1)\, \sigma^-(z_2)\, f(z_3)\rangle = \frac{1}{\sqrt2} z_{12}^{3/8}\, \bar z^{-1/8} (z_{13} z_{23})^{-1/2}$$

$$\langle \sigma_\mp(z_\infty)\, \sigma_\pm(1)\, \sigma_\pm(z)\, \sigma_\mp(0)\rangle = \frac{1}{\sqrt2} |z_\infty|^{-1/4}\, |1-z|^{-1/4}\, |z|^{-1/4} \times (1+|z|+|1-z|)^{1/2}$$

FOR $N=3$

$$A_3 = g\sqrt2 \left\langle V^f_{1(-1/2)}(z_1)\, V^f_{2(-1/2)}(z_2)\, V^b_{3(-1)}(z_3)\right\rangle.$$

Top-quark Yukawa coupling:

Flat-directions — $\langle\phi\rangle \Leftrightarrow N=1$ vacua.

→ analysis of Higgs mass spectrum.

only 2-light higgs doublets

$\bar h_2$ OR $\bar h_1$ and h_{45}.

only t $U_2 Q_2 \bar h_2 \to$ at $N=3$.

Only top-quark $N=3$ mass term.

$$U^f_{2(-1/2)} = e^{(-1/2 c)} S_\alpha\, e^{\frac{i}{2}\chi_{34}}\, e^{i\frac12 \xi_2}\, \sigma_2^+ \sigma_6^+\, e^{i\frac12 \bar\xi_2}\, e^{\frac{i}{2}\bar\eta_2}\, e^{i\sqrt{16}\,\bar w_6}$$

$$Q^f_{2,-1/2} = e^{-1/2 c} S_\beta\, e^{i/2 \chi_{34}}\, e^{-1/2 \xi_2}\, \sigma_2^+ \sigma_6^+\, e^{-1/2 \bar\xi_2}\, e^{+1/2 \bar\eta_2}\, e^{i\sqrt{16}\,\bar w_6}$$

$$\bar h_{2,-1}\quad e^{-c} \qquad e^{i\chi_{34}} \qquad\qquad e^{-i\bar\eta_2}\, e^{i\sqrt{10}\,\bar W_{10}}$$

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The next step is to assemble the correlators,

$$A_3 = g\sqrt{2} \Big\{ \langle e^{-c/2}(z_1)\, e^{-c/2}(z_2)\, e^{-c}(z_3) \rangle \langle S_\alpha(z_1) S_\beta(z_2) \rangle$$

$$\times \langle e^{i\frac{\chi_{34}}{2}}(z_1)\, e^{\frac{i}{2}\chi_{34}}(z_2)\, e^{-i\chi_{34}}(z_3) \rangle \langle e^{\frac{i}{2}\xi_2}(z_1)\, e^{-\frac{i}{2}\xi_2}(z_2) \rangle$$

$$\times \langle \sigma_2^+(z_1)\, \sigma_2^+(z_2) \rangle \langle \sigma_6^+(z_1)\, \sigma_6^+(z_2) \rangle \langle e^{\frac{i}{2}\bar{\xi}_2}(z_1)\, e^{\frac{i}{2}\bar{\xi}_2}(z_2)$$

$$\times \langle e^{i\bar{\bar{J}}_{3,1}\cdot \bar{W}_{3,1}}(z_1)\, e^{i\bar{\bar{J}}_{3,2}\cdot \bar{W}_{3,2}}(z_2) \rangle \langle e^{i\bar{J}_{12}\cdot \bar{W}_{12}}(\bar{z}_3) \rangle$$

$$\times \langle e^{\frac{i}{2}\bar{\eta}_2}(z_1)\, e^{\frac{i}{2}\bar{\eta}_2}(z_2)\, e^{-i\bar{\eta}_2}(z_3) \rangle$$

$$\times \langle \prod_{i=1}^{4} e^{\frac{i}{2}K_i X_i}\, e^{\frac{i}{2}K_i \bar{X}_i} \rangle$$

since $K_1 + K_2 + K_3 = 0$ & $K_1^2 = K_2^2 = K_3^2 = 0$

$\Rightarrow K_1 \cdot K_2 = K_1 \cdot K_3 = K_2 \cdot K_3 = 0.$

& $z_1 = \infty \quad z_2 = 1 \quad z_3 = 0 \Leftarrow SL(2,C)$ invarian

$\Rightarrow A_3 = g\sqrt{2}.$

$\Rightarrow \lambda_+ (M_p) = g\sqrt{2} \rightarrow$ g is the gauge coup

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bottom quark & tau lepton mass terms.

must be obtained from non-renormalizable terms

PLB274 (1992) 47: $N=4$.

$$d_L^2\, Q_2\, h'_{\alpha\beta}\, \bar{\bar{\Phi}}_2 + e_L^2\, L_2\, h'_{45}\, \bar{\Phi}_2$$

$\bar{\Phi}_2 \rightarrow$ SO(10) singlet.

similar analysis.

$$A_4 = \frac{g^2}{2\pi}\,\frac{1}{4}\, I \qquad \frac{z_1 \quad z_2 \quad z_3 \quad z_4}{\infty \quad 1 \quad 0 \quad z}$$

$$I = \int d^2z\, |z|^{-\frac{5}{4}-1}\, |1-z|^{-\frac{4}{4}-\frac{z}{4}}\, \left(|1+|z|+|1-z|\right)'$$

polar coordinates $\quad z = re^{i\theta} \quad \bar{z} = re^{-i\theta}$

$I \approx 77.7$ evaluated numerically s=t= (contact

$$\Rightarrow W_4 = \frac{g}{2\pi}\,\frac{1}{4}\,\frac{I}{M}\left(d_L^2\, Q_2\, h'_{45}\, \bar{\Phi}_2 + e_L^2\, L_2\, h'_{45} \right.$$

where. $\frac{1}{2} g\sqrt{2\alpha'} = \frac{\sqrt{8\pi}}{M_{pl}} = \frac{1}{2}\frac{1}{M_s}$ string scale.

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effective yukawa coupling.

Exist: $\mathrm{tr}\, Q_A \neq 0 \Rightarrow D_A = \frac{g^2}{192\pi^2} \mathrm{tr}\, Q_A \neq 0$

$|D_A|^2 \neq 0 \Rightarrow$ SUSY broken

$$D_A = \sum_k Q_k^A |\langle \chi_k \rangle|^2 + \frac{g^2}{192\pi^2} \mathrm{tr}\, Q_A = 0 \checkmark$$

$$D_j = \sum Q_k^j |\langle \chi_k^2 \rangle|^2 = 0 \checkmark$$

also $\langle W \rangle = \left\langle \frac{\partial W}{\partial \varphi_i} \right\rangle = F_i = 0$ ✓

SUSY vacuum.

so $\lambda_n = c_n \left(\frac{\langle \chi \rangle}{M} \right)^{N-3}$.

in PLB 274

$$|\langle \phi_{45} \rangle|^2 = 3 |\langle \Phi_2 \rangle|^2 = 3 \langle \Delta_{13} \rangle^2 = \frac{3g^2}{16\pi^2} \frac{1}{2\alpha'} = \frac{3g^4}{16\pi^2}$$

where $\Delta_{13} = (|\Phi_{13}|^2 - |\phi_{13}|^2)$

we get $\lambda_S = \lambda_T = \frac{1}{32\pi^2} g^3 = 0.25 g^3$

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take $g \sim \frac{1}{\sqrt{2}}$ at M_U

FROM

at μ.

$$m_t(\mu) = \lambda_t(\mu) v_1 = \lambda_t(\mu) \frac{v_0}{\sqrt{2}} \sin\beta.$$

$$m_b(\mu) = \lambda_b(\mu) v_2 = \lambda_b(\mu) \frac{v_0}{\sqrt{2}} \cos\beta.$$

$$v_0 = \frac{2 M_W}{g_2} = 246 \text{ GeV} \qquad \tan\beta = \frac{v_2}{v_1}$$

$$\Rightarrow m_t(M_Z) \simeq \lambda_t(M_Z) \sqrt{\frac{2 M_W^2}{g_2^2(M_W)} - \left(\frac{m_b(M_Z)}{\lambda_b(M_Z)}\right)^2}$$

we calculated λ_b, λ_t at M_U.

we need λ_b, λ_t at M_Z.

take MSSM spectrum. with

$$\frac{d\lambda_t}{dt} = \frac{\lambda_t}{8\pi^2}\left(3\lambda_t^2 + \tfrac{1}{2}\lambda_b^2 - \tfrac{8}{3} g_3^2 - \tfrac{3}{2} g_2^2 - \tfrac{13}{30} g_1^2\right)$$

$$\frac{d\lambda_b}{dt} = \frac{\lambda_b}{8\pi^2}\left(3\lambda_b^2 + \tfrac{1}{2}\lambda_t^2 - \tfrac{8}{3} g_3^2 - \tfrac{3}{2} g_2^2 - \tfrac{7}{30} g_1^2\right).$$

with the one-loop gauge coupling RGE's.

Run λ_b, λ_t from $M_U \to M_Z$

$$\Rightarrow m_t(M_Z) \simeq 175 - 180 \text{ GeV}$$

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top observed — CDF/D0 → January / March } 19

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what we saw last time.

ⓐ strings: → reparameterization invariance!

ⓑ $T_{\alpha\beta} = \frac{\delta S}{\delta g_{\alpha\beta}} = 0 \rightarrow T_{\alpha\beta} = 0$ classically

Fourier modes of E.M. tensor obey Virasoro algebra.

$$[L_m, L_n] = (m-n) L_{m+n}.$$

~~quantization~~

ⓒ string coordinates obey 2D wave equation with B.C. < open / closed

general solution Expand in Fourier modes α_n

quantization → impose equal time comm.

$$[X^\mu(\sigma), \dot{X}^\nu(\sigma')] = T^{-1} \delta(\sigma-\sigma')$$

⇒ α_n $\tilde{\alpha}_n$ creation annihilation operators.

⇒ ~~normal ordering~~ a

⇒ conformal anomaly: Liouville mode e^ϕ does not decouple → break down of conformal invariance

The fourier modes of the energy-momentum tens
~~obey~~ annihilate the physical states.

$$L_n \, |Phys\rangle = 0 \qquad n > 0,$$
$$(L_0 - a)\, |Phys\rangle = 0$$
$$(L_0 - \bar{L}_0)\, |Phys\rangle = 0. \leftarrow \text{for closed strin}$$

From → comm. re. $\eta^{\mu\nu} \sim (^{-1}{}_{1}{}_{1}) \to$ negative norm states.

and residual gauge degree of freedom → anom
Liouville mode e^{ϕ} not decoupled.

Fix remaining freedom → Light cone gauge.

$$X_{\pm} = \cancel{X^0 + X^{D-1}} \; (X^0 + X^{D-1})/\sqrt{2}.$$

⇒ spectrum free of negative norm states.

⇒ Lorentz invariance not manifest.

$D = 26 \qquad a = 1. \Rightarrow$ Lorentz invariant at Quantum lev

<u>Faddeev - Popov</u>. $\quad c_{gh} = -26.$

$$\Rightarrow c_{gh} + D \cdot 1 = 0 \Rightarrow D = 26$$

OSU.14 mass and spin of string states

$(L_0 - a)\ |Phys\rangle = 0 \quad$ ($a=1$ for bosonic stri

$L_0 = \frac{1}{2}\sum_{-\infty}^{\infty} \alpha_{-n}\alpha_n = \frac{1}{2}\alpha_0 + \frac{1}{2}\sum_{n\neq 0} \alpha_{-n}\alpha_n.$

where $X^\mu_R(\sigma,\tau) = \frac{1}{2}X^\mu + \frac{1}{2}l^2 p^\mu(\tau-\sigma) + \frac{i}{2} l \sum_{n\neq 0} \frac{1}{n}\alpha^\mu_n e^{-2i}$

$\alpha_0 = \frac{1}{2}p^\mu \Rightarrow \underbrace{\alpha_0^2 = \frac{1}{4}p^\mu p_\mu = -\frac{1}{4}M}_{L_0 = \tilde{L}_0}$ for str

OR.

Q.M. $\to \quad M^2 = -8a + 8\sum_{n=1}^{\infty} \underbrace{\alpha_{-n}\alpha_n}_{n_L} = -8a + 8\sum_{n=1}^{\infty} \underbrace{\tilde{\alpha}_{-n}\tilde{\alpha}}_{n}$

Virasoro constraint: $M_L^2 = M_R^2$

$M_L^2 = -8a + n_L = -8a + n_R = $