MATH425 - Quantum Field Theory
Set Work: Sheet 7

1. In the Heisenberg picture the operators are defined by

$$\alpha_H = U^\dagger \alpha U,$$

where α is the operator in the standard Schrödinger picture, and the unitary operator U is defined by

$$i\hbar \frac{\partial U}{\partial t} = HU.$$

Show that

$$i\hbar \frac{\partial \alpha_H}{\partial t} = [\alpha_H, H_H].$$

2. Two-particle states are defined by

$$|\mathbf{p}_1, \mathbf{p}_2 >= a^\dagger(\mathbf{p}_1) a^\dagger(\mathbf{p}_1)|0> .$$

Show that

$$< \mathbf{p}_1', \mathbf{p}_2' | \mathbf{p}_1, \mathbf{p}_2 >$$
$$=(2\pi)^6 (2p_1^0)(2p_2^0)\{\delta(\mathbf{p}_1 - \mathbf{p}_1')\delta(\mathbf{p}_2 - \mathbf{p}_2') + \delta(\mathbf{p}_1 - \mathbf{p}_2')\delta(\mathbf{p}_2 - \mathbf{p}_1')\}.$$

3. Show that if we define the number operator

$$N = \frac{1}{(2\pi)^3} \int \frac{d^3\mathbf{p}}{2p^0} a^\dagger(\mathbf{p}) a(\mathbf{p}),$$

then it satisfies

$$[N, a^\dagger(\mathbf{p})] = a^\dagger(\mathbf{p})$$

and hence

$$N|\mathbf{p}_1 \ldots \mathbf{p}_n >= n|\mathbf{p}_1 \ldots \mathbf{p}_n >$$

4. (a) Show that $\overline{\psi}\gamma^5\gamma^\mu\psi$ transforms as a pseudovector under Lorentz transformations.

(b) For

$$L^\mu{}_\nu = \delta^\mu{}_\nu + \epsilon^\mu{}_\nu,$$
$$\text{where} \quad \epsilon_{\mu\nu} + \epsilon_{\nu\mu} = 0,$$

and $\epsilon_{\mu\nu}$ is infinitesimal denote $S(L)$ by

$$S(L) = 1_4 + i\epsilon^{\mu\nu}\Sigma_{\mu\nu}, \quad \text{where} \quad \Sigma_{\mu\nu} = -\Sigma_{\nu\mu}.$$

Show that for a spatial rotation by an angle θ around an axis $\mathbf{n}$ we have to 1st order

$$\psi'(x) = S(L)\psi(L^{-1}x) = (1 - i\theta\mathbf{n}.\mathbf{J})\psi(x)$$

where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ with $\mathbf{L} = \mathbf{x} \times \mathbf{p}$ and $S^k = \frac{1}{2}\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}$.