**MATH425 - Quantum Field Theory**
**Set Work: Sheet 5**

**1.** Show that

$$[\gamma^\mu, \gamma_\lambda \gamma_\rho] = 2(\delta^\mu{}_\lambda \gamma_\rho - \delta^\mu{}_\rho \gamma_\lambda).$$

Hence check that

$$\Sigma_{\lambda\rho} = -\frac{i}{8}[\gamma_\lambda, \gamma_\rho]$$

is a solution of

$$\tfrac{1}{2}(\delta^\mu{}_\lambda \gamma_\rho - \delta^\mu{}_\rho \gamma_\lambda) = i[\gamma^\mu, \Sigma_{\lambda\rho}].$$

**2.** Show that

$$\mathrm{tr}[\gamma_\mu \gamma_\nu] = 4\eta_{\mu\nu}.$$

Now show that

$$\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma = 2\eta_{\mu\nu}\gamma_\rho\gamma_\sigma - 2\eta_{\mu\rho}\gamma_\nu\gamma_\sigma + 2\eta_{\mu\sigma}\gamma_\nu\gamma_\rho - \gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\mu.$$

Hence show that

$$\mathrm{tr}[\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma] = 4[\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}]$$

**3.** Show that

$$\begin{aligned} \gamma_\mu \not{a}_1 \gamma^\mu &= -2(a_1.\gamma), \\ \gamma_\mu \not{a}_1 \not{a}_2 \gamma^\mu &= 4a_1.a_2, \end{aligned}$$

where

$$\not{a}_1 = a_1^\nu \gamma_\nu.$$

**4.** Show that if $j^\mu(x) = \overline{\psi}\gamma^5\gamma^\mu\psi$, then under a Lorentz transformation $j^\mu$ transforms as

$$j'^\mu(x') = L^\mu{}_\nu j^\nu(x),$$

while under a parity transformation $j^\mu$ transforms as

$$j'^\mu(x') = j^\mu(x).$$

$j^\mu(x)$ is called an **axial vector** or **pseudovector**.