**MATH425 - Quantum Field Theory**
**Set Work: Sheet 4**

**1.** (Revision) Prove

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\mathbf{j} = 0,$$
$$\text{where} \quad \mathbf{j} = -\frac{i\hbar}{2m}\left[\psi^* \nabla \psi - \psi \nabla \psi^*\right]$$

using the standard non-relativistic Schrödinger equation.

**2.** Using the same method used to derive Eq. (1.27) in the notes, show that if

$$\phi = \frac{1}{(2\pi)^3} \int d^3\mathbf{p} \frac{1}{2p^0}\left[f_+(\mathbf{p})e^{-ip.x} + f_-(\mathbf{p})e^{ip.x}\right]$$

then

$$||\phi||^2 = \frac{1}{(2\pi)^3} \int d^3\mathbf{p} \frac{1}{2p^0}\left[|f_+(\mathbf{p})|^2 - |f_-(\mathbf{p})|^2\right],$$

where

$$||\phi||^2 = i \int \phi^* \partial_0 \phi d^3\mathbf{x}.$$

**3.** Defining $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, show that

$$\gamma^{5\dagger} = \gamma^5$$
$$\{\gamma^5, \gamma^\mu\} = 0.$$

**4.** By inserting $(\gamma^\mu)^2 = 1$ for some $\mu = 0, 1, 2, 3$, write each of $\gamma^0\gamma^1\gamma^2$ and $\gamma^0\gamma^1\gamma^3$ as a product $\gamma^5\gamma^\nu$ for some $\nu = 0, 1, 2, 3$.