MATH425 - Quantum Field Theory

Set Work: Sheet 11

1. (i) Write down the Feynman rules in momentum space for a real scalar field theory with interaction $\mathcal{L}_I(\phi) = -\frac{\lambda}{3!} : \phi^3 :$

(ii) Suppose the quanta of the field are called ϵ. Consider the interaction

$$\epsilon(p_1) + \epsilon(p_2) \to \epsilon(p_3) + \epsilon(p_4).$$

where p_i are the momentum 4-vectors for each particle. The T-matrix is defined by

$$S = 1 + i(2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4)T,$$

where S is the S-matrix. Draw the connected Feynman diagrams which contribute to $< \mathbf{p}_3\mathbf{p}_4|T|\mathbf{p}_1\mathbf{p}_2 >$ at $O(\lambda^2)$.

(iii) Decomposing p_i as

$$p_i = (p_i^0, \mathbf{p}_i),$$

let θ be the angle between $\mathbf{p}_1$ and $\mathbf{p}_3$ in the centre-of-mass frame. Show that to $O(\lambda^2)$ we have

$$< \mathbf{p}_3\mathbf{p}_4|T|\mathbf{p}_1\mathbf{p}_2 >=$$
$$\lambda^2 \left[\frac{1}{2|\mathbf{p}|^2(1-\cos\theta) + m^2} + \frac{1}{2|\mathbf{p}|^2(1+\cos\theta) + m^2} - \frac{1}{4(m^2 + |\mathbf{p}|^2) - m^2} \right],$$

where $\mathbf{p}$ is the momentum in the centre of mass frame.