MATH425 - Quantum Field Theory

Set Work: Sheet 1

A classical reference on analytical mechanics is: Classical mechanics, Herbert Goldstein, Addison & Wesley, 1980

* **Reading**: Goldstein, Chapters 8,9,10

1. The potential function of a one dimensional harmonic oscillator is given by

$$V(q) = \frac{1}{2}kq^2.$$

a. write down the force $F(q)$.

b. write down the Lagrangian.

c. write down the Euler-Lagrange eqs. of motion

d. write down the Hamiltonian

e. using the Hamilton-Jacobi formalism find a solution for q and p as a function of time.

f. find an expression for the energy

g. how many constants of the motion are there. find them.

2.

Write down the Lagrangian for a particle of mass m in a potential $V(r,\phi)$ when referred to planar polar coordinates (r,ϕ). Hence show that the equations of motion are

$$m\ddot{r} - mr\dot{\phi}^2 = -\frac{\partial V}{\partial r} \qquad mr\ddot{\phi} + 2m\dot{r}\dot{\phi} = -\frac{1}{r}\frac{\partial V}{\partial \phi}.$$

Hence derive the principle of conservation of angular momentum in the plane, and obtain the usual formula v^2/r for centripetal acceleration.

3.

a. Show that if the Hamiltonian is independent of a generallized coordinate q_0, then the conjugate momentum p_0 is a constant ot the motion. Such coordinates are called **cyclic coordinates**. Give two examples of a physical system that has a cyclic coordinate.

b. Show that in 3 dimension spherical polar coordinates the Hamiltonian of a particle of mass m moving in a potential $V(\vec{x})$ is

$$H = \frac{1}{2m}(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2\sin^2\theta}) + V(\vec{x}).$$

show that $p_\phi = $ constant when $\partial V/\partial\phi \equiv 0$ and interpret this result physically.