MATH425 Quantum Field Theory Solutions 11

1. The Feynman rules are:

The Feynman diagrams are:

Let $s = (p_1 + p_2)^2$, $t = (p_1 - p_4)^2$, $u = (p_1 - p_3)^2$. We have

$$< \mathbf{p}_3\mathbf{p}_4|T|\mathbf{p}_1\mathbf{p}_2 >= (-i)\left[\frac{(-i\lambda)^2 i}{s-m^2} + \frac{(-i\lambda)^2 i}{t-m^2} + \frac{(-i\lambda)^2 i}{u-m^2}\right].$$

Then in the centre-of-mass frame we have

$$p_1 = \begin{pmatrix} E \\ \mathbf{p}_1 \end{pmatrix}, \quad p_2 = \begin{pmatrix} E \\ -\mathbf{p}_1 \end{pmatrix}, \quad p_3 = \begin{pmatrix} E' \\ \mathbf{p}_3 \end{pmatrix}, \quad p_3 = \begin{pmatrix} E' \\ -\mathbf{p}_3 \end{pmatrix},$$

where

$$E = \sqrt{\mathbf{p}_1^2 + m^2}, \quad E' = \sqrt{\mathbf{p}_3^2 + m^2}.$$

But energy conservation $\Rightarrow 2E = 2E' \Rightarrow |\mathbf{p}_3| = |\mathbf{p}_1$. So

$$\begin{aligned} s =& (p_1 + p_2)^2 = 4E^2 = 4(\mathbf{p}_1^2 + m^2) \\ t =& (p_1 - p_4)^2 = -(\mathbf{p}_1 + \mathbf{p}_3)^2 = -2|\mathbf{p}_1|^2(1 + \cos\theta), \\ u =& (p_1 - p_3)^2 = -(\mathbf{p}_1 - \mathbf{p}_3)^2 = -2|\mathbf{p}_1|^2(1 - \cos\theta). \end{aligned}$$

So

$$< \mathbf{p}_3\mathbf{p}_4|T|\mathbf{p}_1\mathbf{p}_2 >=$$
$$\lambda^2\left[\frac{1}{2|\mathbf{p}|^2(1-\cos\theta)+m^2}+\frac{1}{2|\mathbf{p}|^2(1+\cos\theta)+m^2}-\frac{1}{4(m^2+|\mathbf{p}|^2)-m^2}\right].$$