MATH425 - Quantum Field Theory
Set Work: Sheet 9

1. The Dirac field may be written

$$\psi(x) = \frac{1}{(2\pi)^3}\sum_{s=1}^{2}\int \frac{d^3\mathbf{p}}{2p^0}[\psi_p^{(s)}(x)a_s(\mathbf{p}) + \tilde{\psi}_p^{(s)}(x)b_s^\dagger(\mathbf{p})],$$

where

$$\begin{aligned}\psi_p^{(s)}(x) =& e^{-ip.x}u_s(p),\\ \tilde{\psi}_p^{(s)}(x) =& e^{ip.x}v_s(p).\end{aligned}$$

With the scalar product $< \psi_1, \psi_2 >= \int \psi_1^\dagger \psi_2 d^3\mathbf{x}$, we have already shown

$$< \psi_p^{(r)}, \psi_{p'}^{(s)} >= \delta_{rs}2p^0(2\pi)^3\delta(\mathbf{p}-\mathbf{p}').$$

Show also using

$$\begin{aligned}u_r(p) =& \sqrt{p^0+m}\begin{pmatrix}\chi_r\\ \frac{\sigma.\mathbf{p}}{p^0+m}\chi_r\end{pmatrix}\\ v_r(p) =& \sqrt{p^0+m}\begin{pmatrix}\frac{\sigma.\mathbf{p}}{p^0+m}\chi_r\\ \chi_r\end{pmatrix}\end{aligned}$$

that

$$< \psi_p^{(r)}, \tilde{\psi}_{p'}^{(s)} >= 0.$$

Use these results together with the basic anticommutation relations

$$\{\psi_\alpha(\mathbf{x},t), \psi_\beta^\dagger(\mathbf{x}',t)\} = \delta(\mathbf{x}-\mathbf{x}')\delta_{\alpha\beta}$$

to show

$$\{b_r(\mathbf{p}), b_s^\dagger(\mathbf{p}')\} = (2\pi)^3\delta_{rs}2p^0\delta(\mathbf{p}-\mathbf{p}').$$

(b) Defining the energy-momentum vector by

$$P^\mu = i\overline{\psi}\gamma^\mu\partial^0\psi,$$

show that P^μ may be rewritten as

$$P^\mu = \sum_{s=1}^{2}\int \frac{d^3\mathbf{p}}{2p^0}p^\mu[a_s^\dagger(\mathbf{p})a_s(\mathbf{p}) - b_s(\mathbf{p})b_s^\dagger(\mathbf{p})].$$