MATH425 - Quantum Field Theory

Set Work: Sheet 8

1. In the Heisenberg picture the operators are defined by

$$\alpha_H = U^\dagger \alpha U,$$

where α is the operator in the standard Schrödinger picture, and the unitary operator U is defined by

$$i\hbar \frac{\partial U}{\partial t} = HU.$$

Show that

$$i\hbar \frac{\partial \alpha_H}{\partial t} = [\alpha_H, H_H].$$

2. Two-particle states are defined by

$$|\mathbf{p}_1, \mathbf{p}_2 >= a^\dagger(\mathbf{p}_1) a^\dagger(\mathbf{p}_1)|0 > .$$

Show that

$$< \mathbf{p}_1', \mathbf{p}_2' | \mathbf{p}_1, \mathbf{p}_2 >$$
$$=(2\pi)^6 (2p_1^0)(2p_2^0)\{\delta(\mathbf{p}_1 - \mathbf{p}_1')\delta(\mathbf{p}_2 - \mathbf{p}_2') + \delta(\mathbf{p}_1 - \mathbf{p}_2')\delta(\mathbf{p}_2 - \mathbf{p}_1')\}.$$

3. Show that if we define the number operator

$$N = \frac{1}{(2\pi)^3} \int \frac{d^3\mathbf{p}}{2p^0} a^\dagger(\mathbf{p}) a(\mathbf{p}),$$

then it satisfies

$$[N, a^\dagger(\mathbf{p})] = a^\dagger(\mathbf{p})$$

and hence

$$N|\mathbf{p}_1 \ldots \mathbf{p}_n >= n|\mathbf{p}_1 \ldots \mathbf{p}_n >$$