**MATH425 - Quantum Field Theory**
**Set Work: Sheet 6**

**1.** Show that

$$\gamma_\mu \not{a}_1 \gamma^\mu = -2(a_1.\gamma),$$
$$\gamma_\mu \not{a}_1 \not{a}_2 \gamma^\mu = 4a_1.a_2,$$

where

$$\not{a}_1 = a_1^\nu \gamma_\nu.$$

**2.** Show that if $j^\mu(x) = \overline{\psi}\gamma^5\gamma^\mu\psi$, then under a Lorentz transformation $j^\mu$ transforms as

$$j'^\mu(x') = L^\mu{}_\nu j^\nu(x),$$

while under a parity transformation $j^\mu$ transforms as

$$j'^\mu(x') = j^\mu(x).$$

$j^\mu(x)$ is called an **axial vector** or **pseudovector**.

**3.** Consider a plane wave

$$\psi(x) = v(p)e^{ip.x}$$

where

$$p^0 = \sqrt{\mathbf{p}^2 + m^2}.$$

Show that this will be a solution to the Dirac equation if

$$v(p) = N(p)\begin{pmatrix} \frac{\sigma.\mathbf{p}}{p^0+m}\chi \\ \chi \end{pmatrix},$$

where $\chi$ is a 2-vector and $N(p)$ is arbitrary. Suppose we pick two orthonormal 2-vectors $\chi_{1,2}$ satisfying

$$\chi_r^\dagger \chi_s = \delta_{rs}.$$

Show that if we pick $N(p) = \sqrt{p^0 + m}$, then we have

$$\overline{v_r(p)}v_s(p) = -2m\delta_{rs}.$$