MATH425 - Quantum Field Theory

Set Work: Sheet 2

1. In the interaction picture the Hamiltonian is given by

$$H = H_0 + H_I$$

where the free Hamiltonian $H_0^I = H_0^S$ is the same as in the Schrödinger picture. A state in the Interaction picture is related to a state in the SChrödinger picture by

$$|A,t\rangle_I = U_0|A,t\rangle_S$$

where,

$$U_0 \equiv U_0(t,t_0) = \mathrm{e}^{-i\frac{H_0(t-t_0)}{\hbar}},$$

and an operator in the interaction picture is related to an operator in the Schrödinger picture by

$$O^I(t) = U_0^\dagger O^S U_0.$$

(i) show that an operator in the interaction picture obeys

$$i\hbar\frac{dO^I(t)}{dt} = \left[O^I(t), H_0\right]$$

(ii) Use (1) to show that a state in the interaction picture obeys

$$i\hbar\frac{d}{dt}\frac{|A,t\rangle_I}{} = H_I^I|A,t\rangle_I\ .$$

2. The defining equation for the Lorentz group may be written

$$L^T\eta L = \eta. \qquad (1)$$

Consider a 2-dimensional spacetime for which $\eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Show that the standard Lorentz transformation

$$L = \begin{pmatrix} \gamma & -\frac{\gamma v}{c} \\ -\frac{\gamma v}{c} & \gamma \end{pmatrix},$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, satisfies the above condition.