MATH425 - Quantum Field Theory
Set Work: Sheet 4

1. Show that

$$[\gamma^\mu, \gamma_\lambda\gamma_\rho] = 2(\delta^\mu{}_\lambda\gamma_\rho - \delta^\mu{}_\rho\gamma_\lambda).$$

Hence check that

$$\Sigma_{\lambda\rho} = \frac{i}{8}[\gamma_\lambda, \gamma_\rho]$$

is a solution of

$$\tfrac{1}{2}(\delta^\mu{}_\lambda\gamma_\rho - \delta^\mu{}_\rho\gamma_\lambda) = i[\gamma^\mu, \Sigma_{\lambda\rho}].$$

2. Defining $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, show that

$$\gamma^{5\dagger} = \gamma^5$$
$$\{\gamma^5, \gamma^\mu\} = 0.$$

3. By inserting $(\gamma^\mu)^2 = 1$ for some $\mu = 0, 1, 2, 3$, write each of $\gamma^0\gamma^1\gamma^2$ and $\gamma^0\gamma^1\gamma^3$ as a product $\gamma^5\gamma^\nu$ for some $\nu = 0, 1, 2, 3$.

4. Show that

$$\mathrm{tr}[\gamma_\mu\gamma_\nu] = 4\eta_{\mu\nu}.$$

Now show that

$$\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma = 2\eta_{\mu\nu}\gamma_\rho\gamma_\sigma - 2\eta_{\mu\rho}\gamma_\nu\gamma_\sigma + 2\eta_{\mu\sigma}\gamma_\nu\gamma_\rho - \gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\mu.$$

Hence show that

$$\mathrm{tr}[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = 4[\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}]$$