**MATH431 - Modern Particle Physics**

**Set Work: Sheet 8; Due:**

**1.** Suppose that we assign the proton and the neutron to the Isospin doublet in the fundamental triplet representation of $SU(3)$, and identify the electric charge with the combination

$$Q_{electric} = \alpha T_{3_{Isospin}} + \beta Y_{hypercharge},$$

where $T_{3_{Isospin}}$ and $Y_{hypercharge}$ are the diagonal generators of $SU(3)$ with the normalization used in the class.

(a.) Determine the coefficients $\alpha$ and $\beta$.

(b.) Given the coefficients found in (a.) work out the charge assignments for the sextet, octet and decuplet representations of $SU(3)$.

**2.** Consider the simple unitary group $SU(5)$.

(a.) How many diagonal generators of the Lie algebra are there? Write down a representation of the diagonal generators in the terms of $5 \times 5$ hermitian matrices.

(b.) What is the dimension of the group?

(c.) What is the fundamental representation of $SU(5)$? Write down its decomposition in terms of the maximal subgroup $SU(3) \times SU(2) \times U(1)$.

(d.) Find the product and the decomposition under the maximal subgroup $SU(3) \times SU(2) \times U(1)$ of the fundamental times the anti-fundamental representations of $SU(5)$.

**3.** (a) Write down the weight lattice of the spinorial 16 representation of $SO(10)$.

(b) Show how the spinorial 16 representation of $SO(10)$ decomposes under the $SU(5) \times U(1)$, $SO(6) \times SO(4)$ and $SU(3) \times SU(2) \times U(1)^2$ subgroups.

(c) Identify how the Standard Model states fit into the spinorial 16 of $SO(10)$.

**4.** (a) Show that orbital angular momentum is a constant of the motion for a free non–relativistic particle?

(b) Show that orbital angular momentum is not a constant of the motion for a free Dirac particle?

(c) Show that total angular momentum of a free Dirac particle is a constant of the motion.