MATH431 - Modern Particle Physics

Set Work: Sheet 7; Due:

1. Derive the *Gordon decomposition* of the Dirac transition current:

$$\bar{\psi}_f \gamma^\mu \psi_i = \frac{1}{2m} \bar{\psi}_f [(p_f + p_i)^\mu + i\sigma^{\mu\nu}(p_f - p_i)_\nu] \psi_i \ ,$$

where $\sigma^{\mu\nu} = \frac{1}{2} i(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$. [Hint: Use the Dirac equations $\bar{\psi}_f(\not{p}_f - m) = (\not{p}_i - m)\psi_i = 0$.]

2. Consider an electron in a positive constant magnetic field along the z-axis.

(a) write down the vector potential.

(b) Write down the Dirac equation in terms of the two spinor components $\psi = (\phi, \chi)$

(c) Assuming a solution of the form

$$\psi = (\phi(\vec{x}), \chi(\vec{x}))e^{-iEt}$$

solve the Dirac equation in the presence of the constant magnetic field and find the energy eigenvalues.

3. Show that a unitary matrix U can be written as U^{iH}. What are the condition that the matrix H must satisfy?

4. Consider the simple unitary group $SU(4)$.

(a.) How many diagonal generators of the Lie algebra are there? Write down a representation of the diagonal generators in the terms of 4×4 hermitian matrices.

(b.) What is the dimension of the group? Write down a representation of the generators in terms of 4×4 hermitian matrices.

(c.) What is the fundamental representation of $SU(4)$? Write down its decomposition in terms of a maximal subgroup.

(d.) Draw the graphic illustration of the fundamental representation.

(e.) Find the product and the decomposition under the maximal subgroup of the fundamental times the anti-fundamental representations of $SU(4)$.

(f.) Find the product and the decomposition under the maximal subgroup of the fundamental times the fundamental representations of $SU(4)$.

(g.) Discuss the interpretation of the decomposition of the $SU(4)$ group, and its fundamental representation, in terms of the Standard Model subgroups. What can you say about the $U(1)$ charges of the Cartan generators?