**MATH431 - Modern Particle Physics**
**Set Work: Sheet 3;**

**1.** Consider the Poincare group in 1+2 dimensions, with the infinitesimal line element

$$ds^2 = dt^2 - dx^2 - dy^2$$

a. Write in matrix form the metric for this line element and its inverse.

b. Write all the transformations under which this line element is invariant. Write down the generators associated with each transformation.

c. The Pauli-Lubanski vector in four dimensions is given by

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\sigma\rho} J_{\nu\sigma} P_\rho$$

Write down the four components of the Pauli-Lubanski vector in the case of the 3 dimesnional line element given above, for massless and massive particle states.

**2.** The Pauli–Lubanski vector in four dimensions is given by

$$W_\sigma = -\frac{1}{2}\epsilon_{\sigma\mu\nu\lambda} J^{\mu\nu} P^\lambda,$$

where $\epsilon_{\sigma\mu\nu\lambda}$ is asymmetric under odd permutation of the indices; $J^{\mu\nu}$ are the generator of Lorentz transformations; and $P^\lambda$ is the momentum four vector.

a. Show that $W_\sigma P^\sigma = 0$

b. The commutation relations of the Pauli–Lubanski with the generators of the Poincare group are given by

$$[J_{\mu\nu}, W_\rho] = i\left(\eta_{\nu\rho} W_\mu - \eta_{\mu\rho} W_\nu\right),$$
$$[P_\nu, W_\rho] = 0$$

Show that the generators of the Poincare transformations commute with $W_\mu W^\mu$.

**3.** Write down the Lagrangian for a particle of mass $m$ in a potential $V(r, \phi)$ when referred to planar polar coordinates $(r, \phi)$. Hence show that the equations of motion are

$$m\ddot{r} - mr\dot{\phi}^2 = -\frac{\partial V}{\partial r} \qquad mr\ddot{\phi} + 2m\dot{r}\dot{\phi} = -\frac{1}{r}\frac{\partial V}{\partial \phi}.$$

Hence derive the principle of conservation of angular momentum in the plane, and obtain the usual formula $v^2/r$ for centripetal acceleration.

**4.** a. Show that if the Hamiltonian is independent of a generallized coordinate $q_0$, then the conjugate momentum $p_0$ is a constant ot the motion. Such coordinates are called **cyclic coordinates**. Give two examples of a physical system that has a cyclic coordinate.

b. Show that in 3 dimension spherical polar coordinates the Hamiltonian of a particle of mass $m$ moving in a potential $V(\vec{x})$ is

$$H = \frac{1}{2m}(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2 \sin^2 \theta}) + V(\vec{x}).$$

show that $p_\phi =$ constant when $\partial V/\partial\phi \equiv 0$ and interpret this result physically.