MATH431 - Modern Particle Physics
Set Work: Sheet 9;

1. (a.) Show that the substitution of the Lagrangian

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^{\mu} A_{\mu}$$

into the Euler Lagrange equation for A_μ give the Maxwell equation

$$\partial_\mu F^{\mu\nu} = j^\nu$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Hence show that the current j^ν is conserved.

(b.) With the addition of the term $\frac{1}{2} m^2 A_\mu A^\mu$, show that the modified Lagrangian leads to the equation of motion

$$(\partial_\mu \partial^\mu + m^2) A^\mu = j^\nu .$$

2. The Lagrangian for three interacting real fields ϕ_1, ϕ_2, ϕ_3 is

$$L = \frac{1}{2} (\partial_\mu \phi_i)^2 - \frac{1}{2} \mu^2 \phi_i^2 - \frac{1}{4} \lambda (\phi_i^2)^2$$

with $\mu^2 < 0$ and $\lambda > 0$, and where a summation over i is implied. Show that it describes a massive field of mass $\sqrt{-2\mu^2}$ and two massless Goldstone bosons.

3. Suppose we have two complex scalar fields Φ_1 and Φ_2 that form a doublet of $SU(2)_I$. Write down the Lagrangian and show that $m_1 \neq m_2$ entails explicit breaking of the symmetry. Can we write a Higgs potential that preserve the symmetry? Describe how to break the symmetry.

4. Consider the Lagrangian

$$L(q(t), \dot{q}(t), t)$$

and variation of

$$q(t) \quad \rightarrow \quad q(t) \ + \ \delta(q(t)) = q(t) \ + \ \epsilon h(q(t), t)$$

where ϵ is an infinitesimal constant. Show that if the Lagrangian is invariant under the variation then the charge defined as

$$\epsilon Q \equiv \frac{\partial L}{\partial \dot{q}} \delta q$$

is conserved.