**MATH431 - Modern Particle Physics**
**Set Work: Sheet 6; Due:**

**1.** The Dirac wave function for the ground state of the hydrogen atom has the following for (in the standard Dirac matrix representation):

$$\psi(r,\theta,\phi) = R(r)\begin{pmatrix} 1 \\ 0 \\ ia\cos\theta \\ iae^{i\phi}\sin\theta \end{pmatrix},$$

where $a \approx \alpha/2$.

(a)Investigate whether $\psi$ is an eigenstate of $L_z$.

(b)Find the expectation value of $L_z$ and comment on the result.

(c)Show that $\psi$ is an eigenstate of $J_z$ and find its eigenvalue.

[Don't forget to normalize for one electron]

**2.** (a) Consider the Dirac equation in four dimensions

$$(i\gamma^\mu\partial_\mu - m)\,\psi(x) = 0$$

where $\psi$ is a Dirac spinor. Show that each of the four components of the Dirac spinor satisfies the Klein-Gordon equation.

(b) Derive the conservation equation $\partial_\mu J_V^\mu = 0$ for the four vector current density $J_V^\mu = \bar{\psi}\gamma^\mu\psi$, using the covariant form of the Dirac equation and the relation $(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$.

(c) Show that the axial 4-vector current density $J_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$ is not conserved but instead satisfies the covariant equation

$$\partial_\mu J_A^\mu = 2im\bar{\psi}\gamma^5\psi\ .$$

**3.** Derive the *Gordon decomposition* of the Dirac transition current:

$$\bar{\psi}_f\gamma^\mu\psi_i = \frac{1}{2m}\bar{\psi}_f[(p_f+p_i)^\mu + i\sigma^{\mu\nu}(p_f-p_i)_\nu]\psi_i\ ,$$

where $\sigma^{\mu\nu} = \frac{1}{2}i(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$. [Hint: Use the Dirac equations $\bar{\psi}_f(\not{p}_f - m) = (\not{p}_i - m)\psi_i = 0$.]

**4.** Consider an electron in a positive constant magnetic field along the $z$-axis.

(a) write down the vector potential.

(b) Write down the Dirac equation in terms of the two spinor components $\psi = (\phi, \chi)$

(c) Assuming a solution of the form

$$\psi = (\phi(\vec{x}), \chi(\vec{x}))e^{-iEt}$$

solve the Dirac equation in the presence of the constant magnetic field and find the energy eigenvalues.