

MATH431 - Modern Particle Physics

Set Work: Sheet 5;

1. Two-particle states are defined by

$$|\mathbf{p}_1, \mathbf{p}_2\rangle = a^\dagger(\mathbf{p}_1)a^\dagger(\mathbf{p}_2)|0\rangle.$$

Show that

$$\begin{aligned} & \langle \mathbf{p}'_1, \mathbf{p}'_2 | \mathbf{p}_1, \mathbf{p}_2 \rangle \\ &= (2\pi)^6 (2p_1^0)(2p_2^0) \{ \delta(\mathbf{p}_1 - \mathbf{p}'_1) \delta(\mathbf{p}_2 - \mathbf{p}'_2) + \delta(\mathbf{p}_1 - \mathbf{p}'_2) \delta(\mathbf{p}_2 - \mathbf{p}'_1) \}. \end{aligned}$$

2. (a) Let A_μ be the electromagnetic vector potential. The electromagnetic field strength tensor is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Show that Maxwell's equation in four vector notation can be derived from the electromagnetic Lagrangian given by

$$L_{e.m.} = -\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}.$$

Show that $L_{e.m.}$ is invariant under the transformation $A_\mu \rightarrow A_\mu - \partial_\mu \Lambda$, where Λ is a scalar function.

- (b) Show that imposing a local $U(1)$ symmetry forbids the photon from attaining a mass.

3. Defining $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, show that

$$\begin{aligned} \gamma^{5\dagger} &= \gamma^5 \\ \{\gamma^5, \gamma^\mu\} &= 0. \end{aligned}$$

4. By inserting $(\gamma^\mu)^2 = 1$ for some $\mu = 0, 1, 2, 3$, write each of $\gamma^0\gamma^1\gamma^2$ and $\gamma^0\gamma^1\gamma^3$ as a product $\gamma^5\gamma^\nu$ for some $\nu = 0, 1, 2, 3$.

5. Show that

$$\text{tr}[\gamma_\mu\gamma_\nu] = 4\eta_{\mu\nu}.$$

Now show that

$$\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma = 2\eta_{\mu\nu}\gamma_\rho\gamma_\sigma - 2\eta_{\mu\rho}\gamma_\nu\gamma_\sigma + 2\eta_{\mu\sigma}\gamma_\nu\gamma_\rho - \gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\mu.$$

Hence show that

$$\text{tr}[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = 4[\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}]$$