

MATH431 - Modern Particle Physics

Set Work: Sheet 7; Due:

1. Show that a unitary matrix U can be written as U^{iH} . What are the condition that the matrix H must satisfy?
2. Consider the simple unitary group $SU(4)$.
 - (a.) How many diagonal generators of the Lie algebra are there? Write down a representation of the diagonal generators in the terms of 4×4 hermitian matrices.
 - (b.) What is the dimension of the group? Write down a representation of the generators in terms of 4×4 hermitian matrices.
 - (c.) What is the fundamental representation of $SU(4)$? Write down its decomposition in terms of a maximal subgroup.
 - (d.) Draw the graphic illustration of the fundamental representation.
 - (e.) Find the product and the decomposition under the maximal subgroup of the fundamental times the anti-fundamental representations of $SU(4)$.
 - (f.) Find the product and the decomposition under the maximal subgroup of the fundamental times the fundamental representations of $SU(4)$.
3. Consider the electromagnetic field in three spacetime dimensions.
 - (a) Find the reduced Maxwell equations in three dimensions by starting with Maxwell's equations and the force law in four dimensions, using the ansatz $E_z = B_x = B_y = 0$, and assuming that no field can depend on the z direction.
 - (b) Write down the field strength tensor in three dimensions in terms of the three dimensional scalar and vector potentials. Write down the field strength tensor in three dimensions in terms of the three dimensional electric and magnetic fields.