



April 2021 CLASS TEST

Introduction to Modern Particle Physics

TIME ALLOWED: 1 hour

INSTRUCTIONS TO CANDIDATES: In this paper bold-face quantities like **x** represent three-dimensional vectors.

Full marks can be obtained for complete answers to the two questions.

1. Consider the infinitesimal line element,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dx^2 + dy^2.$$

- (a) Write the metric $g_{\mu\nu}$ and its inverse in an explicit matrix form.

[2 marks]

- (b) Find the set of independent transformations of the form

$$\begin{aligned} x &\rightarrow x + \epsilon A(x, y) \\ y &\rightarrow y + \epsilon B(x, y), \end{aligned}$$

where ϵ is an infinitesimal constant and the functions A and B have to be determined by the requirement that ds^2 is invariant. State what each transformation represents in space time.

[18 marks]

2. (a.) Show that the substitution of the Lagrangian

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^\mu A_\mu$$

into the Euler Lagrange equation for A_μ give the Maxwell equation

$$\partial_\mu F^{\mu\nu} = j^\nu$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Hence show that the current j^ν is conserved.

[15 marks]

- (b.) With the addition of the term $\frac{1}{2} m^2 A_\mu A^\mu$, show that the modified Lagrangian leads to the equation of motion

$$(\partial_\mu \partial^\mu + m^2) A^\mu = j^\mu.$$

[5 marks]