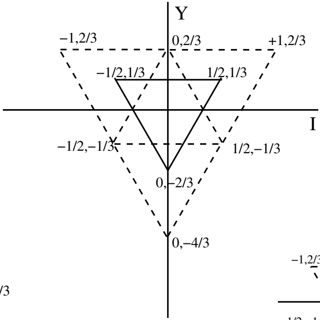
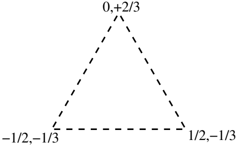
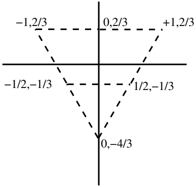
from the previous lectures ...







The product $3 \times 3 = 6 + \bar{3} \quad \rightarrow \quad$ the sextet and $\bar{3}$ representations of $SU(3)$.

Inside the sextet we have

$$6 \quad = \quad \begin{cases} (-1, \frac{2}{3}) \qquad\qquad (0, \frac{2}{3}) \qquad\qquad (+1, \frac{2}{3}) & \rightarrow \quad SU(2)_I \text{ triplet} \\ \qquad (-\frac{1}{2}, -\frac{1}{3}) \qquad\qquad (\frac{1}{2}, -\frac{1}{3}) & \rightarrow \quad SU(2)_I \text{ doublet} \\ \qquad\qquad (0, -\frac{4}{3}) & \rightarrow \quad SU(2)_I \text{ singlet} \end{cases}$$

which decomposes under $SU(3) \supset SU(2) \times U(1)$ as

$$6 = 3_{\frac{2}{3}} + 2_{-\frac{1}{3}} + 1_{-\frac{4}{3}}$$

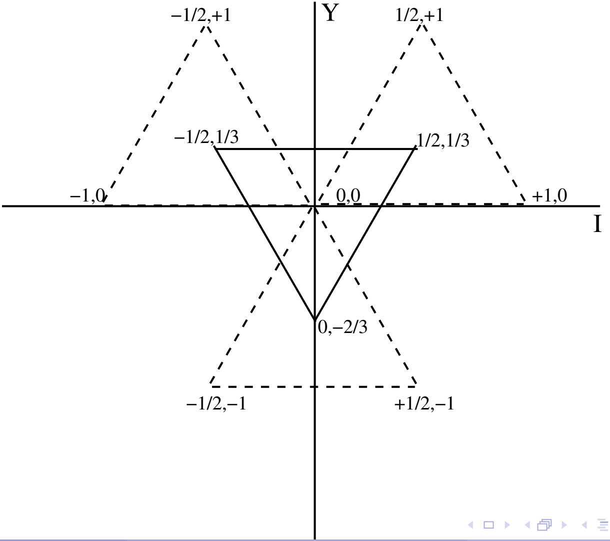
and the $\bar{3}$ decomposes as

$$\bar{3} = 2_{-\frac{1}{3}} + 1_{\frac{2}{3}}$$

we get the $\bar{3}$ representation. Note that $\bar{3} \neq 3$, whereas in $SU(2)$ $\bar{2} = 2$.

there physical particle that fit the 6 & $\bar{3}$ together with P, N?

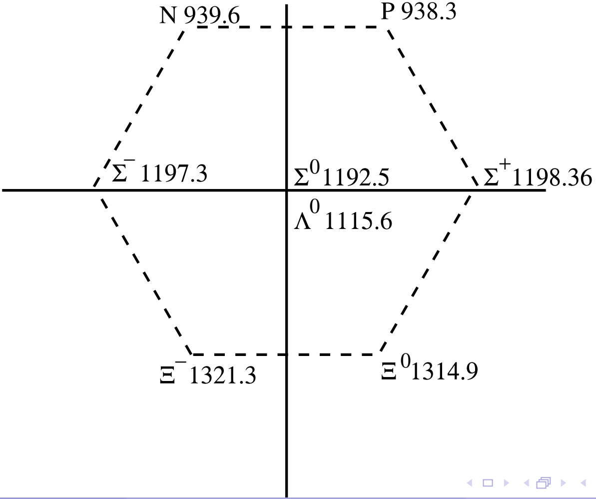
Not yet look at another possibility $3 \times \bar{3}$. Multiply $\bar{3}$ at every point of 3.



$3 \times \bar{3} = 8 + 1 \quad \rightarrow \quad$ octet + singlet

under $SU(2) \times U(1)$ the octet decomposes as $8 = 2_{+1} + 3_0 + 2_{-1} + 1_0$

The octet has a physical assignment

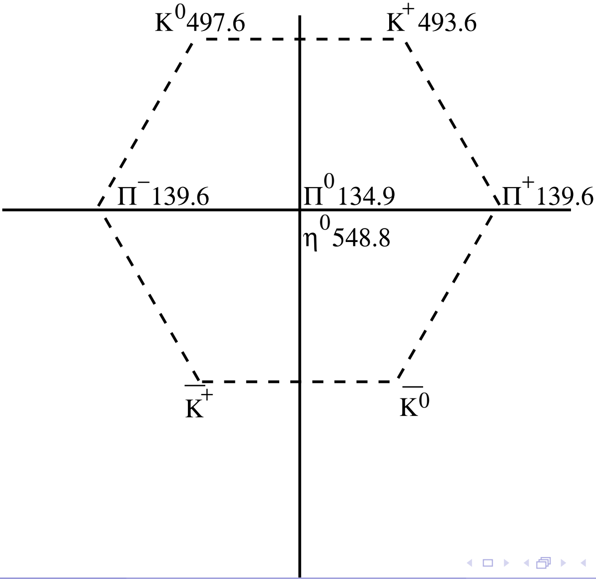


The spin of the particles is $s = \frac{1}{2}$ and baryon number $B = +1$.
We can find a relation between the electric charge and the Isospin T_3 and hypercharge Y

$$Q = \alpha T_3 + \beta Y \qquad \left.\begin{array}{r} Q(P) = +1 = \alpha \cdot (\ \frac{1}{2}) + \beta \cdot 1 \\ Q(N) = \quad 0 = \alpha \cdot (-\frac{1}{2}) + \beta \cdot 1 \end{array}\right\} \Rightarrow \quad \alpha = 1 \ \ , \quad \beta = \frac{1}{2}$$

$Q = T_3 + \frac{1}{2} Y \quad \rightarrow \quad$ holds for the other particles in the octet

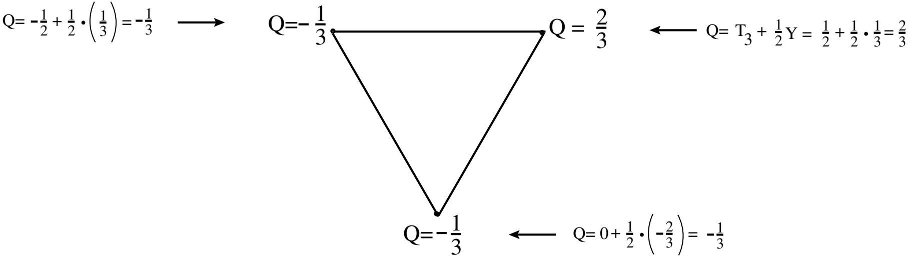
Second example: mesons *spin* $= 0$; Baryon number $= 0$.



Question: which representations can represent particles and why?

$8 \quad \to \quad$ Baryons, mesons

$3,\ \bar{3},\ 6,\ \bar{6}$ don't represent integrally charged particles.



Perhaps the physical representations are those that yield integrally charged states.

Another physical representation: 10

$$\begin{array}{ccccccccccc}
\Delta^- & & \Delta^0 & & \Delta^+ & & \Delta^{++} & & I=\frac{3}{2} & & Y=1\\
& Y^{*-} & & Y^{*0} & & Y^{*+} & & & I=1 & & Y=0\\
& & \Xi^{*-} & & \Xi^{*0} & & & & I=\frac{1}{2} & & Y=-1\\
& & & \Omega^- & & & & & I=0 & & Y=-2
\end{array}$$

$S=\frac{3}{2}$, $B=\frac{3}{2}$

$$\begin{aligned}
m(\Delta) &\sim 1230-1234\mathrm{MeV}\\
&\qquad \updownarrow 150\mathrm{MeV}\\
m(Y^*) &\sim 1382\mathrm{MeV}\\
&\qquad \updownarrow 150\mathrm{MeV}\\
m(\Xi^*) &\sim 1531\mathrm{MeV}
\end{aligned}$$

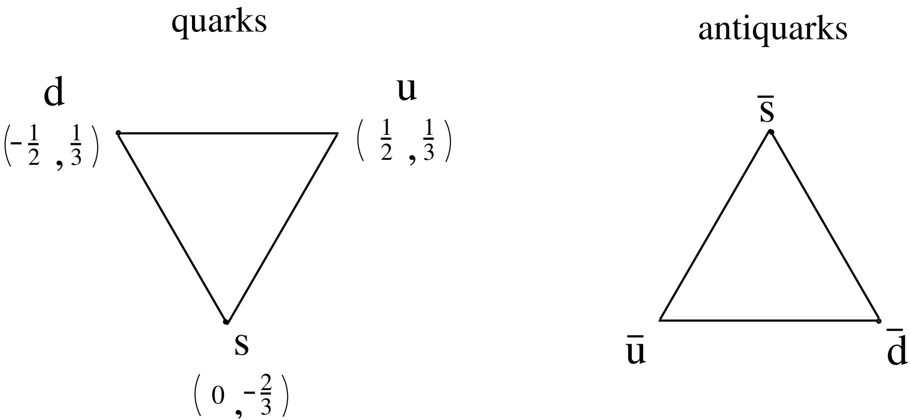
The Ω^- was predicted in Gellman's "The eightfold way" with $m(\Omega^-)\sim 1680\mathrm{MeV}$ and was discovered in 1964.

The $SU(3)$ representations are obtained from products of 3, $\bar{3}$.

Physical	Unphysical
$3 \times \bar{3} = 8 + 1$	$3 \times 3 = 6 + \bar{3}$
$3 \times 3 \times 3 = 10 + 8 + 8 + 1$	
$\bar{3} \times \bar{3} \times \bar{3} = \overline{10} + 8 + 8 + 1$	

Questions: Only some products of 3 and $\bar{3}$ are physical. Why? Is there a physical meaning to the 3, $\bar{3}$?

Gellmann & Zweig: The baryons & mesons are made of more elementary building blocks $\rightarrow$ quarks.



$$
\begin{aligned}
Q(\text{ up }) &= \tfrac{2}{3} & \qquad Q(\text{ antiup }) &= -\tfrac{2}{3} \\
Q(\text{down}) &= -\tfrac{1}{3} & \qquad Q(\text{antidown}) &= \tfrac{1}{3} \\
Q(\text{strange}) &= -\tfrac{1}{3} & \qquad Q(\text{antistrange}) &= \tfrac{1}{3}
\end{aligned}
$$

The spin of the quarks must be $s = \frac{1}{2} \;\Rightarrow\; s(P,\ N) = \frac{1}{2}$.

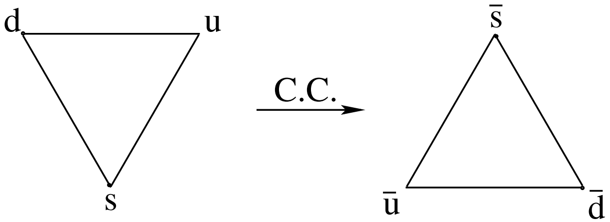
The physical states correspond to bound states of

$$\begin{aligned} \text{quark–antiquark} &\quad\to\quad 3 \times \bar{3} \\ \text{quark–quark–quark} &\quad\to\quad 3 \times 3 \times 3 \\ \text{antiquark–antiquark–antiquark} &\quad\to\quad \bar{3} \times \bar{3} \times \bar{3} \end{aligned}$$

	T_3	Y	$Q = T_3 + \frac{1}{2}Y$
u	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
d	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$
s	0	$-\frac{2}{3}$	$-\frac{1}{3}$

	T_3	Y	$Q = T_3 + \frac{1}{2}Y$
$\bar{u}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$
$\bar{d}$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
$\bar{s}$	0	$\frac{2}{3}$	$\frac{1}{3}$

Under charge conjugation we have



In $SU(2)$ $\;2 = \bar{2}$

In $SU(3)$ $\;3 \neq \bar{3}$