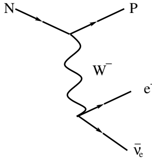
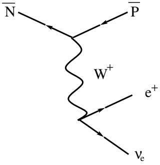
from the previous lectures ...

<u>Unification of E&M and weak interactions</u>

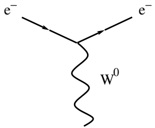
(Glashow 1961; Weinberg; Salam 1968)

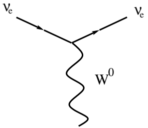
<u>Problems</u>: Weak interactions also involve leptons





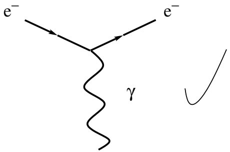
However, if $W^{\pm} \in SU(2)$ we must also have $W^0 \in SU(2)$

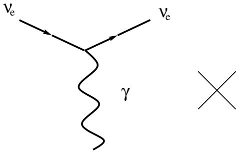




Do these currents exist in nature?

? indetify $W^0 = \gamma \quad \rightarrow \quad \text{photon?}$





ν is neutral $\rightarrow$ does not couple to γ

Must have a new W^0 that couples to ν

The new W^0 must be a mixture of W^3 and γ such that $Q(W^{\pm}) = \pm 1$.

We saw: Weak interactions only couple to left–handed fields whereas E&M couples to both left & right handed fields

$\rightarrow$ only (e_L, ν_L) and (u_L, d_L) interact weakly

(e_L, e_R) and (u_L, u_R, d_L, d_R) interact E&M

$Q(\nu_L) = 0$.

So far no need for ν_R *i.e.* no strong, weak or E&M interactions for ν_R.

Left–handed fields form doublets of $SU(2)_W$.

Right–handed fields are singlets of $SU(2)_W$.

$$
\begin{array}{llllll}
\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L & \begin{pmatrix} u \\ d \end{pmatrix}_L & e_R & u_R & d_R \\
\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L & \begin{pmatrix} c \\ s \end{pmatrix}_L & \mu_R & c_R & s_R \\
\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L & \begin{pmatrix} t \\ b \end{pmatrix}_L & \tau_R & t_R & b_R
\end{array}
$$

The quarks are triplets of $SU(3)_{\mathrm{color}}$.

The leptons are singlets of $SU(3)_{\mathrm{color}}$.

The $SU(2)_W$ doublets have T_3^W quantum numbers.

The $SU(2)_W$ singlets have $T_3^W = 0$.

We have to introduce a $U(1)$ symmetry to incorporate E&M charges

$SU(2)_W \times U(1)_Y$

But $U(1)_Y \neq U(1)_{\mathrm{e.m.}}$

All $SU(2)$ representations must have the same $U(1)_Y$ charge,

but $Q(e_L) \neq Q(\nu_L)$

$\Rightarrow \quad U(1)_Y \neq U(1)_{\text{e.m.}}$

<u>Combination</u> : $\quad Q_{\text{e.m.}} = T_{3_W} + \frac{1}{2} Y$

Find values for Y such that $Q_{\text{e.m.}}$ is reproduced for the different particle states.

	T_3	$\frac{1}{2}Y$	$Q_{e.m.}$		T_3	$\frac{1}{2}Y$	$Q_{e.m.}$
ν_L	$\frac{1}{2}$	$-\frac{1}{2}$	0	u_L	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
e_L	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	d_L	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
e_R	0	-1	-1	u_R	0	$\frac{2}{3}$	$\frac{2}{3}$
ν_R	0	0	0	d_R	0	$-\frac{1}{3}$	$-\frac{1}{3}$

How can we write a four vector current j^μ that will incorporate both the weak & electromagnetic interactions?

$$
\begin{aligned}
J_\mu^+(x) &= \bar{\chi}_L \gamma_\mu \tau_+ \chi_L \qquad , \qquad J_\mu^-(x) = \bar{\chi}_L \gamma_\mu \tau_- \chi_L \\
J_\mu^3(x) &= \bar{\chi}_L \gamma_\mu \frac{1}{2} \tau_3 \chi_L \\
&= (\bar{\nu}_e \bar{e}^-)_L \gamma_\mu \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L = \frac{1}{2} \bar{\nu}_{e_L} \gamma^\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma^\mu e_L
\end{aligned}
$$

where $\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$ and $\chi_L = \binom{\nu_e}{e^-}_L$. We can write this in the form

$$
J_\mu^i = \bar{\chi}_L \gamma_\mu \frac{1}{2} \tau_i \chi_L \quad \text{with} \quad i = 1, 2, 3.
$$

These currents couple to the vector bosons

$$
W^{\mu\pm} = \frac{1}{\sqrt{2}} (W^{\mu 1} \mp i W^{\mu 2})
$$

using the identity $\quad \dfrac{1}{2}(\tau_1 W^1 + \tau_2 W^2) = \dfrac{1}{\sqrt{2}}(\tau^+ W^+ + \tau^- W^-)$

We can write

$$\underbrace{\frac{1}{\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu})}_{\text{charged currents}} + \underbrace{J_\mu^3 W^{3\mu}}_{\text{neutral current}} = \sum_{i=1}^{3} J_\mu^i W^{i\mu}$$

The electromagentic current is given by

$$J_{e.m.}^\mu A_\mu = \underbrace{eQ}_{\text{electric charge coupling}} \bar{\psi}\gamma^\mu\psi A_\mu = \underbrace{e}_{\text{coupling}} \bar{\psi}\gamma^\mu \underbrace{Q}_{\text{charge}} \psi A_\mu$$

where A_μ is the E&M vector boson, *i.e* the photon.

The new $U(1)$ current $\frac{1}{2} \underbrace{g'}_{\text{gauge coupling}} \bar{\psi}\gamma^{\mu} \underbrace{Y}_{\text{hypercharge}} \psi \underbrace{B_{\mu}}_{\text{gauge field}}$

The neutral $SU(2)$ current is $g\bar{\psi}\gamma^{\mu} T_3 \psi W^3_{\mu}$ with $T_3 = \frac{\tau_3}{2}$

To get consistency with the charge assignment we should have

$$Q = T_3 + \frac{1}{2}Y \quad \Rightarrow \quad J^{e.m.}_{\mu} = J^3_{\mu} + \frac{1}{2}J^Y_{\mu}$$

we obtain this by making a rotation on $B_{\mu},\ W^3_{\mu}$.

$$\begin{pmatrix} A_{\mu} \\ Z_{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_{\mu} \\ W^3_{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_W B_{\mu} + \sin\theta_W W^3_{\mu} \\ -\sin\theta_W B_{\mu} + \cos\theta_W W^3_{\mu} \end{pmatrix}$$

or inversely $$\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

$$\theta_W \quad \longrightarrow \quad \text{Weinberg angle}$$

<u>we get</u> :

$$\begin{aligned} & g J^3_\mu W^{3\mu} + \frac{g'}{2} J^Y_\mu B^\mu \\ = \;& g J^3_\mu (\sin\theta_W A^\mu + \cos\theta_w Z^\mu) + \frac{g'}{2} J^Y_\mu (\cos\theta_W A^\mu - \sin\theta_w Z^\mu) \\ = \;& (g\sin\theta_W J^3_\mu + g'\cos\theta_W \frac{J^Y_\mu}{2}) A^\mu + (g\cos\theta_W J^3_\mu - g'\sin\theta_W \frac{J^Y_\mu}{2}) Z^\mu \end{aligned}$$

The first term is the electromagnetic interaction

$$e J^{e.m.}_\mu A^\mu = e(J^3_\mu + \frac{1}{2} J^Y_\mu) A^\mu$$

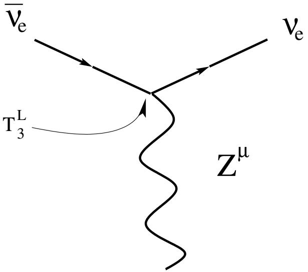
Therefore we have to impose

$$e = g \sin\theta_W = g' \cos\theta_W \quad \Longrightarrow \quad \tan\theta_W = \frac{g'}{g}$$

We can express the neutral current interaction in the form

$$\frac{g}{\cos\theta_W}(J_\mu^3 - \sin^2\theta_W J_\mu^{e.m.})Z^\mu = \frac{g}{\cos\theta_W} J_\mu^{NC} Z^\mu$$

We now have a new neutral current, coupling neutrinos to Z^μ



This neutral current was proposed by Glashow in 1961
and observed at CERN in 1970

$\rightarrow$ We still have a problem

$\rightarrow$ E&M interactions $\rightarrow$ Long range $\rightarrow$ $m_\gamma = 0$

$\rightarrow$ Weak interactions $\rightarrow$ Short range $\rightarrow$ $m_{W^\pm;Z} \neq 0$

How ? $\rightarrow$ symmetry breaking

Lagrangian is invariant, but vacuum $\rightarrow$ symmetry breaking

The vacuum $\rightarrow$ the states of lowest energy