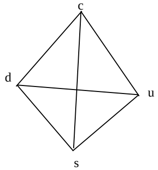
from the previous lectures ...

Nice ... But ...

Problems:

1) $\Delta^{++} = u^{\uparrow}u^{\uparrow}u^{\uparrow}$ fully symmetric $\rightarrow$ conflict with Pauli's spin–statistics
2) charm was discovered $su(3)_{\text{flavour}}$ $\rightarrow$ not enough. $SU(4)_f$ $\quad 4 \; =$



In 1974 J/Ψ particle was discovered with $m \sim 3.1\mathrm{GeV}$. spin $= 0$. $c\bar{c}$.

bottom was discovered in 1978 $m(B-\mathrm{meson}) \approx 10\mathrm{GeV}$. Spin $= 0$. $b\bar{b}$.

top was discovered in 1994 $m_t \sim 175\mathrm{GeV}$.

To resolve the conflict with Pauli's exclusion principle a new quantum number is introduced – colour.

All colour bound states form colour singlets. Hence the Δ^{++} wave function is asymmetric under colour and symmetric under flavour $\times$ soin $\times$ space quantum numbers.

$$(qqq)_{\rm colour\ singlet} = \frac{1}{\sqrt{6}}(RGB - RBG + BRG - BGR + GBR - GRB)$$

This wave–function is asymmetric under exchange of any two colour.

$\rightarrow$ quarks are in the fundamental representation of $SU(3)_C$.

$$q = \begin{pmatrix} R \\ G \\ B \end{pmatrix} \qquad \bar{q} = \begin{pmatrix} \overline{R} \\ \overline{G} \\ \overline{B} \end{pmatrix}$$

$SU(3)_{\rm colour}$ is a new degree of freedom, different from $SU(3)_f$.

$SU(3)_{\rm colour}$ is exact. $SU(3)_f$ is approximate and accidental.

Similarly to $SU(3)_f$ we can introduce $SU(4)_f$. $u,\ d,\ ,s,\ c$.

$SU(3)_f$ violated by mass differences of $O(100\mathrm{MeV})$.

$SU(4)_f$ violated by mass differences of $O(1\mathrm{GeV})$.

The representations would be:

$$
\begin{aligned}
4\times\bar{4} \quad &\rightarrow \quad \text{mesons} \quad (3\bar{3})_{\text{colour}} = 8+1 \\
4\times 4\times 4 \quad &\rightarrow \quad \text{baryons} \quad (3\cdot 3\cdot 3)_{\text{colour}} = 10+8+8+1
\end{aligned}
$$

quarks have spin $1/2 \rightarrow$ fermions

The problem of flavour is an important open question under experimental and theoretical research.

$$SU(3)_{\rm colour} \quad \to \quad \text{Exact symmetry} \;\to\; \text{strong interactions}$$
$$U(1)_{\rm E.M.} \quad \to \quad \text{Exact symmetry} \;\to\; \text{E \& M interactions}$$
<u>under colour</u> : $\quad q \to Uq = \mathrm{e}^{i\vec{\alpha}(x)\cdot\vec{\lambda}}q \quad \vec{\lambda} = (\lambda_1, \cdots, \lambda_8); \vec{\alpha} = (\alpha_1, \cdots, \alpha_8)$

gauge symmetry $\to$ local phase invariance $\to$ gauge bosons $\bar{\psi}(\partial_\mu + i\vec{A}_\mu \cdot \vec{\lambda})\psi$, $\vec{A} = (A_1, \cdots, A_8)$.

The strong interactions correspond to local phase invariance under $SU(3)_{\rm colour}$.

Quarks are observed only as confined states inside hadrons and mesons

$\to$ not observed as free quarks

observed electric charge is integral $\qquad Q(u) = \frac{2}{3} \quad Q(d) = -\frac{1}{3}$

$$Q(c) = \frac{2}{3} \quad Q(s) = -\frac{1}{3}$$

The bound states are (qqq) and $q\bar{q}$ $\;\Rightarrow$ only integrally charged combinations are observed

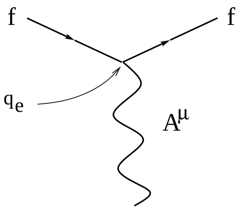
What about the weak interactions?

$$\text{E\&M} \quad \rightarrow \quad U(1) \quad \text{Abelian local symmetry}$$

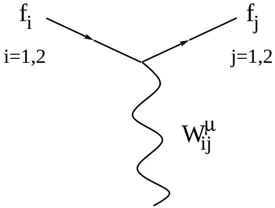
$$\left.\begin{array}{lll} \text{Weak} & \rightarrow & SU(2) \quad \text{local symmetry} \\ \text{Strong} & \rightarrow & SU(3) \quad \text{local symmetry} \end{array}\right\} \rightarrow \text{non–Abelian}$$

Consider the E&M vs weak interactions

U(1)



SU(2)



$$f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \quad \text{doublet}$$

The E&M interactions don't change the identity of the particle

In the weak interactions f_1 may be different from f_2

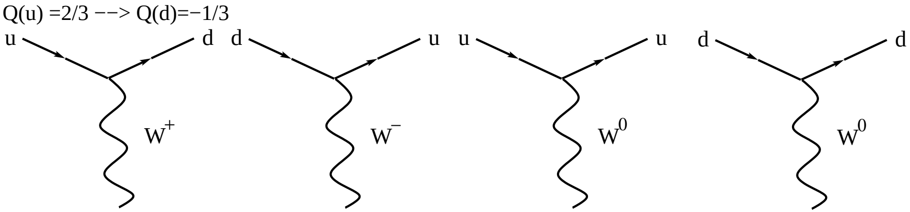
The gauge bosons W_i^μ are in the adjoint representation

$n \times \bar{n} = (n^2 - 1) + 1 \;\rightarrow SU(n)$ adjoint representation

$$W_\alpha^\mu \qquad \mu = 0, 1, 2, 3 \quad \text{Lorentz index}$$

$$\alpha = 1, 2, 3 \quad \text{gauge group index}$$

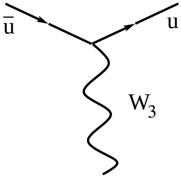
Couplings:

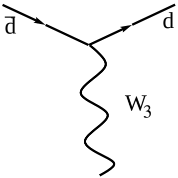


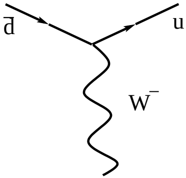
The current in the Lagrangian has the form

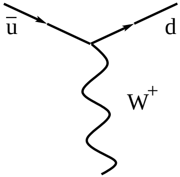
$$
\begin{aligned}
& \begin{pmatrix} \bar{u} & , & \bar{d} \end{pmatrix} \left(\sum_{i=1}^{3} \tau_i W_i^{\mu} = \vec{\tau} \cdot \vec{W}^{\mu} \right) \begin{pmatrix} u \\ d \end{pmatrix} \\
= & \begin{pmatrix} \bar{u} & , & \bar{d} \end{pmatrix} \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} W_1^{\mu} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} W_2^{\mu} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} W_3^{\mu} \right] \begin{pmatrix} u \\ d \end{pmatrix} \\
= & \begin{pmatrix} \bar{u} & , & \bar{d} \end{pmatrix} \begin{bmatrix} W_3^{\mu} & W_1^{\mu} - iW_2^{\mu} \\ W_1^{\mu} + iW_2^{\mu} & -W_3^{\mu} \end{bmatrix} \begin{pmatrix} u \\ d \end{pmatrix} =
\end{aligned}
$$

$$
W_3^{\mu} \left(\bar{u} u - \bar{d} d \right) \quad + \quad (W_1^{\mu} + iW_2^{\mu}) \bar{d} u \quad + \quad (W_1^{\mu} - iW_2^{\mu}) \bar{u} d
$$









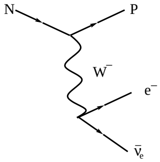
In $SU(3)$:

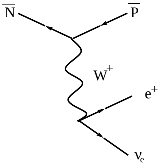
$$\begin{pmatrix} \bar{u}_1 & , & \bar{u}_2 & , \bar{u}_3 \end{pmatrix} \left(\sum_{i=1}^{8} \lambda_j A_j^{\mu} = \vec{\lambda} \cdot \vec{A}^{\mu} \right) \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

where λ_j are the Gellmann matrices with $j = 1, ..., 8$

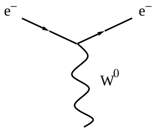
Unification of E&M and weak interactions (Glashow 1961; Weinberg; Salam 1968)

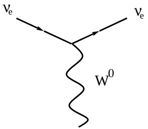
Problems: Weak interactions also involve leptons





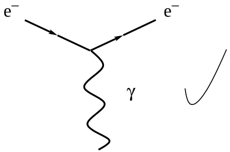
However, if $W^{\pm} \in SU(2)$ we must also have $W^0 \in SU(2)$

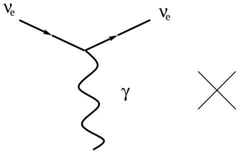




Do these currents exist in nature?

? indetify $W^0 = \gamma \quad \rightarrow \quad \text{photon?}$





ν is neutral $\rightarrow$ does not couple to γ

Must have a new W^0 that couples to ν

The new W^0 must be a mixture of W^3 and γ such that $Q(W^{\pm}) = \pm 1$.

<u>We saw</u>: Weak interactions only couple to left–handed fields whereas E&M couples to both left & right handed fields

$\rightarrow$ only (e_L, ν_L) and (u_L, d_L) interact weakly

(e_L, e_R) and (u_L, u_R, d_L, d_R) interact E&M

$Q(\nu_L) = 0$.

So far no need for ν_R *i.e.* no strong, weak or E&M interactions for ν_R.