

## MATH328 Modern Particle Physics Solutions 8

1a

$$3 = (2, 1/3) + (1, -2/3)$$

under  $SU(2) \times U(1)$ . The proton and neutron form an isospin doublet with  $SU(2)_I$  charge  $+1/2$  and  $-1/2$ , respectively. Then,

$$\begin{aligned} +1 &= \alpha \frac{1}{2} + \beta \frac{1}{3} \\ 0 &= \alpha \left(-\frac{1}{2}\right) + \beta \frac{1}{3} \end{aligned}$$

which gives  $\alpha = +1$ ,  $\beta = +3/2$

1b The decomposition of the sextet, octet and decuplet of  $SU(3)$  in terms of  $SU(2) \times U(1)$  is:

$$\begin{aligned} 6 &= \{(3, 2/3) + (2, -1/3) + (1, -4/3)\} \\ 8 &= \{(2, +1) + (3, 0) + (1, 0) + (2, -1)\} \\ 10 &= \{(4, +1) + (3, 0) + (2, -1) + (1, -2)\} \end{aligned}$$

The electric charges of the states are:

$$\begin{aligned} 6 &= \{(2, 1, 0) + (0, -1) + (-2)\} \\ 8 &= \{(2, 1) + (1, 0, -1) + (0) + (-1, -2)\} \\ 10 &= \{(3, 2, 1, 0) + (1, 0, -1) + (-1, -2) + (-3)\} \end{aligned}$$

**2a.** 4 diagonal generators.

$$\begin{aligned} \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \lambda_8 &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ \lambda_{15} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \lambda_{24} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{pmatrix}. \end{aligned}$$

This basis corresponds to the decomposition  $SU(5) \rightarrow SU(4) \times U(1)$ . Another basis

$$\begin{aligned}\lambda_3 &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, & \lambda_8 &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ \lambda_{15} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \\ \lambda_{24} &= \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix},\end{aligned}$$

which corresponds to the decomposition  $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$

2b.  $D = 24$ .

2c.

$$5 = (3, 1, 1/3) + (1, 2, -1/2)$$

under  $SU(3) \times SU(2) \times U(1)$ .

2d.

$$\bar{5} = (\bar{3}, 1, -1/3) + (1, 2, 1/2)$$

$$\begin{aligned}5 \times \bar{5} &= \{(3, 1, 1/3) + (1, 2, -1/2)\} \times \{(\bar{3}, 1, -1/3) + (1, 2, 1/2)\} = \\ &24 + 1 = \{(8, 1, 0) + (1, 3, 0) + (1, 1, 0) + (3, 2, 5/6) + (\bar{3}, 2, -5/6)\} + (1, 0)\end{aligned}$$