

MATH431 - Modern Particle Physics

Set Work: Sheet 6; Due:

1. The Dirac wave function for the ground state of the hydrogen atom has the following for (in the standard Dirac matrix representation):

$$\psi(r, \theta, \phi) = R(r) \begin{pmatrix} 1 \\ 0 \\ ia \cos \theta \\ ia e^{i\phi} \sin \theta \end{pmatrix},$$

where $a \approx \alpha/2$.

- (a) Investigate whether ψ is an eigenstate of L_z .
(b) Find the expectation value of L_z and comment on the result.
(c) Show that ψ is an eigenstate of J_z and find its eigenvalue.
[Don't forget to normalize for one electron]

2. Derive the conservation equation $\partial_\mu J_V^\mu = 0$ for the four vector current density $J_V^\mu = \bar{\psi} \gamma^\mu \psi$, using the covariant form of the Dirac equation and the relation $(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0$.

Show that the axial 4-vector current density $J_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$ is not conserved but instead satisfies the covariant equation

$$\partial_\mu J_A^\mu = 2im \bar{\psi} \gamma^5 \psi.$$

3. Derive the *Gordon decomposition* of the Dirac transition current:

$$\bar{\psi}_f \gamma^\mu \psi_i = \frac{1}{2m} \bar{\psi}_f [(p_f + p_i)^\mu + i\sigma^{\mu\nu} (p_f - p_i)_\nu] \psi_i,$$

where $\sigma^{\mu\nu} = \frac{1}{2}i(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$. [Hint: Use the Dirac equations $\bar{\psi}_f(\not{p}_f - m) = 0$ and $(\not{p}_i - m)\psi_i = 0$.]

4. Consider an electron in a positive constant magnetic field along the z -axis.
(a) write down the vector potential.
(b) Write down the Dirac equation in terms of the two spinor components $\psi = (\phi, \chi)$
(c) Assuming a solution of the form

$$\psi = (\phi(\vec{x}), \chi(\vec{x})) e^{-iEt}$$

solve the Dirac equation in the presence of the constant magnetic field and find the energy eigenvalues.