

MATH431 - Modern Particle Physics

Set Work: Sheet 8; Due:

1. Suppose that we assign the proton and the neutron to the Isospin doublet in the fundamental triplet representation of $SU(3)$, and identify the electric charge with the combination

$$Q_{electric} = \alpha T_{3_{Isospin}} + \beta Y_{hypercharge},$$

where $T_{3_{Isospin}}$ and $Y_{hypercharge}$ are the diagonal generators of $SU(3)$ with the normalization used in the class.

- (a.) Determine the coefficients α and β .
- (b.) Given the coefficients found in (a.) work out the charge assignments for the sextet, octet and decuplet representations of $SU(3)$.

2. Consider the simple unitary group $SU(5)$.

- (a.) How many diagonal generators of the Lie algebra are there? Write down a representation of the diagonal generators in the terms of 5×5 hermitian matrices.

- (b.) What is the dimension of the group?

- (c.) What is the fundamental representation of $SU(5)$? Write down its decomposition in terms of the maximal subgroup $SU(3) \times SU(2) \times U(1)$.

- (d.) Find the product and the decomposition under the maximal subgroup $SU(3) \times SU(2) \times U(1)$ of the fundamental times the anti-fundamental representations of $SU(5)$.

3. (a) Write down the weight lattice of the spinorial 16 representation of $SO(10)$.

- (b) Show how the spinorial 16 representation of $SO(10)$ decomposes under the $SU(5) \times U(1)$, $SO(6) \times SO(4)$ and $SU(3) \times SU(2) \times U(1)^2$ subgroups.

- (c) Identify how the Standard Model states fit into the spinorial 16 of $SO(10)$.