MATH328 Modern Particle Physics Solutions 8

1a

$$3 = (2, 1/3) + (1, -2/3)$$

under $SU(2) \times U(1)$. The proton and neutron form an isospin doublet with $SU(2)_I$ charge $+1/2$ and $-1/2$, repectively.Then,

$$\begin{aligned} +1 &= \alpha\frac{1}{2} + \beta\frac{1}{3} \\ 0 &= \alpha(-\frac{1}{2}) + \beta\frac{1}{3} \end{aligned}$$

which gives $\alpha = +1$, $\beta = +3/2$

1b The decomposition of the sextet, octet and decuplet of $SU(3)$ in terms of $SU(2) \times U(1)$ is:

$$\begin{aligned} 6 &= \{(3, 2/3) + (2, -1/3) + (1, -4/3)\} \\ 8 &= \{(2, +1) + (3, 0) + (1, 0) + (2, -1)\} \\ 10 &= \{(4, +1) + (3, 0) + (2, -1) + (1, -2)\} \end{aligned}$$

The electric charges of the states are:

$$\begin{aligned} 6 &= \{(2, 1, 0) + (0, -1) + (-2)\} \\ 8 &= \{(2, 1) + (1, 0, -1) + (0) + (-1, -2)\} \\ 10 &= \{(3, 2, 1, 0) + (1, 0, -1) + (-1, -2) + (-3)\} \end{aligned}$$

2a. 4 diagonal generators.

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} , \quad \lambda_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} ,$$
$$\lambda_{15} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} , \quad \lambda_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{pmatrix} .$$

This basis corresponds to the decomposition $SU(5) \to SU(4) \times U(1)$. Another basis

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad , \quad \lambda_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad ,$$

$$\lambda_{15} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad ,$$

$$\lambda_{24} = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} \quad ,$$

which corresponds to the decomposition $SU(5) \to SU(3) \times SU(2) \times U(1)$

2b. $D = 24$.

6c.

$$5 = (3, 1, 1/3) + (1, 2, -1/2)$$

under $SU(3) \times SU(2) \times U(1)$.

6d.

$$\bar{5} = (\bar{3}, 1, -1/3) + (1, 2, 1/2)$$

$$\begin{aligned} 5 \times \bar{5} = \{(3, 1, 1/3) + (1, 2, -1/2)\} \times \{(\bar{3}, 1, -1/3) + (1, 2, 1/2)\} = \\ 24 + 1 = \{(8, 1, 0) + (1, 3, 0) + (1, 1, 0) + (3, 2, 5/6) + (\bar{3}, 2, -5/6)\} + (1, 0) \end{aligned}$$