$$[J_1, J_2] \neq (\theta, \phi)$$

$$\left(\cos\phi \frac{\partial}{\partial\theta} - \frac{\cos\theta}{\sin\theta}\sin\phi \frac{\partial}{\partial\phi}\right)\left(\sin\phi \frac{\partial}{\partial\theta} + \frac{\cos\theta}{\sin\theta}\cos\phi \frac{\partial}{\partial\phi}\right) f$$

$$- \left(\sin\phi \frac{\partial}{\partial\theta} + \frac{\cos\theta}{\sin\theta}\cos\phi \frac{\partial}{\partial\phi}\right)\left(\cos\phi \frac{\partial}{\partial\theta} - \frac{\cos\theta}{\sin\theta}\sin\phi \frac{\partial}{\partial\phi}\right) f =$$

$$= \cos\phi\sin\phi \frac{\partial^2}{\partial\theta^2} + \cos^2\phi\left(-\frac{1}{\sin^2\theta}\frac{\partial}{\partial\phi} + \frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}\frac{\partial}{\partial\phi}\right)$$

$$- \frac{\cos\theta}{\sin\theta}\left(\sin^2\phi \frac{\partial}{\partial\phi}\frac{\partial}{\partial\theta} + \sin\phi\cos\phi \frac{\partial}{\partial\theta}\right)$$

$$- \left(\frac{\cos\theta}{\sin\theta}\right)^2\left(-\sin^2\phi \frac{\partial}{\partial\phi} + \sin\phi\cos\phi \frac{\partial}{\partial\phi}\frac{\partial}{\partial\phi}\right)$$

$$- \sin\phi\cos\phi \frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta} + \sin^2\phi\left(-\frac{1}{\sin^2\theta}\frac{\partial}{\partial\phi} + \frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi}\frac{\partial}{\partial\theta}\right)$$

$$- \frac{\cos\theta}{\sin\theta}\cos\phi\left(-\sin\phi \frac{\partial}{\partial\theta} + \cos\phi \frac{\partial}{\partial\phi}\frac{\partial}{\partial\theta}\right)$$

$$+ \left(\frac{\cos\theta}{\sin\theta}\right)^2\cos\phi\left(\cos\phi \frac{\partial}{\partial\phi} + \sin\phi \frac{\partial}{\partial\phi}\frac{\partial}{\partial\phi}\right) =$$

$$= \cos^2\phi\left(\frac{-1}{\sin^2\theta}\right)\frac{\partial}{\partial\phi} + \left(\frac{\cos\theta}{\sin\theta}\right)^2\sin^2\phi \frac{\partial}{\partial\phi}$$

$$+ \sin^2\phi\left(\frac{-1}{\sin^2\theta}\right)\frac{\partial}{\partial\phi} + \left(\frac{\cos\theta}{\sin\theta}\right)^2\cos^2\phi \frac{\partial}{\partial\phi} =$$

$$= \frac{-1}{\sin^2\theta}\frac{\partial}{\partial\phi} + \left(\frac{\cos\theta}{\sin\theta}\right)^2 \frac{\partial}{\partial\phi} = \frac{-1}{\sin^2\theta}\left(1 - \cos^2\theta\right) = -\frac{\partial}{\partial\phi}$$

$$= \left[iJ_1, iJ_2\right] = -J_3 = i\,\epsilon_{123}\, iJ_3$$