

**MATH431 - Modern Particle Physics**  
**Set Work: Sheet 9;**

1. (a.) Show that the substitution of the Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu$$

into the Euler Lagrange equation for  $A_\mu$  give the Maxwell equation

$$\partial_\mu F^{\mu\nu} = j^\nu$$

where  $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ . Hence show that the current  $j^\nu$  is conserved.

- (b.) With the addition of the term  $\frac{1}{2}m^2 A_\mu A^\mu$ , show that the modified Lagrangian leads to the equation of motion

$$(\partial_\mu \partial^\mu + m^2)A^\mu = j^\mu.$$

**2.**

The Lagrangian for three interacting real fields  $\phi_1, \phi_2, \phi_3$  is

$$L = \frac{1}{2}(\partial_\mu \phi_i)^2 - \frac{1}{2}\mu^2 \phi_i^2 - \frac{1}{4}\lambda(\phi_i^2)^2$$

with  $\mu^2 < 0$  and  $\lambda > 0$ , and where a summation over  $i$  is implied. Show that it describes a massive field of mass  $\sqrt{-2\mu^2}$  and two massless Goldstone bosons.

**3.**

Suppose we have two complex scalar fields  $\Phi_1$  and  $\Phi_2$  that form a doublet of  $SU(2)_I$ . Write down the Lagrangian and show that  $m_1 \neq m_2$  entails explicit breaking of the symmetry. Can we write a Higgs potential that preserve the symmetry? Describe how to break the symmetry.