from the previous lectures ...

The Dirac equation $$i\hbar\frac{1}{c}\frac{\partial}{\partial t}\Psi(\vec{x},t) = (-i\hbar\vec{\alpha}\cdot\vec{\nabla} + \beta mc)\Psi(\vec{x},t)$$

Massless Dirac particles $\Rightarrow$ for massless particles with $m = 0$ the 2 components spinors χ and ϕ are eigenstates of the helicity operator.

γ^5 is the helicity operator for massless particles

In the case of massless particles we can decompose the Dirac equation into two equations for the two helicity eigenstates.

In the chiral basis, $\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\chi = +1\chi \quad ; \quad \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\phi = -1\phi$

$$\text{denote} \qquad \chi = \psi_R \quad ; \quad \phi = \psi_L$$

$$\psi = \begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix} \qquad \psi_L,\ \psi_R \text{ are called Weyl spinors}$$

We define the operators $P_{L,R} = \left(\frac{1 \pm \gamma^5}{2}\right)$

we have $P_L\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} = \psi_L \qquad P_R\psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} = \psi_R$

The weak interactions were observed experimentally to have the vector minus axial–vector form, $(V - A)$, *i.e.*

$$(J_V^\mu - J_A^\mu)_{fi} = \bar{\psi}_f \gamma^\mu (1 - \gamma^5)\psi_i$$

In the Standard Model only left–handed fields interact via the weak interactions.

The kinetic terms containing the derivatives involve $L \leftrightarrow L$ and $R \leftrightarrow R$ terms, whereas the mass terms involve $L \leftrightarrow R$ and $R \leftrightarrow L$ terms.

This is a crucial result for modern particle physics.

Majorana fields

So far we encountered Weyl and Dirac spinors.

A Majorana spinor is a Dirac spinor in which ψ_L and ψ_R are not independent.

$$\underline{\text{Rather}} \quad \Psi_M = \begin{pmatrix} \psi_L \\ i\sigma^2\psi_L^* \end{pmatrix} \quad \to \text{same \# of D.o.f as Weyl spinor}$$

A Majorana spinor is invariant under charge conjugation

$$\Psi_M^C \;=\; \Psi_M$$

in a Majorana spinor we have $\psi_R = i\sigma_2\psi_L^*$.

Majorana spinor $\to$ neutral field $\to$ its own anti–particle $\Rightarrow$ $m\bar{\Psi}_M\Psi_M$.

Classification of elementary particles

We assembled some of the ingredients needed to classify elementary particles.

We saw : spin: - 0 scalars; $\frac{1}{2}$ fermions; $+1$ gauge bosons; $+2$ graviton;

mass: from 0 to $175 GeV$

Spin & mass are the Casimir labels of the Poincare group

charge – electric – weak – strong

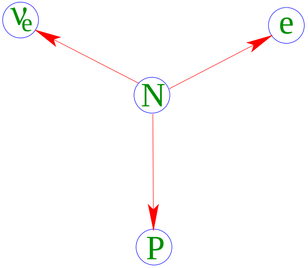
where the charges depend on the particle interactions.

	Light	heavy
Additional properties :	Leptons	Hadrons
	don't interact strongly	interact strongly

$1940's$	electron	neutrino	proton	neutron
fermions	e	ν_e	P	N
			$\tau \geq 10^{32}$years	$\tau_{\frac{1}{2}} \sim 15$min
bosons	γ			

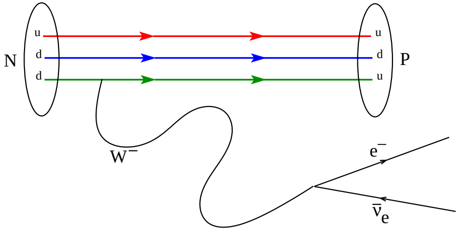
The electron neutrino was suggested By Pauli to account for observed spread of energy in nuclear β–decay. It was observed experimentally in 1956.

Lifetime depends on global & local conservation rules for different interactions.



The process of beta decay

In Fermi theory $G_F \bar{P}\gamma^\mu N \bar{e}\gamma^\nu \nu_e \quad \leftarrow \quad$ four Fermi interaction, with $G_F \sim 10^{-5}$.



In modern particle physics the interaction is mediated by a heavy vector boson, with $G_F \sim \frac{1}{M_W^2}$ and $M_W = 80.379 GeV$.

Isospin symmetry $\rightarrow$ classify elementary particles $\rightarrow$ broken symmetry

1. Proton and neutron form a doublet of Isospin
   $s = \frac{1}{2} \qquad m = 939\mathrm{MeV};\ 938\mathrm{MeV} \qquad e - \mathrm{charge} + 1,\ 0$
   if we turn off eletromagnetic interactions we cannot distinguish between the proton and the neutron

$SU(2)_I$ – Isospin – global continuous $SU(2)$ symmetry, which is exact if we ignore E&M interactions. Isospin symmetry is approximate in nature versus E&M which is exact.

In the 1950's a slew of particles (resonances) were discovered

All the particles that interacted strongly formed families of Isospin interactions

Examples:

2 pions

$m(\pi^0) \sim 139\mathrm{MeV} \quad ; \quad m(\pi^\pm) \sim 139.5\mathrm{MeV}$

spin $= 0$

Isospin $= +1 \to$ triplet

3 Kaons

$m(K^+) \sim 439.7\mathrm{MeV} \quad ; \quad m(K^0) \sim 497,8\mathrm{MeV}$

Spin $= 0$

Isospin $= \frac{1}{2} \to$ doublet

4 P^- and $\bar{N}^0$

$m(P^-) \sim 938\mathrm{MeV} \quad ; \quad m(\bar{N}^0) \sim 939\mathrm{MeV}$

Spin $= \frac{1}{2}$

Isospin $= \frac{1}{2}$

5 Sigmas

$m(\Sigma^+) \sim 1189.36\mathrm{MeV} \quad ; \quad m(\Sigma^0) \sim 1192.46\mathrm{MeV} \quad ; \quad m(\Sigma^-) \sim 1197.34\mathrm{MeV}$

Spin $= \frac{1}{2}$

Isospin $= +1$

6 Λ^0

$m(\Lambda^0) = 1115.6\mathrm{MeV}$

Spin $= \frac{1}{2}$ Isospin $= 0$

7 η

$m(\eta) = 458.8\mathrm{MeV}$

Spin $= 0$ Isospin $= 0$

Classification: states with same spin and comperable mass form Isospin families

Additionally, the resonances are classified by their lifetime and decay products.

The observed resonances decay via their strong, electromagnetic, and weak interactions. The decays are typified by the decay rates which are inversely propotional to their lifetime.

$$\begin{aligned} \text{Strong} &\sim 10^{-24} \text{ sec} \\ \text{E\&M} &\sim 10^{-18} \text{ sec} \\ \text{Weak} &\sim 10^{-8} \text{ sec} \end{aligned}$$

Hadrons – strongly interacting

baryons. spin $= n + \frac{1}{2} \quad n = 0, 1, \ldots$

mesons. spin $= n \quad n = 0, 1, \ldots$

Leptons – not strongly interacting

charged $\rightarrow \; e, \; \mu, \; , \tau$

neutral $\rightarrow \; \nu_e, \; \nu_\mu, \; \nu_\mu$

gauge bosons. spin 1 $\quad \gamma; \; W^{\pm}, \; Z; \; G$

All interactions respect the familiar conservation laws, *e.g.* $\rightarrow$ charge, energy, momentum, angular momentum.

In addition: some additional conservation laws must be imposed *e.g.* the process

$$P \rightarrow e^{+} \pi^{0}$$

respects conservation of charge, angular momentum and energy, but is not observed in nature, with $\tau_P \geq 10^{32}\mathrm{years}$.

Proton decay is forbidden in the renormalisable Standard Model but is predicted to occur in many extensions of the Standard Model. Consequently, there are many experimental searches looking for proton decay. Furthermore, some degree of proton instability is necessary to create an excess of matter over anti–matter. Without such excess, all of the protons would have annihilated with anti–protons in the early universe and there would have been non left to form galaxies, stars, planets, and us. On the other had “we know it in our bones” that the proton has to be lond lived. Otherwise, we would have decayed long ago.

$\Rightarrow$ introduce conserved baryon charge

$B(P) = +1,\ B(\bar{P}) = -1,\ B(L) = 0 = B(M)$

$$\Rightarrow \quad P \ \not\rightarrow \ e^{+} \ \pi^{0}$$