

## MATH431 - Modern Particle Physics

### Set Work: Sheet 8; Due:

1.

Suppose that we assign the proton and the neutron to the Isospin doublet in the fundamental triplet representation of  $SU(3)$ , and identify the electric charge with the combination

$$Q_{electric} = \alpha T_{3_{Isospin}} + \beta Y_{hypercharge},$$

where  $T_{3_{Isospin}}$  and  $Y_{hypercharge}$  are the diagonal generators of  $SU(3)$  with the normalization used in the class.

(a.) Determine the coefficients  $\alpha$  and  $\beta$ .

(b.) Given the coefficients found in (a.) work out the charge assignments for the sextet, octet and decuplet representations of  $SU(3)$ .

2. Consider the simple unitary group  $SU(5)$ .

(a.) How many diagonal generators of the Lie algebra are there? Write down a representation of the diagonal generators in the terms of  $5 \times 5$  hermitian matrices.

(b.) What is the dimension of the group?

(c.) What is the fundamental representation of  $SU(5)$ ? Write down its decomposition in terms of the maximal subgroup  $SU(3) \times SU(2) \times U(1)$ .

(d.) Find the product and the decomposition under the maximal subgroup  $SU(3) \times SU(2) \times U(1)$  of the fundamental times the anti-fundamental representations of  $SU(5)$ .