

$$[J_1, J_2] \neq (0, 0)$$

$$\begin{aligned}
 & \left(\cos \phi \frac{\partial}{\partial \theta} - \frac{\cos \theta}{\sin \theta} \sin \phi \frac{\partial}{\partial \phi} \right) \left(\sin \phi \frac{\partial}{\partial \theta} + \frac{\cos \theta}{\sin \theta} \cos \phi \frac{\partial}{\partial \phi} \right) f \\
 & - \left(\sin \phi \frac{\partial}{\partial \theta} + \frac{\cos \theta}{\sin \theta} \cos \phi \frac{\partial}{\partial \phi} \right) \left(\cos \phi \frac{\partial}{\partial \theta} - \frac{\cos \theta}{\sin \theta} \sin \phi \frac{\partial}{\partial \phi} \right) f = \\
 & = \cos \phi \sin \phi \frac{\partial^2}{\partial \theta^2} + \cos^2 \phi \left(-\frac{1}{\sin^2 \theta} \frac{\partial}{\partial \phi} + \frac{\cos \theta}{\sin \theta} \frac{\partial}{\partial \theta} \frac{\partial}{\partial \phi} \right) \\
 & - \frac{\cos \theta}{\sin \theta} \left(\sin^2 \phi \frac{\partial}{\partial \phi} \frac{\partial}{\partial \theta} + \sin \phi \cos \phi \frac{\partial}{\partial \theta} \right) \\
 & - \left(\frac{\cos \theta}{\sin \theta} \right)^2 \left(-\sin^2 \phi \frac{\partial}{\partial \phi} + \sin \phi \cos \phi \frac{\partial}{\partial \phi} \frac{\partial}{\partial \theta} \right) \\
 & - \sin \phi \cos \phi \frac{\partial}{\partial \theta} \frac{\partial}{\partial \theta} + \sin^2 \phi \left(-\frac{1}{\sin^2 \theta} \frac{\partial}{\partial \phi} + \frac{\cos \theta}{\sin \theta} \frac{\partial}{\partial \phi} \frac{\partial}{\partial \theta} \right) \\
 & - \frac{\cos \theta}{\sin \theta} \cos \phi \left(-\sin \phi \frac{\partial}{\partial \theta} + \cos \phi \frac{\partial}{\partial \phi} \frac{\partial}{\partial \theta} \right) \\
 & + \left(\frac{\cos \theta}{\sin \theta} \right)^2 \cos \phi \left(\cos \phi \frac{\partial}{\partial \phi} + \sin \phi \frac{\partial}{\partial \phi} \frac{\partial}{\partial \theta} \right) = \\
 & = \cos^2 \phi \left(-\frac{1}{\sin^2 \theta} \right) \frac{\partial}{\partial \phi} + \left(\frac{\cos \theta}{\sin \theta} \right)^2 \sin^2 \phi \frac{\partial}{\partial \phi} \\
 & + \sin^2 \phi \left(-\frac{1}{\sin^2 \theta} \right) \frac{\partial}{\partial \phi} + \left(\frac{\cos \theta}{\sin \theta} \right)^2 \cos^2 \phi \frac{\partial}{\partial \phi} = \\
 & = -\frac{1}{\sin^2 \theta} \frac{\partial}{\partial \phi} + \left(\frac{\cos \theta}{\sin \theta} \right)^2 \frac{\partial}{\partial \phi} = -\frac{1}{\sin^2 \theta} (1 - \cos^2 \theta) = -\frac{\partial}{\partial \phi} \\
 & = [iJ_1, iJ_2] = -J_3 = i \epsilon_{123} J_3
 \end{aligned}$$