MATH431 - Modern Particle Physics
Set Work: Sheet 9;

1. (a.) Show that the substitution of the Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu$$

into the Euler Lagrange equation for A_μ give the Maxwell equation

$$\partial_\mu F^{\mu\nu} = j^\nu$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Hence show that the current j^ν is conserved.

(b.) With the addition of the term $\frac{1}{2}m^2 A_\mu A^\mu$, show that the modified Lagrangian leads to the equation of motion

$$(\partial_\mu\partial^\mu + m^2)A^\mu = j^\nu.$$

2. The Lagrangian for three interacting real fields ϕ_1, ϕ_2, ϕ_3 is

$$L = \frac{1}{2}(\partial_\mu\phi_i)^2 - \frac{1}{2}\mu^2\phi_i^2 - \frac{1}{4}\lambda(\phi_i^2)^2$$

with $\mu^2 < 0$ and $\lambda > 0$, and where a summation over i is implied. Show that it describes a massive field of mass $\sqrt{-2\mu^2}$ and two massless Goldstone bosons.

3. Suppose we have two complex scalar fields Φ_1 and Φ_2 that form a doublet of $SU(2)_I$. Write down the Lagrangian and show that $m_1 \neq m_2$ entails explicit breaking of the symmetry. Can we write a Higgs potential that preserve the symmetry? Describe how to break the symmetry.

4. The Lagrangian density for an interacting complex scalar field

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2),$$

is

$$\mathcal{L} = (\partial_\mu\phi)^*(\partial^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

with $\mu^2 < 0$ and $\lambda > 0$.

(a) What are the transformations under which $\mathcal{L}$ is invariant?

(b) Show that it describes a massive field of mass $\sqrt{-2\mu^2}$ and one massless Goldstone boson.