from the previous lectures ...

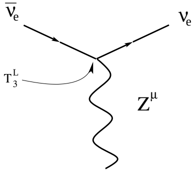
Starting from : $SU(2)_W \times U(1)_y$

$$g J^3_\mu W^{3\mu} + \frac{g'}{2} J^Y_\mu B^\mu$$

$$\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \qquad \theta_W \to \text{Weinberg angle}$$

$$\longrightarrow \quad e J^{e.m.}_\mu A^\mu + \frac{g}{\cos\theta_W} J^{NC}_\mu Z^\mu$$

impose $\quad e = g\sin\theta_W = g'\cos\theta_W \implies \tan\theta_W = \frac{g'}{g}$



Problem

$\rightarrow$ E&M interactions $\rightarrow$ Long range $\rightarrow$ $m_\gamma = 0$

$\rightarrow$ Weak interactions $\rightarrow$ Short range $\rightarrow$ $m_{W^\pm;Z} \neq 0$

How ? $\rightarrow$ symmetry breaking

Lagrangian is invariant, but vacuum $\rightarrow$ symmetry breaking

The vacuum $\rightarrow$ the states of lowest energy

The Higgs mechanism

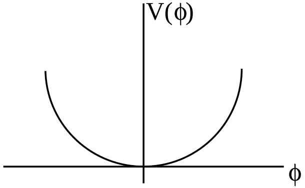
Consider a real scalar field with the Lagrangian

$$\mathcal{L} = T - V = \frac{1}{2}(\partial_\mu \phi)^2 - (\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4)$$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \qquad \text{with} \quad \lambda > 0$$

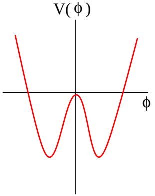
This Lagrangian is invariant under the transformation $\phi \quad \to \quad -\phi$

For $\mu^2 > 0$ the potential looks like



The Lagrangian describes a self–interacting scalar field with coupling λ and mass μ. The ground state correspond to $\langle\phi\rangle = 0$ and it obeys the reflection symmetry of the Lagrangian.

For $\mu^2 < 0$ the potential looks like



The potential has two minima at $\dfrac{\partial V}{\partial \phi} = \phi(\mu^2 + \lambda\phi^2) = 0$

$$\Rightarrow \quad \langle\phi\rangle = \pm v \quad \text{with} \quad v = \sqrt{\frac{-\mu^2}{\lambda}}$$

The extremum $\phi = 0$ does not correspond to the minimum of the energy. We perform perturbative calculation around the classical minimum $\phi = v$ or $\phi = -v$.

we write $$\phi(x) = v + \eta(x)$$

$\eta(x)$ represents quantum fluctuations about this minimum.

Substituting into the Lagrangian we obtain

$$\begin{aligned}
\mathcal{L}' &= \frac{1}{2}\partial_\mu(v+\eta)\partial^\mu(v+\eta) - \frac{1}{2}\left(\mu^2(v+\eta)^2 + \frac{\lambda}{4}(v+\eta)^4\right) \\
&= \frac{1}{2}(\partial_\mu\eta)^2 - \left(\frac{\mu^2}{2}(v^2+2\eta v+\eta^2) + \frac{\lambda}{4}(v^4+4v^3\eta+6v^2\eta^2+4v\eta^3+\eta^4)\right) \\
&= \frac{1}{2}(\partial_\mu\eta)^2 + \frac{\lambda v^2}{2}(v^2+2\eta v+\eta^2) - \frac{\lambda}{4}(v^4+4v^3\eta+6v^2\eta^2+4v\eta^3+\eta^4) \\
&= \frac{1}{2}(\partial_\mu\eta)^2 - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{\lambda\eta^4}{4} + \mathrm{const}
\end{aligned}$$

The field η has a mass term with the correct sign

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

The higher terms in η are self–interaction terms. We do perturbation theory around a stable minimum $\phi = v + \eta$.

η is a massive field

The reflection symmetry is broken by the choice of the vacuum.

Consider now a complex scalar field

$$\phi = \frac{(\phi_1 + i\phi_2)}{\sqrt{2}}.$$

$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

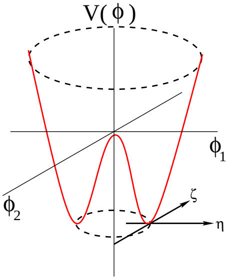
$\mathcal{L}$ is invariant under the global $U(1)$ symmetry

$$\phi \to e^{i\alpha} \phi$$

For λ and $\mu^2 < 0$ we rewrite the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left((\partial_\mu \phi_1)^2 + (\partial_\mu \phi_2)^2\right) - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

There is now a circle of minima



$$\text{at} \qquad \phi_1^2 + \phi_2^2 = v^2 \quad \text{with} \quad v^2 = -\frac{\mu^2}{\lambda}$$

We translate the field ϕ to $\langle\phi_1\rangle = v$, $\langle\phi_2\rangle = 0$.

Expand the Lagrangian around the vacuum with

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\zeta(x))$$

$$\begin{aligned}\mathcal{L}' &= \frac{1}{2}\left((\partial_\mu\eta)^2 + (\partial_\mu\zeta)^2\right) + \mu^2\eta^2 + \text{constant} \\ &\qquad +(\text{cubic \& quartic terms in } \eta \text{ \& } \zeta)\end{aligned}$$

The term $\mu^2\eta^2$ is a mass term for the η field $m_\eta = \sqrt{-2\mu^2}$ as before.

There is no corresponding mass term for ζ $\rightarrow$ massless scalar field

Goldstone theorem $\rightarrow$ spontaneously broken continuous global symmetry

$\rightarrow$ Goldstone boson

Consider now a complex scalar field coupled to a continuous $U(1)$ symmetry

$$\phi(x) \to \mathrm{e}^{i\alpha(x)}\phi(x)$$

As we saw local phase invariance requires that we replace ∂_μ by

$$D_\mu = \partial_\mu - ieA_\mu$$

and the gauge field A_μ transforms as

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha$$

The gauge invariant Lagrangian is

$$\mathcal{L} = (\partial_\mu + ieA_\mu)\phi^*(\partial^\mu - ieA^\mu)\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

For $\mu^2 > 0 \;\rightarrow\;$ Lagrangian for charged self–interacting scalar field with mass μ

For $\mu^2 < 0$ expand
$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\zeta(x))$$

$$\begin{aligned} \rightarrow \quad \mathcal{L}' \quad &= \quad \frac{1}{2}\left((\partial_\mu \eta)^2 + (\partial_\mu \zeta)^2\right) - \lambda v^2 \eta^2 + \frac{1}{2} e^2 v^2 A_\mu A^\mu \\ & \quad -evA_\mu \partial^\mu \zeta - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \text{interaction terms} \end{aligned}$$

The particle spectrum appears to be

massless Goldston boson ζ with $m_\zeta = 0$

massive scalar η with $m_\eta = \sqrt{2\lambda v^2}$

massive vector boson A^μ with $m_A = ev$

However, this interpretation should be revised massless $A^\mu \;\rightarrow\;$ 2T physical degrees of freedom

massive $A^\mu \;\rightarrow\;$ 2T + 1L physical degrees of freedom

where did the third degree of freedom come from ? Note that to lowest order

$$\phi = \frac{1}{\sqrt{2}}(v + \eta(x) + i\zeta(x)) \cong \frac{1}{\sqrt{2}}\,(v + \eta(x))\,\mathrm{e}^{i\frac{\zeta(x)}{v}}$$

$\rightarrow$ use a different set of fields $h,\ \theta,\ A_\mu$

$$\phi \rightarrow \frac{1}{\sqrt{2}}(v + h(x))\mathrm{e}^{i\frac{\theta(x)}{v}}$$

$$A_\mu \rightarrow A_\mu + \frac{1}{ev}\partial_\mu\theta$$

substitute into $\mathcal{L}$. We get

$$\begin{aligned}\mathcal{L}'' \;=\; & \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 + \frac{1}{2}e^2 v^2 A_\mu A^\mu - \lambda v h^3 - \frac{1}{4}\lambda h^4 \\ & + \frac{1}{2}e^2 A_\mu A^\mu h^2 + v e^2 A_\mu A^\mu h - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

The Goldstone boson disappeared altogether

$\rightarrow$ The Goldstone boson is absorbed as the longitudenal mode of A_μ

$\rightarrow$ only 2 physical fields h and A^μ.