/4/06.1 | MPS. 138 | ~~F~~Friday | Abingdon

we saw earlier.

	T_3	$\frac{1}{2}Y$	SU(3)	SU(2)	$U(1)_Y$
$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ ν_L	$\frac{1}{2}$	$-\frac{1}{2}$	1	2	-1
e_L	$-\frac{1}{2}$	$-\frac{1}{2}$	1	2	-1
e_R	0	-1	1	1	-1
ν_R	0	0	1	1	0
$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_L	$\frac{1}{2}$	$\frac{1}{6}$	3	2	$\frac{1}{3}$
d_L	$-\frac{1}{2}$	$\frac{1}{6}$	3	2	$\frac{1}{3}$
u_R	0	$\frac{2}{3}$	3	1	$\frac{4}{3}$
d_R	0	$-\frac{1}{3}$	3	1	$-\frac{2}{3}$.

we also saw.

$$SU(5) \supset SU(3) \times SU(2) \times U(1).$$

$$A = \left(\begin{array}{ccc|cc} x & x & x & z & z \\ x & x & x & z & z \\ x & x & x & z & z \\ \hline w & w & w & y & y \\ w & w & w & y & y \end{array}\right)$$

4 diagonal generators. $\mathrm{Tr} A = 0$

$$\text{Dimension} = 5^2 - 1 = 24.$$

$$24 = \underbrace{(8,1)_0}_{\substack{\text{generators} \\ \text{of } SU(3)}} + \underbrace{(3,2)_\alpha + (\bar{3},2)_{-\alpha}}_{\text{lepto quarks.}} + \underbrace{(1,3)_0}_{\substack{\text{generators} \\ \text{of } SU(2)}} + \underbrace{(1,1)_0}_{U(1).}$$

The U(1) generator. $\begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix}$.

$\boxed{\text{And}}$ $\left(\begin{array}{ccc|cc} 1 & & & & \\ & -1 & & & \\ & & 0 & & \\ \hline & & & & \\ & & & & \end{array}\right)$ $\left(\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & & \\ & & -2 & & \\ \hline & & & & \\ & & & & \end{array}\right)$ $\left(\begin{array}{ccc|cc} & & & & \\ & & & & \\ & & & & \\ \hline & & & 1 & \\ & & & & -1 \end{array}\right)$

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This means that the five decomposes as.

$$5 = \underbrace{\left\{ \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix} \begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix} \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}\right.}_{3} \underbrace{\left. \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix} \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix} \right\}}_{2}.$$

$$5 = (3,1)_{-2} + (1,2)_{3}.$$

Does this fit the SM Reps ?

$$\begin{matrix} d_R & + & L & \\ (3,1)_{-2/3} & + & \underbrace{(1,2)_{-1}}_{} & \text{tr } U(1) \neq 0. \ X. \end{matrix}$$

Ratio OK. ✓

try $\bar{5} = (\bar{3},1)_{2} + (1,2)_{-3}$

$$\begin{matrix} d_R & & L \\ (\bar{3},1)_{-2/3} & + & (1,2)_{-1} \end{matrix}$$

Again does not fit d_R & L have the same sign for $U(1)_Y$.

but L - Left handed.

d_R - Right handed.

$\Rightarrow$ different Reps of the Lorentz group.

particle in a unified representation must have the same quantum numbers under the Lorentz group.

/06.3| MPS.140

This means that all state in a unified rep. must have the same helicity → either left or right handed but not both.

⇒ $d_R \rightarrow d_L^c = (\bar{3},1)_{+2/3}$.

ow $d_L^c + L$

$(\bar{3},1)_{+2/3} + (1,2)_{-1} \subset \bar{5}$ of $SU(5)$.

$\boxed{U(1)\text{ charge} = *1/3}$

what about the other states

$$5\cdot 5 = \left[(3,1)_{-2} + (1,2)_{3}\right]\left[(3,1)_{-2} + (1,2)_{3}\right] =$$

$$= \left[(6,1)_{-4} + (\bar{3},1)_{-4} + (3,2)_{+1} + (3,2)_{+1} + (1,3)_{6} + (1,1)_{6}\right.$$

$$= \left[(3,2)_{+1} + (\bar{3},1)_{-4} + (1,1)_{6}\right] + \left[(6,1)_{-4} + (3,2)_{+1} + (1,3)_{6}\right]$$

$$= \quad 10_A \quad + \quad 15_S.$$

n| $SU(2)$: $2\cdot 2 = 3_S + 1_A$

$SU(3)$: $3\cdot 3 = 6_S + \bar{3}_A$

$SU(4)$: $4\cdot 4 = 10_S + 6_A$

m $SU(5)$: $5\cdot 5 = 15_S + 10_A$.

$$10 = (3,2)_{1/3} + (\bar{3},1)_{-4/3} + (1,1)_{2}$$

$\quad Q \qquad U_L^c \qquad e_L^c$

4/06.4 | MPS, 141

In SU(5) each standard Model generation fits into $10 + \bar{5}$ of SU(5)
if we add a right handed neutrino then
have $\bar{5} + 10 + 1 = 16$.
Right handed neutrinos are needed if neutrinos are massive
massive neutrinos → neutrino oscilations $\sim \Delta m_\nu^2$
explains why only 1/3 of expected neutrinos are seen from the sun
neutrino oscilations → neutrinos change flavor.

$$\bar{\nu}_e \to \bar{\nu}_\mu, \bar{\nu}_\tau$$

998 → neutrinos have mass. $\Delta m^2 \sim 10^{-3}$ eV.

if ν_R exist then we can have
Left-Right symmetry → spontaneously broken.

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_R \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$SU(2)_L \quad SU(2)_R. \quad SU(2)_L \quad SU(2)_R.$$

$$T_{3L} \quad T_{3R} \quad T_{3L} \quad T_{3R}.$$

$$Q_{em.} = T_{3L} + T_{3R} + U(1)_{PS}$$

PS = Pati-Salam.

$$Q_{em} = T_{3L} + T_{3R} + \frac{1}{2} Y_{P.S.}$$

29/4/05. 5 | MPS. 142.

$Q(u_L) = \frac{2}{3} = \frac{1}{2} + 0 + \frac{1}{2}b \Rightarrow b = \frac{1}{3}$

$Q(u_R) = \frac{2}{3} = 0 + \frac{1}{2} + \frac{1}{2}b \Rightarrow b = \frac{1}{3}$

$Q(e_L) = -1 = \frac{1}{2} + 0 + \frac{1}{2}b \Rightarrow b = -1$

$Q(e_R) = -1 = 0 + \frac{1}{2} + \frac{1}{2}b \Rightarrow b = -1$

$Q(d_L) = -\frac{1}{3} = -\frac{1}{2} + 0 + \frac{1}{2}b \Rightarrow b = \frac{1}{3}$

etc.

$$U(1)_{P.S.} = \begin{pmatrix} \frac{1}{3} & & & \\ & \frac{1}{3} & & \\ & & \frac{1}{3} & \\ & & & -1 \end{pmatrix} = B - L$$

$$Q_{em} = T_{3L} + T_{3R} + \frac{1}{2}(B-L)$$

$$SU(3)_C \times U(1)_{B-L} \times \underbrace{SU(2)_L \times SU(2)_R}_{SO(4)}$$

$$\downarrow$$

$$SU(4)_{P.S.} \times SU(2)_L \times SU(2)_R$$

$\begin{pmatrix} u_L & \nu_L \\ d_L & e_L \end{pmatrix}$	4	2	1
$\begin{pmatrix} u_R & \nu_R \\ d_R & e_R \end{pmatrix}$	$\bar{4}$	1	2.

$$SU(4) \sim SO(6)$$

$$SO(6) \times SO(4) \sim SO(10)$$

$SO(n)$. Simple orthogonal $n \times n$ matrices.

→ Rotation group in n dimensions.

2D $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\theta$

06.1 | MPS.143

$SO(2) \sim U(1)$

$SO(3) \sim SU(2)$

$SO(4) \sim SU(2) \times SU(2)$

Similar to what we did for unitary matrices.

...gonal $R^T R = I$. $\begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \eta & \rho \\ \zeta & \chi & \omega \end{pmatrix} \rightarrow SO(3)$

9 parameters $-$ 6 conditions. $= 3$

1·1 1·2 1·3

2·2 2·3

3·3

$SO(n) \quad n^2 - (1+2 \cdots + n) = n^2 - \frac{n(n+1)}{2} = \frac{n(n-1)}{2}.$

...e number of $SO(n)$ group generators $\frac{n(n-1)}{2}$.

For $SO(2n)$ Rank $= n$.

$SO(10)$ Rank $= 5$.

In $SO(10) \supset \underset{R=4}{SU(5)} \times \underset{R=1}{U(1)}$.

$$16 = \underset{\{u_L^c, Q, e_L^c\}}{10} + \underset{\{d_L^c, L\}}{\bar{5}} + \underset{\{N_L^c\}}{1}.$$

in $SO(10)$ all the Standard Model generations plus the right handed neutrino fit into a single Representation $\rightarrow$ 16 of $SO(10)$.

4/6.2 | MPS.144

Fermion masses.

Dirac mass term $\quad m\bar{\psi}\psi = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right).$

For the electron $\quad m_e\left(\underbrace{(\bar{\nu}_e,\bar{e})_L}_{(1,2)_{+1/2}}\ \underbrace{e_R}_{(1,1)_{-1}} + \bar{e}_R\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right),$

not invariant under $SU(2)_w \times U(1)_Y$

Yukawa couplings
couples fermions to the Higgs $\quad \lambda_e\left[\underbrace{(\bar{\nu}_e,\bar{e})_L}_{(1,2)_{+1/2}}\ \underbrace{\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}}_{(1,2)_{1/2}}\ \underbrace{e_R}_{(1,1)_{-1}} + \bar{e}_R\,(\phi^-,\phi^0)\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right]$

invariant under $SU(2)\times U(1)$.

$$\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \simeq \begin{pmatrix}0\\ v+h(x)\end{pmatrix}. \qquad \langle\phi\rangle = \begin{pmatrix}0\\ v\end{pmatrix}.$$

then $\quad \mathcal{L} = (\lambda_e v)\left(\bar{e}_L e_R + \bar{e}_R e_L\right)$

$$m_e = \lambda_e v.$$

λ_e: dimensionless parameter.

Extend to three generations.

$$\lambda_e = \begin{pmatrix} 3\times 3 \end{pmatrix} \qquad \lambda_u = \begin{pmatrix} 3\times 3 \end{pmatrix} \qquad \lambda_d = \begin{pmatrix} 3\times 3 \end{pmatrix}.$$

good agreement between theory and experiment,

$m_e = \lambda_e v \qquad m_u = \lambda_u v \qquad \lambda_d = \lambda_d v.$

no flavor changing neutral currents.

0/4/06.3 | MPS. 145T

in Grand unified theories.

Higgs. : $(1,2)_{+1} \rightarrow [(3,1)_{-\frac{2}{3}} + (1,2)_{+1}] = 5$

$[(\bar{3},1)_{\frac{2}{3}} + (1,\bar{2})_{-1}] = \bar{5}$

$\bar{5}_f \; 10_f \; \bar{5}_H$

$\lambda_{f_1} \; [(\bar{3},1)_{2/3} + (1,2)_{-1}] \cdot [(3,2)_{1/3} + (\bar{3},1)_{-4/3} + (1,1)_2] \; [(\bar{3},1)_{2/3} + (1,2)_{-1}]$

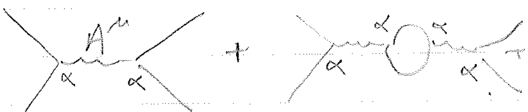
$\rightarrow \lambda_f \; (1,2)_{-1} (1,1)_2 \; (1,2)_{-1} \rightarrow (\lambda_e v_1) = m_e$

$\lambda_f \; (\bar{3},1)_{2/3} \; (3,2)_{1/3} \; (1,2)_{-1} \rightarrow (\lambda_d v_1) = m_d$

" GUT scale.

continued

Penomenological aspects



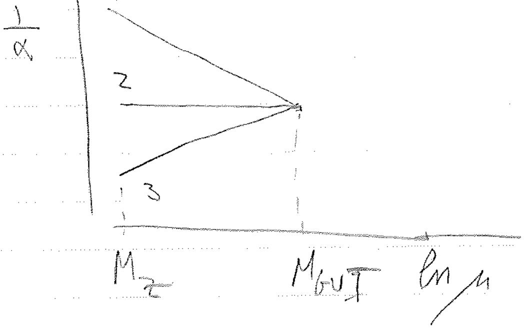
$$\boxed{\alpha = \frac{g^2}{4\pi}}$$

$U(1)$ $\quad \frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_1(GUT)} - \frac{1}{2\pi} b_1 \ln\left(\frac{M_{GUT}}{\mu}\right)$

$SU(2)$ $\quad \frac{1}{\alpha_2(\mu)} = \frac{1}{\alpha_2(GUT)} - \frac{1}{2\pi} b_2 \ln\left(\frac{M_{GUT}}{\mu}\right)$

$SU(3)$ $\quad \frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_3(GUT)} - \frac{1}{2\pi} b_3 \ln\left(\frac{M_{GUT}}{\mu}\right)$

At M_{GUT} $\quad \alpha_1 = \alpha_2 = \alpha_3$



we had that $\tan\theta_W = \frac{g'}{g}$

$$\Rightarrow \sin^2\theta_W = \frac{g'^2}{g^2 + g'^2}$$

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$10_f 10_f \bar{5}_h$.

$$\left[(3,2)_{\frac{1}{3}} + (\bar{3},1)_{-4/3} + (1,1)_2\right]\left[(3,2)_{1/3} + (\bar{3},1)_{4/3} + (1,1)_2\right]\left[(\bar{3},1)_{-2/3} + (1,2)_{+1}\right] =$$

$\supset (3,2)_{1/3}\ (\bar{3},1)_{-4/3}\ (1,2)_{+1} \longrightarrow \lambda_u v = m_u$.

$\quad Q \qquad \bar{U} \qquad h^*$

in $SU(5)$ fermion masses

$\bar{5}_f 10_f \bar{5}_H \rightarrow m_e, m_d \qquad m_e = m_d$

$10_f 10_f 5_H. \qquad m_u$

$SO(10)$ $\quad 16 = (10 + \bar{5} + 1) =$ spinor $= m$

$\quad 10 = (5 + \bar{5}) =$ vector

$16_f 16_f 10_H \Rightarrow m_t = m_b = m_\tau$

$\boxed{m_u = m_d = m_e}$

In E_6: Adjoint: $\rightarrow SO(10) \times U(1)$

$78 \qquad 45_0 + 16_x + \overline{16}_{-x} + 1$

$\therefore 27 \rightarrow 16_{1/2} + 10_{-1} + 1_2$

matter + Higgs + singlet.

14/06, 4 | MPS.146 |

In the SM the normalization of $U(1)$ generator B_μ is arbitrary.

in $SU(5)$ $\quad U(1) \subset SU(5)$,

$\Rightarrow$ normalization of $U(1)$ = normalization of $SU(3)$, $SU(2)$,

$$\Rightarrow U(1)_{\tilde{Y}} = \sqrt{\frac{3}{5}}\, U(1)_Y,$$

$$\mathcal{L} \sim g_Y B_\mu = \sqrt{\frac{3}{5}}\, g_{\tilde{Y}} \sqrt{\frac{5}{3}}\, B_{\tilde{Y}}$$

$$\Rightarrow g_Y = \sqrt{\frac{3}{5}}\, g_{\tilde{Y}}$$

$$g_Y^2 = \frac{3}{5} g_{\tilde{Y}}^2$$

$$\sin^2\theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{\frac{3}{5} g_{\tilde{Y}}^2}{g_2^2 + \frac{3}{5} g_{\tilde{Y}}^2}$$

end of lecture 33.

At M_{GUT}, $\quad g_2 = g_{\tilde{Y}}$

$$\Rightarrow \sin^2\theta_W\Big|_{M_{GUT}} = \frac{\frac{3}{5}}{1+\frac{3}{5}} = \frac{3/5}{8/5} = \frac{3}{8}$$

$$\Rightarrow \sin^2\theta_W\Big|_{M_Z} \simeq 0.23$$ $\to$ in good agreement with the data.

similarly $\quad \lambda_e = \lambda_d \big|_{M_{GUT}} \qquad m_e = m_d \big|_{M_{GUT}}$

$\Rightarrow m_b/m_e \big|_{M_b} \sim 2.7$ in good agreement with data.

KSP.14

In this way we generate masses for the charged-leptons, up-quarks, down quarks

$\lambda_e \begin{pmatrix} \nu \\ e \end{pmatrix} E h \qquad \lambda_u \begin{pmatrix} u \\ d \end{pmatrix} U \tilde{h} \quad , \quad \lambda_d \begin{pmatrix} u \\ d \end{pmatrix} D h .$

note that it is $\tilde{h}$ which gives up quark mass terms. $\tilde{h} = (i\tau^2 h^*)$.

The fermion masses are then

$$m_{e,u,d} \sim \lambda_{e,u,d} \, v$$

quark mixing. $Q_j = \begin{pmatrix} u_j \\ d_j \end{pmatrix}_L$; u_{jR} d_{jR} $j=1,2,3$

In the charged-lepton sector we can always rotate to a basis in which the mass eigenstates coincide with gauge eigenstates.

The most general quark Yukawa interactions.

$$\mathcal{L} = \lambda_u^{ij} \, \bar{U}_i (\tilde{h}^+ Q_j) + \lambda_d^{ij} \, \bar{d}_i \, (h^+ Q_j) + h.c$$

where λ_u and λ_d are non-diagonal matrices.

from the vevs of h and $\tilde{h}$ we get the mass terms for the $+2/3$ and $-1/3$ charge quarks.

$$(\bar{U}_1, \bar{U}_2, \bar{U}_3)_R \, \mathcal{M}_u \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L + h.c. \qquad (\bar{d}_1, \bar{d}_2, \bar{d}_3)_R \, \mathcal{M}_d \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L + h.c.$$

$$\mathcal{M}_u^{ij} = \frac{v}{\sqrt{2}} \lambda_u^{ij} \qquad \mathcal{M}_d^{ij} = \frac{v}{\sqrt{2}} \lambda_d^{ij}$$

X15

We can rotate to the mass eigen basis by performing a bi-unitary transformation.

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}.$$

we can transform $\mathcal{M}^u$ and $\mathcal{M}^d$ to a diagonal basis

$$U_R^{-1} \mathcal{M}^u U_L = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}.$$

$$D_R^{-1} \mathcal{M}^d D_L = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}.$$

where U_R, U_L, D_R, D_L are unitary matrices. and $m_{u,c,t}$ $m_{d,s,b}$ are the physical quark masses.

the weak eigenstates U_1, U_2, U_3 / $d_1\ d_2\ d_3$ are linear combinations of the mass eigenstates u, c, t / d, s, b.

In the charged current sector, of the weak Interactions we have,

$$\left(\overline{U_1, U_2, U_3}\right)_L \gamma^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L = (u, c, t)_L U_L^{+} D_L \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

we will therefore in general have generation mixing of the mass eigenstates.

described by the matrix.

$$V = U_L^\dagger D_L$$

In the neutral sector however we have,

$$\overline{(u_1\, u_2\, u_L)}_L \gamma^\mu \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L = \overline{(u_1 c_1 t)}_L U_L^\dagger U_L \gamma^\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$$

unitarity $\Rightarrow$ $U_L^\dagger U_L = \underline{1}$ $\Rightarrow$

GIM mechanism $\Rightarrow$ no Flavor changing neutral current

<u>Mixing matrix parametrization.</u>

For a 3X3 unitary matrix there are 9 independent parameters (9 are removed by unitarity).

we can remove additional five by redifinition of the quark fields.

We are left with 4 physical parameters.

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 c_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}.$$

where $0 \leq \theta_i \leq \pi/2$ $\quad 0 \leq \delta \leq 2\pi$

three mixing angle and one phase.

17) This is the KM mixing matrix.

The measurement of its entries constitute much of the theoretical and experimental activity in particle physics.

gauge / charges fixed by → 1 in SO(10).

The Standard Model → data.

st m_e m_u m_d
nd m_μ m_c m_s
rd m_τ m_t m_b

} 9 masses.

$\boxed{CKM}$: $\theta_1, \theta_2, \theta_3, \delta \Rightarrow 4$

Gauge couplings: $\alpha_3(M_Z), \alpha_2(M_Z), \alpha_Y(M_Z)$. 3

$\alpha_3(M_Z)$ $\sin^2\theta_W(M_Z)$ $\alpha_{e.m.}(M_Z)$.

Higgs! (λ, v) ; (G_F, m_h) 2

Strong CP $\bar{\theta} \leq 10^{-10}$ $\left(\frac{\theta}{16\pi^2} \mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$ $\frac{1}{19}$

more if we include ν_R.

This is the structure + data that we would like to derive from

String theory.

1/66.5 | MPS.147. |

I_n SO(10) $(5+\bar{5}) = 10$

λ_f $16_f \cdot 16_f \cdot 10_h \rightarrow \lambda_e = \lambda_d = \lambda_u$

3

$E_8 \quad D = 248$
$\quad\quad R = 8$

only one Rep. Adjoint $= 248$

$E_8 \rightarrow SO(16)$. $\quad (I_1\, I_2\, \ldots\, {}_{3\,4\,5\,6\,7\,8})$
$\quad\quad\quad (I\tfrac{1}{2} \ldots \ldots I\tfrac{1}{2})$

$248 \rightarrow 128_{Sp.} + 120_{Ad.}$

enomenological Aspects of Grand unified theories.

uge coupling unification.

$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(5).$$

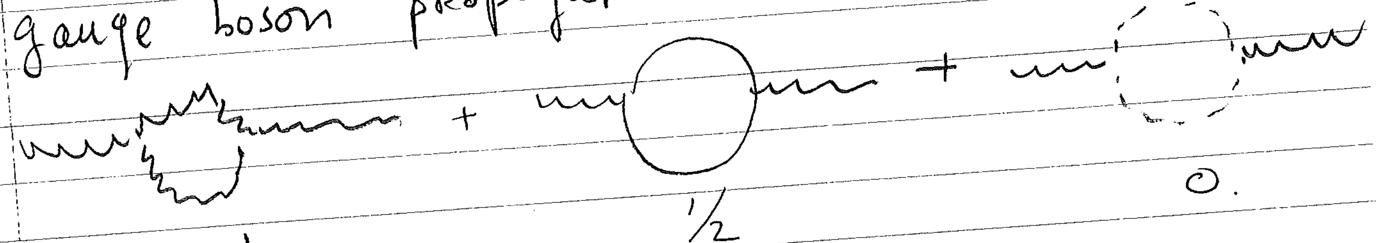
$$g_3 \quad g_2 \quad g_Y \rightarrow g_5.$$

coupling constants Run. !!!

Because of Renormalization we must define the coupling constants at a particular scale μ. However, at a different μ scale the coupling will, in general, change. This dependence of the couplings on the scale μ is governed by the Renormalization Groups equations.

$$\frac{d g_n}{d(\ln \mu)} = - b_n g_n^3$$

where b_n are the beta function coefficients and are obtained by calculating the radiative corrections to the gauge boson propagator



$S = 1$

FOR $SU(n)$.

$$b_n = \frac{1}{16\pi^2}\left[-\frac{11}{3}N + \frac{1}{3}n_f + \frac{1}{6}n_s\right]$$

n_f = number of fermion flavors.

n_s = number of scalar flavors.

Since we unified all the group generators into a single group, they should all have the same normalization. This is fine for the NA groups $SU(2), SU(3)$, since the normalization of these generators is fixed by the structure functions. However, the normalization of $U(1)$ is

).25

is arbitrary.

To fix it's normalization we must insure that the $U(1)_Y$ generator has the same normalization as the non-Abelian generators. We take the trace over a single rep. and demand equality.

$$\frac{\mathrm{Tr}\, T_3^2}{\mathrm{Tr}(cY)^2} = \frac{\left(\frac{1}{4}\right)\cdot 3}{c^2\left(6\cdot\frac{1}{36} + 3\cdot\frac{4}{9} + 1\right)} = \frac{3/2}{c^2\left(\frac{1}{6}+\frac{8}{6}+\frac{6}{6}\right)} = \frac{3/2}{c^2\left(15/6\right)}$$

$$= \frac{3/2}{c^2\, 15/6} = \frac{18}{c^2\, 30} = \frac{3}{c^2 5}$$

$$\mathrm{tr}\, Y^2 = 5/3\ \mathrm{tr}\, T_3^2$$

we must therefore define

$$U(1)_{Y'} = \sqrt{\frac{3}{5}}\ U(1)_Y$$

The Lagrangian must remain unchanged.

$$i g_Y Y \rightarrow i g_{Y'} Y'$$

So

$$g_{Y'} = g_1 = \sqrt{\frac{5}{3}}\, g_Y.$$

2.26) recall $\sin^2\theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{\frac{3}{5} g_1^2}{g_2^2 + \frac{3}{5} g_1^2}$

At the unification scale we have.

$g_1 = g_2 = g_3$.

Then, $\sin^2\theta_W = \frac{3/5}{8/5} = \frac{3}{8}$

hence unification predicts $\sin^2\theta_W = 3/8$ at M_{GUT}.

From the RGE's we get.

$$\frac{1}{g_1^2(\mu)} = \frac{1}{g_{GUT}^2} + \frac{b_1}{2\pi} \ln \frac{\mu}{M_{GUT}}$$

$$\frac{1}{g_2^2(\mu)} = \frac{1}{g_{GUT}^2} + \frac{b_2}{2\pi} \ln \frac{\mu}{M_{GUT}}$$

$$\frac{1}{g_3^2(\mu)} = \frac{1}{g_{GUT}^2} + \frac{b_3}{2\pi} \ln \frac{\mu}{M_{GUT}}.$$

we can manipulate these eq. and get.

$$\ln\left(\frac{M_X}{\mu}\right) = \frac{\pi}{11}\left[\frac{1}{\alpha(\mu)} - \frac{8}{3\alpha_s(\mu)}\right]$$

$$\sin^2\theta_W = \frac{3}{8} - \frac{55}{24\pi}\alpha(\mu) \ln\left(\frac{M_X}{\mu}\right)$$

p.27

where α, α_s are the electromagnetic and strong coupling at scale μ.

use experimental values for α, α_s.

get $\quad \sin^2\theta_W \sim 0.21$

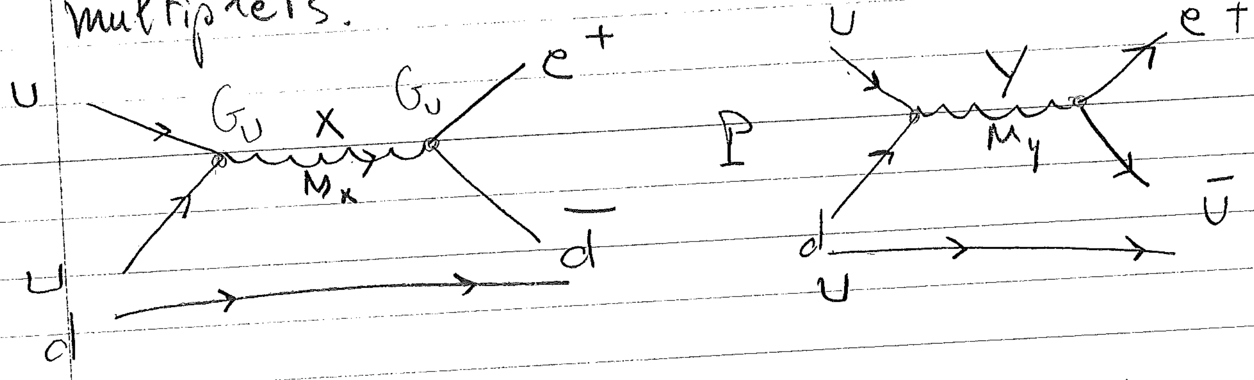
$$M_X \sim 4 \cdot 10^{14}\ \text{GeV}.$$

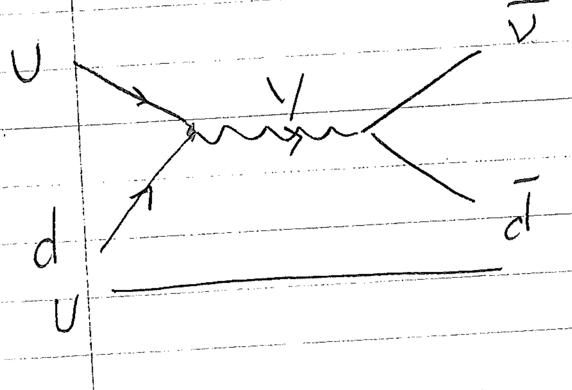
experimentally $\sin^2\theta_W(M_Z) \sim 0.231 \longrightarrow$ amazing !!!

<u>Proton decay</u>

The most important consequence of GUTs is Proton decay.

Proton decay is a Result of embedding quarks and leptons in the same multiplets.



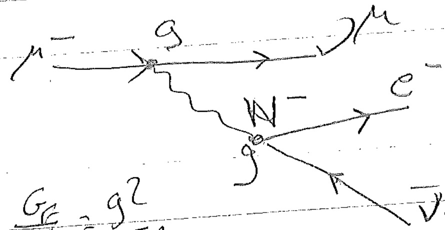


dominant mode

$$p \rightarrow e^+ \pi^0.$$

.28 What is the proton life time?

From the diagram.



$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$

$\Gamma(\mu \to e \nu \bar{\nu}) = G_F^2 m_\mu^5 = \frac{m_\mu^5}{M_W^4}$

$\sim \tau^{-1} \sim \frac{1}{M_X^4}$

dimensional grounds $\tau \propto \frac{1}{\alpha_G^2} \frac{M_X^4}{M_p^5}$

inserting $M_X \sim 10^{14}$ GeV $M_p \sim 1$ GeV.

we obtain Roughly $\tau_p \sim 10^{30}$ years

Experimentally. $\tau_p \geq 10^{32-33}$ years.

Most extensions of SM $\rightarrow$ Proton decay.

In SM. 1) Renormalizability. 2) B & L conserved global symmetries $\Big\} \Rightarrow$ no P-decay

Extensions $\rightarrow$ cut off $\rightarrow$ non-renormalizable interactions.

$\Rightarrow$ most extensions $\rightarrow$ Proton decay.

$\Rightarrow M_X \longrightarrow \Uparrow \geq 10^{15-16}$ GeV.

solutions are often ad-hoc.!!! (e.g. symmetries)

$M_P \sim 10^{19}$ GeV. $\rightarrow$ natural gravity scale.

End lecture 3