MATH191: Problem Solution Sheet 2

1. To determine the inverse function, solve the equation $y = f(x)$ for x:

$$\begin{aligned} y &= \frac{2x}{x+1} \\ y(x+1) &= 2x \\ yx - 2x &= -y \\ x(y-2) &= -y \\ &= \frac{-y}{y-2} = \frac{y}{2-y}. \end{aligned}$$

Thus $f^{-1}(y) = y/(2-y)$, or $f^{-1}(x) = x/(2-x)$.

2. a) $2x + 1$ is increasing

   b) $(x+1)^2$ is neither increasing nor decreasing on its maximal domain.

   c) $x^3 - 4$ is increasing.

3. a) $\sin((-x)^3) = \sin(-x^3) = -\sin(x^3)$, because both x^3 and sin are odd functions, so $\sin(x^3)$ is odd.

   b) $(\cos(-x))/(-x) = -(\cos(-x))/x = -(\cos(x))/x$, because cos is even, so $(\cos(x))/x$ is odd.

4. a) $\sin(-\frac{\pi}{3}) = -\sin(\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$.

   b) $\cos^{-1}(0) = \frac{\pi}{2}$.

   c) $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.

5. a) One solution is $\alpha = \tan^{-1}(-1) = -\pi/4$. The general solution is therefore

$$\theta = -\frac{\pi}{4} + n\pi \qquad n \in \mathbb{Z}.$$

   b) One solution is $\alpha = \cos^{-1}(2/5) = 1.1593$ to 4 decimal places. The general solution is therefore

$$\theta = \pm 1.1593 + 2n\pi \qquad n \in \mathbb{Z}.$$

   c) One solution is $\alpha = \sin^{-1}(1/2) = \pi/6$. The general solution is therefore

$$\theta = \begin{cases} \frac{\pi}{6} + 2n\pi & n \in \mathbb{Z} \\ (\pi - \frac{\pi}{6}) + 2n\pi & n \in \mathbb{Z}. \end{cases}$$

   (The second solution can also be written as $\frac{5\pi}{6} + 2n\pi$: either form is equally good.)

6.

$$
\begin{aligned}
\sin(3\theta) &= \sin(2\theta+\theta) \\
&= \sin(2\theta)\cos\theta + \cos(2\theta)\sin\theta \qquad [\text{formula for } \sin(A+B)] \\
&= (2\sin\theta\cos\theta)\cos\theta + (1-2\sin^2\theta)\sin\theta \qquad [\text{formulae for } \sin(2\theta) \text{ and } \cos(2\theta)] \\
&= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta \\
&= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta \qquad [\sin^2\theta + \cos^2\theta = 1] \\
&= 3\sin\theta - 4\sin^3\theta.
\end{aligned}
$$

When $\theta = \pi/2$, we have $\sin\theta = 1$, so the RHS of the formula is $3(1) - 4(1^3) = -1$, which is equal to $\sin(3\pi/2)$.

When $\theta = \pi/6$, we have $\sin\theta = 1/2$, so the RHS of the formula is $3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3 = \frac{3}{2} - \frac{4}{8} = 1$, which is equal to $\sin(3\pi/6)$ (i.e. to $\sin(\pi/2)$.)