MATH191: Problem Solution Sheet 1

1.

a) 0, 0.5 and 1 belong to $[0,1]$: they are real numbers which are ≥ 0 and ≤ 1.

b) 0.5 and 1 belong to $(0,1]$: they are real numbers which are ≥ 0 and < 1.

c) Only 0.5 belongs to $(0,1)$: it is a real number which is > 0 and < 1.

d) 0 and 0.5 belong to $(-\infty,1)$: they are real numbers which are < 1.

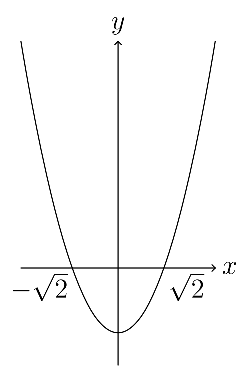
So (Y means in set, N means not in set)

	-1	0	0.5	1	π
$[0,1]$	N	Y	Y	Y	N
$(0,1]$	N	N	Y	Y	N
$(0,1)$	N	N	Y	N	N
$(-\infty,1)$	Y	Y	Y	N	N

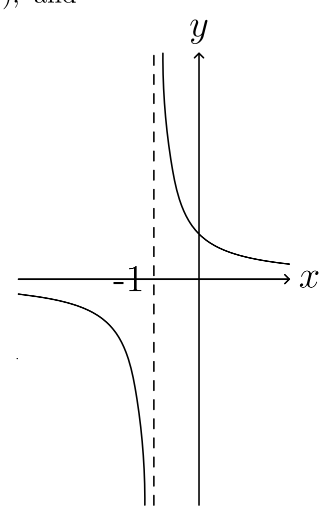
2.

a) $f(x) = x^2 - 2$ has maximal domain $\mathbb{R}$ and range $[-2,\infty)$. It has two zeros, at $x = -\sqrt{2}$ and $x = \sqrt{2}$.

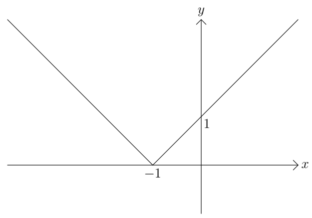
b) $f(x) = 1/(1+x)$ has maximal domain $\mathbb{R} - \{-1\}$ (or $(-\infty,-1) \cup (-1,\infty)$), and range $\mathbb{R} - \{0\}$ (or $(-\infty,0) \cup (0,\infty)$). It has no zeros.



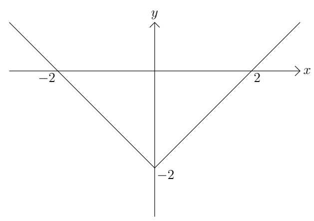
a)



c) $f(x) = |x+1|$ has maximal domain $\mathbb{R}$ and range $[0, \infty)$. It has one zero, at $x = -1$.



d) $f(x) = |x| - 2$ has maximal domain $\mathbb{R}$ and range $[-2, \infty)$. It has two zeros, at $x = 2$



and $x = -2$.

3. In this question, remember that for a rational function $f(x) = g(x)/h(x)$, if $g(x)$ and $h(x)$ have no common zeros, the maximal domain of $f(x)$ is all real numbers except the zeros of $h(x)$, and the zeros of $f(x)$ are just the zeros of $g(x)$.

a) Here $g(x) = x + 2$, which has one zero at $x = -2$, and $h(x) = x - 3$ which has one zero at $x = 3$.

So maximal domain of $f(x)$ is $\mathbb{R} - \{3\}$ (or $(-\infty, 3) \cup (3, \infty)$) and $f(x)$ has one zero, at $x = -2$.

b) Here $g(x) = (x-1)(x-2)$, $h(x) = (x-2)(x+2)$. So $f(x)$ can be written more simply as $(x-1)/(x+2)$.

The maximal domain of $f(x)$ is all real numbers other than 2, -2. (It's quite acceptable to answer like this, but if you like the maximal domain can be written as $(-\infty, -2) \cup (-2, 2)(-2, \infty)$, or as $\mathbb{R} - \{2, -2\}$.) Since $f(x)$ is not defined at $x = 2$ it has only one zero, at $x = 1$. I shall accept an answer that the maximal domain is all numbers other than -2 – in which case the function is defined at 2, but there is still only one zero, at -2

4.

a) $f(-x) = (-x)^2 + 3 = x^2 + 3 = f(x)$. Thus $f(x)$ is even.

b)

$$f(-x) = \frac{(-x)^2 + 2}{-x + 1} = \frac{x^2 + 2}{1 - x}.$$

This is not equal to either $f(x)$ or $-f(x)$ (except when $x = 0$), so $f(x)$ is neither even nor odd.

c) $f(x)$ is a polynomial with only odd powers, so it is odd. A sum of multiples of odd functions is odd. If you want to go through the calculation,

$$\begin{aligned} f(-x) &= (-x)^{13} - 19(-x)^5 + \frac{1}{18}(-x) \\ &= -x^{13} - 19(-x^5) - \frac{1}{18}x \\ &= -x^{13} + 19x^5 - \frac{1}{18}x \\ &= -f(x) \end{aligned}$$