MATH191: Problem Sheet 8

Due *Monday 26th November*

1. Let $f(x)$ be defined by

$$f(x) = \begin{cases} x^2 + 2x & \text{if } x \leq 0 \\ \frac{x}{x-2} & \text{if } x \in (0,2) \cup (2, \infty). \end{cases}$$

Differentiate $f(x)$ for $x \neq 0$, 2. Find and classify any stationary points, determine any zeros of $f(x)$, and any horizontal and vertical asymptotes. Is $f(x)$ continuous at $x = 0$? Is it differentiable at $x = 0$? Sketch the graph of $f(x)$.

2. This question is intended to be a final revision of differentiation, which is the main skill necessary for success in this course. If you find it difficult, then you need additional practice before the exam. Differentiate the following functions:

a) $e^x \cos x$; b) $\dfrac{\cos x}{e^x}$; c)$e^{\cos x}$; d) $\sin(x^2 + 2x)$; e) $(x^2 - 1)^{-3/2}$.

3. Evaluate the following integrals, giving your answers exactly if possible, otherwise to 3 decimal places:

a) $\displaystyle\int_0^3 (x^2 + x + 1)\, dx$; b) $\displaystyle\int_1^2 \left(e^{-x} + \frac{1}{x}\right) dx$; c) $\displaystyle\int_{-\pi/2}^{\pi/2} (\cos(4x) + \sin(2x))\, dx$.

4. Find the indefinite integral $\int f(x)\, dx$ of each of the following functions $f(x)$.

a) $x^4 + 2$; b) $\sin(2x - 1) + \sinh(2x - 1)$; c) $\dfrac{1}{\sqrt{x}} + \dfrac{2}{x^2}$; d) $\cos^2 x$.

(*Hint: For part d), use the trigonometric identity* $\cos(2x) = 2\cos^2(x) - 1$.)

I will collect solutions at the lecture on Monday 19th November. Any solutions which are not handed in then, or by 5pm that day in the box outside Office 120 in the Maths Building Theoretical Physics Wing will not be marked.