MATH191: Problem Sheet 7

Due Monday 19th November

1. The equation $x^2 + 4y^2 = 4$ describes an ellipse, centred on the origin, with horizontal diameter 4 and vertical diameter 2. Calculate the slopes of the tangents to this ellipse at $(x, y) = (1, \sqrt{3}/2)$ and at $(x, y) = (1, -\sqrt{3}/2)$.

2. Consider the curve defined by

$$2x^2y + xy^2 - 3x = 0.$$

Find an expression for $\frac{dy}{dx}$ in terms of x and y, and hence give the equation of the tangent to the curve at the point (x, y) = (1, 1).

3. Find and classify the stationary points of the following functions f(x). In each case, sketch the graph of the function, indicating any crossings of the axes. You may not be able to give zeros of f exactly.

a)
$$x^3 - x^2 - x - 2;$$
 b) $x^4 - 4x^3 + 4x^2 + 7;$ c) $\frac{1}{1 + x^2}$.

I will collect solutions at the lecture on Monday 19th November. Any solutions which are not handed in then, or by 5pm that day in the box outside Office 120 in the Maths Building Theoretical Physics Wing will not be marked.