MATH191: Problem Sheet 6

Due *Monday 5th November*

1. Find the Maclaurin series of:

$$\text{a) } f(x) = \cos 2x; \qquad \text{b) } f(x) = \cos^2 x$$

Think before doing b): there is a trick which helps.

2. Use L'Hôpital's rule to evaluate the following limits:

$$\text{a) } \lim_{x \to -1} \frac{x^2 + 4x + 3}{x + 1}; \qquad \text{b) } \lim_{x \to 0} \frac{e^x - e^{2x}}{x}; \qquad \text{c) } \lim_{x \to 1} \frac{\ln x}{x - 1}.$$

3. Use the definitions

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

to prove the identity

$$\cosh(x + y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y).$$

4. Sketch the graphs of

$$\text{a) } f(x) = e^{-x/2}; \qquad \text{b) } f(x) = e^{-x^2/4},$$

indicating clearly any crossings of axes, and any horizontal or vertical asymptotes.

I will collect solutions at the lecture on Monday 5th November. Any solutions which are not handed in then, or by 5pm that day in the envelope outside Office 120 in the Theoretical Physics Wing of the Maths Building will not be marked.