MATH191: Problem Sheet 5

Due *Monday 29th October*

1. Differentiate the following functions. This is a no mercy question: you will get no marks for an incorrect answer, no matter how minor the mistake. There is no need to simplify your answers – and if you simplify them incorrectly, you will get no marks.

a) $x^2 - 3x + 2 - \cos x$; b) $x^{-3/2}$; c) $(x^2+1)\cos x$; d) $\sqrt{x}\sin(3x+1)$;

e) $\dfrac{x}{x^2+1}$; f) $\dfrac{x\sin x}{x^2+x+1}$; g) $\sin(2x+1)$; h) $\dfrac{1}{\sqrt{x^2+1}}$;

i) $x\cos(3x-2)$; j) $\dfrac{\sin((2x+1)^3)}{x}$.

2. Let $f(x) = x^2 + 2x - 2$. Find the exact solutions to the equation $f(x) = 0$, involving $\sqrt{3}$. Apply the Newton-Raphson method to $f(x)$, with a starting value $x_0 = 1$, to compute $f(x_n)$ and x_n for $0 \le n \le 3$. Give each answer to 6 decimal places. Check your answer by evaluating $\sqrt{3}$ on your calculator.

3. By sketching the graphs of $y = x^3$ and $y = 2 - 2x$ on the same axes, explain why the equation

$$f(x) = x^3 + 2x - 2$$

has exactly one solution, which is in $(0, 1)$

Use the Newton-Raphson formula with an initial guess $x_0 = 1$ to compute $f(x_n)$ and x_n for $0 \le n \le 3$. Give each answer to 6 decimal places.

A suggested method for computing the x_i and $f(x_i)$ is as follows, starting with x_0 and using the university calculator keys :

1. 1 sto A

   This stores $x_0 = 1$ in A.

2. alpha A x^2 +2 alpha A -2 sto B *This displays $f(x_0)$ and stores it in B.*

3. 2 alpha A + 2 sto C *This displays $f'(x_0)$ and stores it in C.*

4. $A - B \div C$ sto D *This displays $x_1 = x_0 - (f(x_0)/f'(x_0)$ and stores it in D.*

5. sto A *This then stores x_1 in A, replacing x_0.*

Repeat steps b) to e) as many times as desired.

I will collect solutions at the lecture on Monday 29th October. Any solutions which are not handed in then, or by 5pm that day in the envelope outside Office 120 in the Theoretical Physics Wing of the Maths Building will not be marked.)